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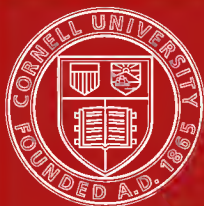
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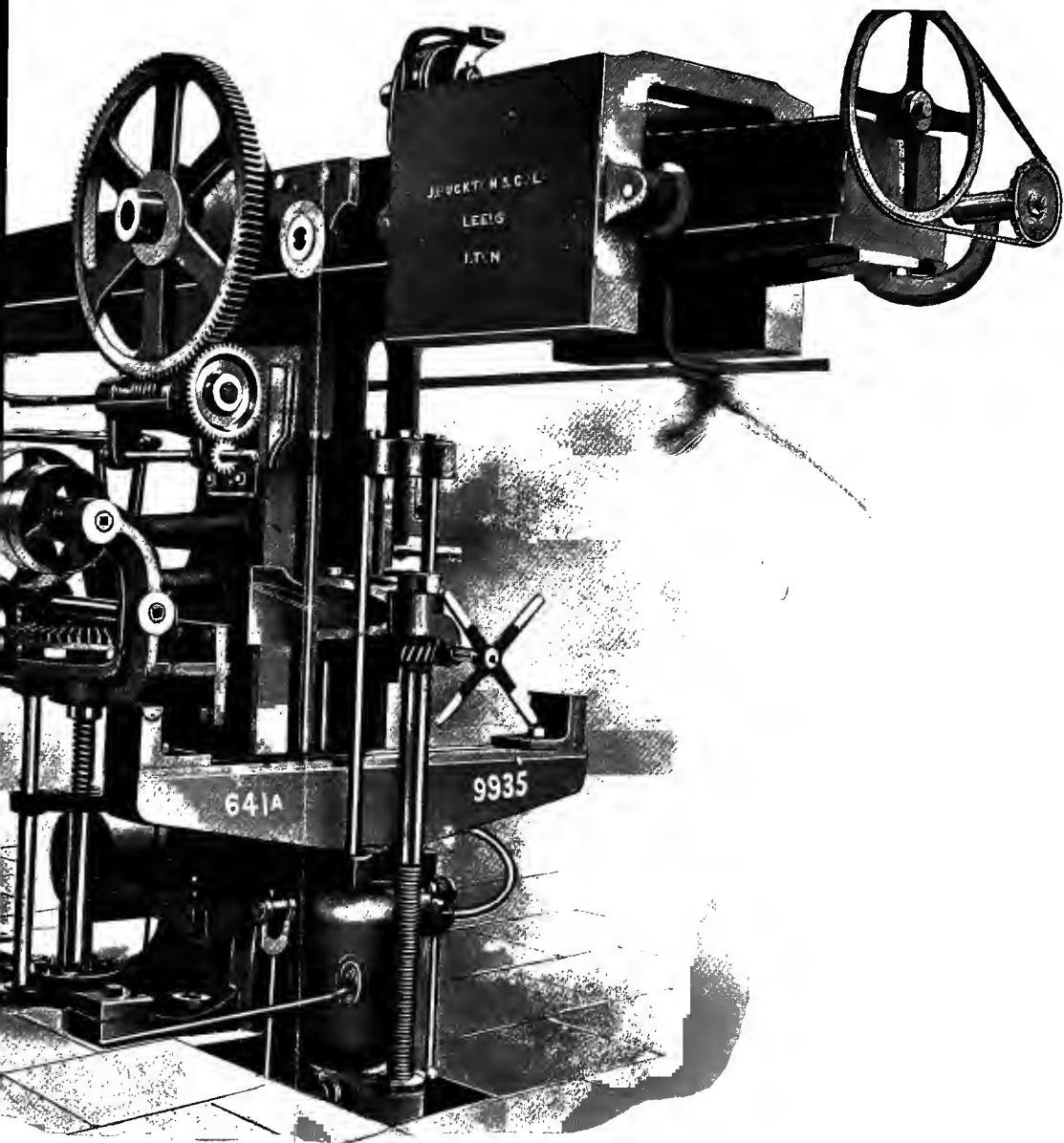
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Engineering, and for Engineers Generally.*

BY

ANDREW JAMIESON, M.INST.C.E.,

FORMERLY PROFESSOR OF ENGINEERING IN THE GLASGOW AND WEST OF SCOTLAND
TECHNICAL COLLEGE; MEMBER OF THE INSTITUTION OF ELECTRICAL ENGINEERS;
FELLOW OF THE ROYAL SOCIETY, EDINBURGH; AUTHOR OF TEXT-BOOKS
ON STEAM AND STEAM ENGINES, APPLIED MECHANICS AND
MECHANICAL ENGINEERING, MAGNETISM AND
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With Numerous Diagrams, Special Plates, and
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P R E F A C E

TO THE SEVENTH EDITION,

It has been found necessary to still further subdivide this wide and all-important subject of Advanced Applied Mechanics and Mechanical Engineering.

In order to do so with the least departure and derangement of the previous volumes and editions, it has been advisable and convenient to follow the recent subdivision of this subject as stated in the "Rules and Syllabus of Examinations applying to the Election of Associate Members of The Institution of Civil Engineers."

Moreover, this particular method of subdivision is practised by several Universities and Technical Colleges. It is also being advocated by Teachers in connection with the Boards of Education, and, to a certain extent, by those connected with the City and Guilds of London Examinations in Mechanical Engineering.

Consequently, Volume I. will deal with "Applied Mechanics" proper, Volume II. will discuss and give practical illustrations of "Strength and Elasticity of Materials," Volume III. will be confined to "The Theory of Structures," Volume IV. to "Hydraulics, Hydraulic and Refrigerating Machinery," whilst Volume V. will be greatly enlarged, and treat upon "The Theory of Machines."

Separate Contents and Index have been carefully arranged for each Volume. These enable students to find the details and pages where the different subjects are treated. The Author's system of Engineering Symbols, Abbreviations, and Index Letters have been printed at the beginning of each volume.

It is thus hoped, that the size and cost of each volume will suit the requirements of every Student of Engineering.

The Author has again to thank his chief assistant, Mr John Ramsay, A.M.Inst.C.E., for his help, and his Publishers for the special care with which they have prepared these five volumes. The Author is specially indebted to Professor Alexander MacLay, B.Sc., C.E., and the Technical Publishing Company, Ltd., Manchester, for permission to abstract certain parts and figures from his excellent book on "*Locs* in Mechanical Drawing, including Point Paths in Mechanisms." These additions appear in Vol. V. upon the "Theory of Machines," along with the new Lectures dealing with the "Kinematics of Machinery."

The Author will feel much obliged to Engineers, Teachers, his B.Sc. and C.E. "Correspondence Students," also to any other Students of Engineering, for hints which may tend to further enhance the attractiveness and usefulness of these works.

ANDREW JAMIESON.

Consulting Engineer and Electrician.
16 ROSSLYN TERRACE, KELVINSIDE,
GLASGOW, *September, 1909.*

INSTRUCTIONS FOR ANSWERING HOME EXERCISES.

(*As used in my Engineering and Electrical Science Correspondence System.*)

1. Use ordinary foolscap paper, and write on the left side *only*, leaving the facing page blank for my corrections and remarks.

2. Put the date of the Exercises at the left-hand top corner; your Name and Address in full, the name of the Subject or Section, as well as number of Lecture or Exercise, in the centre of the first page. The number of each page should be put in the right-hand top corner.

3. Leave a margin $1\frac{1}{2}$ inches wide on the left-hand side of each page, and in this margin place the *number* of the question *and nothing more*. Also, leave a clear space of *at least* 2 inches deep between your answers.

4. Be sure you understand *exactly* what the question requires you to answer, then give *all* it requires, but *no more*. If unable to fully answer any question, write down your own best attempt and state your difficulties.

5. Make your answers concise, clear, and exact, and always accompany them, if possible, by an *illustrative sketch*. Try to give (1) Side View, (2) Plan, (3) End View. Where asked, or advisable, give Sections, or Half Outside Views and Half-Sections for (1), (2), and (3).

6. Make all sketches large, open, and in the centre of the page. Do not crowd any writing about them. Simply print sizes and index letters (or names of parts), with a bold Sub-heading of what each figure or set of figures represent.

7. The character of the sketches will be carefully considered in awarding marks to the several answers. Neat sketches and "*index letters*," having the first letter of the name of the part, will always receive more marks than a bare written description.

8. All students are strongly recommended to *first* thoroughly study the Lecture, *second* to work out each answer in scroll, and *third* to compare it with the question and the text-book in order to see that each item has been answered. The final copy should be done without *any aid or reference* to the scroll copy or to *any text-book*. This acts as the best preparation for sitting at Examinations.

9. Reasonable and easily intelligible contractions (*e.g.*, mathematical, mechanical or electrical, and chemical symbols) are permitted.

10. Each corrected answer which has the symbols R.W. marked there-upon must be carefully re-worked as *one* of the set of answers for the following week.

ANDREW JAMIESON.

CONTENTS.

STRENGTH OF MATERIALS.—STRESS, STRAIN, ELASTICITY, FACTORS OF SAFETY, RESILIENCE, CYLINDERS, CHAINS, SHAFTS, BEAMS, AND GIRDERS.

LECTURE I.

PAGES

Stress and Strain, and Bodies under Tension.

Stress—Definition of Intensity of Stress—Relation between Normal and Tangential Stresses—Strain—Example I.—Coefficient or Young's Modulus of Elasticity—Limit of Elasticity—Work done in Stretching a Bar—Resilience—Example II.—Sudden Pull or Live Load—Shrunk Rings—Example III.—Strength of Thin Cylinders—Helical Seams—Strength of Thick Cylinders—Example IV.—Strength of Suspended Chains and Wires—Example V.—The Catenary Curve with Origin at a Definite Distance below its Vertex—Questions, ...	1-27
--	------

LECTURE II.

Strength of Beams and Girders.

Strength of Beams and Girders—Definitions of Shearing Force and Bending Moment—Beam Fixed at one end and Loaded at the other—Beam Fixed at one end and Loaded Uniformly—Beam Supported at both ends and Loaded in the middle—Example I.—Beam Supported at ends and Loaded anywhere—Beam Supported at both ends and Loaded Uniformly—Examples II. and III.—Floating Beams—Travelling Loads—Two Loads Moving at a Fixed Distance apart—Example IV.—Distributed Travelling Load—Questions, ...	28-53
---	-------

LECTURE III.

Deflection of Beams and Girders.

- Resistance of Beams to Flexure—Examples I., II., III., and IV.
 —Thin Wrought-Iron Girders—Example V.—Curvature
 and Deflection of Beams—Example VI.—Uniform Beams
 on Three Supports—Uniform Beam Fixed at one end and
 Supported at the other—Beams Fixed at both ends and
 Loaded at Centre—Beams Fixed at both ends and Loaded
 Uniformly—Tables—Example VII.—Questions, . . . 54-93

LECTURE IV.

Strength of Shafts.

- Torsional Strength of Shafts—Examples I., II., and III.—
 Strength of Shafts subjected to Combined Twisting and
 Bending—Theorem—Examples IV. and V.—Stiffness of
 Shafts—Angle of Twist—Example VI.—Table of Powers
 Transmitted by Shafts—Questions, . . . 94-112

LECTURE V.

*Strength and Elasticity of Materials. Testing and
Testing Machines.*

- Short History of Mechanical Testing Machines—Advantages of
 Laboratory Testing to Young Engineers—Mechanical *plus*
 Chemical and Microscopic Tests of Engineering Materials—
 Different Mechanical Tests—Necessity for a Careful Prior
 Investigation of the Kind of Stresses to which the Struc-
 ture may be Subjected—Investigations of Accidents and
 Breakages—Prof. Barr's Wire-Testing Machine—Line
 Diagrams of Three Leading Types of Testing Machines—
 Thirty-ton Single-Lever Vertical Testing Machine, with
 Autographic Recorder—Fifty-ton Single-Lever Testing
 Machine—Buckton & Co.'s 300-ton Universal Testing

Machine for Full-sized Structural Members—Equipment of this Machine for Different Kinds of Tests—A Convenient Method of Obtaining and Regulating the Pressure and Flow of Water to the Ram of a Testing Machine—Proportions of Test Bars and their Attachment in Tensile Tests—Transverse Bar-Testing Machines—Cross Bending, &c.—Admiralty Rules for Testing Materials for Machinery—French Methods of Testing Materials of Construction—Publications dealing with Engineering Tests and Materials—Questions, ...	113-138
--	---------

LECTURE VI.

Strength and Elasticity of Materials. Testing and Testing Machines.

Shearing, Planing, Boring, Punching, Drifting, and Riveting, Hammering to a Point or Edge—Fatigue of Metals—Historical Notes—Wöhler's Testing Machines, with an Example—General and Particular Deductions from Wöhler's Experiments—Gerber's Formula—Elastic Limits—Heating due to Stress—Rest and Heating Relieve Stress—Microscopic Examination of the Inter-Molecular Crystalline Action in Metals due to Fatigue—Prof. Ewing's Conclusions—Prof. Reynolds' and Dr. J. H. Smith's Method of Testing Metals by Tensile and Compressive Reversals of the Mean Stress—Determination of the Tensile, Compressive, and Range of the Reversal Stresses, with Formulæ and Results on Mild Steels—Impact Tests—Seaton and Jude's Impact Testing Machine—Advantages of the Impact Test—Cumulative Effects of Small and Medium Shocks—Effect of Allowing a Factor of Safety for both Fatigue and for Impact—Drop or Falling Tests—Torsional Tests—Torsion of Rods and Wires—Hardness, Temper, and Brittleness Tests—Methods of Determining the Hardness of a Metal—Workshop Test—Calvert and Johnson's Test for Alloys—Prof. Unwin's Test—Comparison between Unwin's Indentation Test and the Scratch Method—Questions, ...	139-159
--	---------

LECTURE VII.

Stress-Strain Diagrams and Elasticity of Materials.

Autographic Apparatus for Recording Stress-Strain Diagrams	
—Prof. Hele Shaw's Apparatus for Automatically Drawing Stress-Strain Diagrams—Prof. Kennedy's and H. G. Ashcroft's Instrument for obtaining Stress-Strain Diagrams—Automatically-Drawn Stress-Strain Diagrams, giving the Elastic, Working, and Ultimate Strengths of a Material—Elastic-Strain Diagrams—Plasticity—How to obtain a Measure of the Work done per Cubic Inch by aid of the Autographic Diagram, when subjecting a Test-Piece to a Tensile Stress—Influence of Time on Stress and Strain as shown by the Diagram—Torsion Tests—To find the Work done per Cubic Inch by aid of an Autographic Diagram when Subjecting a Test-Piece to a Torsion Test—Searle's Apparatus for Measuring the Extension of a Wire and of Determining Young's Modulus of Elasticity—Prof. Ewing's Extensometer for Measuring the Elastic Extension, and Young's Modulus of Elasticity of Specimens of Metal under Tensile Tests—Testing Machine with Extensometer attached for Measuring the Elastic Extension of Rods—Measurement of Young's Modulus in Wires—Measuring Young's Modulus by Bauschinger's Extensometer—Extensometer applied to Measure the Elastic Compression of Short Blocks—Apparatus for Determining the Deflection of Beams and Cantilevers—Measurement of Young's Modulus of Elasticity by Deflection of Beams and Cantilevers—Measurement of the Modulus of Rigidity by the Torsion of Wires—Measurement of the Modulus of Rigidity by Torsional Oscillations—Questions, . . .	160-189

LECTURE VIII.

Strength and Elasticity of Columns.

Short Columns and the Effects of Imperfect Loading—Rankine's Rules for the Strength of Columns of Medium Length—Proof of Gordon's Formula for Columns—Tables of Constants for Rankine's and Gordon's Formulæ—Example I.—Long Columns—Proof of Euler's Formula for Long Columns—Example II.—Questions, . . .	190-202
---	---------

APPENDIX A.

- | | |
|--|---------|
| 1. General Instructions by The Board of Education for their Examinations on Applied Mechanics, | 204 |
| 2. General Instructions by The City and Guilds of London Institute for their Examinations on Mechanical Engineering, | 205 |
| 3. Rules and Syllabus of Examinations by The Institution of Civil Engineers for Election of Associate Members, | 206-209 |

APPENDIX B.

- | | |
|---|---------|
| 1. Board of Education's Examination Papers in Applied Mechanics, Stages 2 and 3; City and Guilds of London Institute's Honours Examination Papers in Mechanical Engineering; The Institution of Civil Engineers' Examination Papers in Strength of Materials, arranged in the order of the Lectures in this Volume, | 210-238 |
| 2. All new Answers to Questions for their respective Lectures are tabulated under the two main headings—Board of Education and City and Guilds, or Institution of Civil Engineers, | 239-245 |

APPENDIX C.

- | | |
|--|---------|
| The Latest Exam. Papers pertaining to Strength of Materials and set by the governing bodies enumerated under Appendix A, | 246-249 |
|--|---------|

APPENDIX D.

- | | |
|---|---------|
| Tables of Constants, Logarithms, Antilogarithms, and Functions of Angles, | 250-255 |
| INDEX, | 257-264 |

CONTENTS OF VOLUMES.

- VOL. I.**—Applied Mechanics.—The Principle of Work and its Applications; Friction, Power Tests, with Efficiencies of Machines.—Velocity and Acceleration.—Motion and Energy.—Energy of Rotation and Centrifugal Force.
- VOL. II.**—Strength of Materials.—Stress, Strain, Elasticity, Factors of Safety, Resilience, Cylinders, Shafts, Beams and Girders, Testing Machines, and Testing of Materials of Construction.
- VOL. III.**—Theory of Structures and Graphic Statics, with Applications to Roofs, Cranes, Beams, Girders, and Bridges.
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MECHANICAL ENGINEERING SYMBOLS, ABBREVIATIONS, AND INDEX LETTERS

USED IN VOLUMES I. TO V.

OF PROFESSOR JAMIESON'S "APPLIED MECHANICS."

Prefatory Note.—It is very tantalising, as well as a great inconvenience to Students and Engineers, to find so many different symbol letters and terms used for denoting one and the same thing by various writers on mechanics. It is a pity, that British Civil and Mechanical Engineers have not as yet *standardised* their symbols and nomenclature as Chemists and Electrical Engineers have done. The Committee on Notation of the Chamber of Delegates to the International Electrical Congress, which met at Chicago in 1893, recommended a set of "Symbols for Physical Quantities and Abbreviations for Units," which have ever since been (almost) universally adopted throughout the world by Electricians.* This at once enables the results of certain new or corroborative investigations and formulæ, which may have been made and printed anywhere, to be clearly understood anywhere else, without having to specially interpret the precise meaning of each symbol letter.

In the following list of symbols, abbreviations and index letters, the *first* letter of the chief noun or most important word has been used to indicate the same. Where it appeared necessary, the *first* letter or letters of the adjectival substantive or qualifying words have been added, either as a following or as a subscript or suffix letter or letters. For certain specific quantities, ratios, coefficients and angles, small Greek letters have been used, and I have added to this list the complete Greek alphabet, since it may be refreshing to the memory of some to again see and read the names of these letters, which were no doubt quite familiar to them when at school.

* These "Symbols for Physical Quantities and Abbreviations for Units" will be found printed *in full* in the form of a table at the commencement of Munro and Jamieson's *Pocket-Book of Electrical Rules and Tables*. If a similar recommendation were authorised by a committee composed of delegates from the chief Engineering Institutions, it would be gladly adopted by "The Profession" in the same way that the present work of "The Engineering Standards Committee" is being accepted.

TABLE OF MECHANICAL ENGINEERING QUANTITIES, SYMBOLS, UNITS
AND THEIR ABBREVIATIONS.

(As used in Vols. I. to V. of Prof. Jamieson's "Applied Mechanics.")

Quantities.	Symbols.	Defining Equations.	Practical Units.	Abbreviations of the Practical Units.
FUNDAMENTAL.				
Length, . . .	L, l	...	{ Yard, . . .	yd.
			{ Foot, . . .	ft.
Mass, . . .	M, m	...	{ Inch, . . .	in.
			{ Pound, . . .	lb.
Time, . . .	T, t	...	{ Second, . . .	s.
			{ Minute, . . .	m.
			{ Hour, . . .	h.
GEOMETRIC.				
Surface, . . .	S, s	$S = L^2$	{ Square foot, . . .	sq. ft.
			{ Square inch, . . .	sq. in.
Volume, . . .	V	$V = L^3$	{ Cubic foot, . . .	cb. ft.
			{ Cubic inch, . . .	cb. in.
Angle, $\angle$. . .	$\left\{ \begin{array}{l} \alpha, \beta \\ \theta, \phi \end{array} \right\}$	$\alpha = \frac{\text{arc}}{\text{radius}}$	{ Degree, . . .	1°
			{ Minute, . . .	1'
			{ Second, . . .	1"
			{ Radian = $\frac{180^\circ}{\pi}$. . .	rn.
MECHANICAL.				
Velocity, . . .	v	$v = \frac{L}{T}$	Foot per second, . . .	$\frac{\text{ft.}}{\text{s.}}$
Angular velocity, . . .	ω	$\omega = \frac{v}{L} = \frac{\theta}{t}$	{ Revs. per second, . . .	r.p.s.
			{ Revs. per minute, . . .	r.p.m.
Acceleration, . . .	a, g	$a = \frac{v}{T}$	{ Radians per second, . . .	$\frac{\omega}{\text{s.}}$
			Foot per sec. per sec. . .	$\frac{\text{ft.}}{\text{s}^2}$
Force, . . .	F, f	...	{ Pound weight (gravitational unit), . . .	lb. wt. (or lb.)
	W, w	$F = Ma$	{ Poundal (absolute unit), . . .	pdl.
Pressure (per unit area), . . .	p	$p = \frac{F}{s}$	Pound per sq. inch, . . .	lb. □"
Work, . . .	(Wh)	$Wh = FL$	Foot-pound, . . .	ft.-lb.
Potential energy, . . .	E_p	$E_p = Wh$	Foot-pound, . . .	ft.-lb.
Kinetic energy, . . .	E_k	$E_k = \frac{Wv^2}{2g}$	Foot-pound, . . .	ft.-lb.
Power or activity, . . .	HP	$H.F. = \frac{Wh}{T}$	{ Horse power, . . .	H.P.
			{ Ft.-lb. per min., . . .	ft.-lb./m.
Moment of inertia, . . .	I	$I = Mk^2$	{ Ft.-lb. per sec., . . .	ft.-lb./s.
				lb.-ft. ²
Density, . . .	ρ	$\rho = \frac{M}{V}$	{ Pound per cb. ft., . . .	$\frac{\text{lb.}}{\text{ft.}^3}$
			{ Pound per cb. in., . . .	$\frac{\text{lb.}}{\text{in.}^3}$

OTHER SYMBOLS AND ABBREVIATIONS IN VOLS. I. TO V.

A for Areas.	x, y, z for Unknown quantities.
B, b ,, Breadths.	Z ,, Modulus of section.
C, c, k ,, Constants, ratios.	Z_t ,, ,, tension.
c.g. ,, Centre of gravity.	Z_c ,, ,, compression.
D, d ,, Diameters depths, deflections.	
D_1, D_2, D_3 ,, Drivers in gearing.	Δ, δ, d for Differential signs which are prefixed to another letter; then the two together represent a very small quantity.
E ,, Modulus of elasticity.	e, e ,, Represents base of Napierian Logs = 2.7182; for example, $\log_e 3 = 1.1$.
e ,, Velocity ratio in wheel gearing.	η ,, Efficiency.
F_1, F_2, F_3 ,, Followers in gearing.	λ ,, Length ratio of ship to model.
f, f_s ,, Forces of shear and tension.	μ ,, Coefficient of friction.
H, h ,, Heights, heads.	π ,, Circumference of a circle $\div$ its diameter.
H.P., h.p. ,, Horse-power.	ρ ,, Radius of curvature, radian.
B.H.P. ,, Brake horse-power.	
E.H.P. ,, Effective ,,	Σ for Symbol for sum total of a number of quantities.
L.H.P. ,, Indicated ,,	$\int_0^x$,, Sign of integration or summation between limits 0 and x .
k ,, { Radius of gyration, or, Coef. of discharge in hydraulics.	$\sim$,, Sign for the difference between two quantities.
N, n ,, Numbers—e.g., number of revs. per min., number of teeth, &c.	$\square$,, Sign for square—e.g., 10 $\square'' = 10$ square inches.
P, Q ,, Push or pull forces.	— ,, Sign over two letters, $\overline{PQ}$ for a force acting from P to $\rightarrow Q$, means that they represent a vector quantity, which has (1) magnitude, (2) direction, (3) sense.
$R_1 R_2$,, Reactions, resultants, radii, resistances.	$\supseteq$,, Sign for equal to or greater than.
s ,, { Seconds, space, surface.	$\leq$,, Sign for equal to or less than.
s ,, { Displacement, distance.	
SF ,, Shearing force.	
TM ,, Torsional moment.	
TR ,, Torsional resistance.	
BM ,, Bending moment.	
MR ,, Moment of resistance.	
RM ,, Resisting moment.	
T_d, T_s ,, Tensions on driving and slack sides of belts or ropes, &c.	
W_L, W_T, W_U ,, Lost, total, and useful work.	

GREEK ALPHABET.

A	α	Alpha.	I	ι	Iota.	P	ρ	Rho.
B	β	Beta.	K	κ	Kappa.	Σ	σ or ς	Sigma.
Γ	γ	Gamma.	Λ	λ	Lambda.	T	τ	Tau.
Δ	δ	Delta.	M	μ	Mu.	Υ	υ	Upsilon.
E	ϵ	Epsilon.	N	ν	Nu.	Φ	ϕ	Phi.
Z	ζ	Zeta.	Ξ	ξ	Xi.	X	χ	Chi.
H	η	Eta.	O	$\omicron$	Omicron.	Ψ	ψ	Psi.
Θ	θ	Theta.	Π	π	Pi.	Ω	ω	Omega.

VOLUME II.

ON

STRENGTH OF MATERIALS.—STRESS, STRAIN, ELASTICITY, FACTORS OF SAFETY, RESILIENCE, CYLINDERS, CHAINS, SHAFTS, BEAMS, AND GIRDERS.

LECTURE I.

CONTENTS.—Stress—Definition of Intensity of Stress—Relation between Normal and Tangential Stresses—Strain—Example I.—Coefficient or Young's Modulus of Elasticity—Limit of Elasticity—Work done in Stretching a Bar—Resilience—Example II.—Sudden Pull or Live Load—Shrunk Rings—Example III.—Strength of Thin Cylinders—Helical Seams—Strength of Thick Cylinders—Example IV.—Strength of Suspended Chains and Wires—Example V.—The Catenary Curve with Origin at a Definite Distance below its Vertex—Questions.

Stress.—When a piece of material is subjected to the action of external forces they tend to cause the material to change its shape or form. The particular way in which the change takes place depends upon the manner in which the load is applied. This tendency gives rise to certain forces within the material which offer resistance to the change. These internal forces are generally called *stresses*; but the term *Stress* which we have now to consider has a somewhat more definite meaning. By the principle of the equality of action and reaction, we know that so long as no rupture of the material takes place, the algebraic sum of the components of the internal forces in the direction of the load at any section of the material must be equal to the load. This principle enables us to express the *internal* in terms of the *external* forces. It is a fundamental fact that, for a given load, the amount of resistance to be contributed by each individual fibre or part composing a section will be less or greater, according as the number of such fibres or parts is greater or less; or as we usually regard it, according as there is more or less area of section. This introduces us to the conception of *distributed* force, and paves the way towards gaining definite and clear ideas regarding the strength of materials.

DEFINITION.—Intensity of stress is the resistance or reaction due to a load per unit area of section. For brevity it is usually called the *Stress*. Stresses may be of three different kinds, depending on the direction of the applied force with reference to the section on which the stress is estimated.

(1) If the applied force is normal or at right angles to the section, and acting *away* from it, the stress is called *tensile*.

(2) If acting *towards* the section, the stress is termed *compressive*

(3) If the direction of the applied force be *parallel* to the section, then the stress is named a *shearing* stress.

It is evident that if the applied force be acting in a direction inclined to the given section, it will cause both a shearing and a direct stress, the latter being tensile or compressive, according as the force is directed away from or towards the section.

When the applied force acts in such a way that we know that its effect is uniformly distributed over the section we are considering, then we estimate the stresses as follows:—

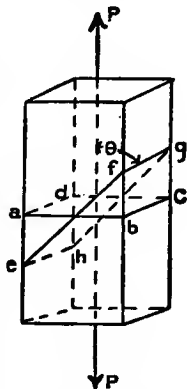
Let P_n = The applied load (or its component) acting normally to the section in lbs. or tons.

„ A = The area of the section (usually in square inches).

„ f = The direct stress, which may be either tensile or compressive.

„ P_t = The applied load (or its component) acting tangentially to the section in lbs. or tons.

„ f_s = The shearing stress.



ILLUSTRATING NORMAL AND TANGENTIAL STRESSES.

$$\left. \begin{array}{l} \text{Then,} \\ \text{And,} \end{array} \right\} \begin{array}{l} f = \frac{P_n}{A} \\ f_s = \frac{P_t}{A} \end{array} \dots \dots \dots (I)$$

Relation between Normal and Tangential Stresses.—Let $abcd$ be the section of a bar normal to the direction of the applied force P , and $efgh$ another section making an angle θ , with the direction of P ; and let the area of $abcd$ be A square inches.

Thus, the stress on $abcd$ is:—

$$f = \frac{P}{A};$$

But, on the area $efgh$, we have a normal force:—

$$P_n = P \sin \theta,$$

And a tangential force:—

$$P_t = P \cos \theta.$$

$$\text{Now, the area } efgh = \frac{\text{area } abcd}{\sin \theta} = \frac{A}{\sin \theta}$$

beyond this, the material does not do so, but retains a *permanent set*. In the following investigations the stress, in all cases, is assumed to be within the elastic limit:—

$$\text{Consequently, } \frac{\text{Stress}}{\text{Strain}} = E \text{ (a constant).} \quad \text{ (III)}$$

This E is termed Young's Modulus of Elasticity, or more appropriately by some writers the Coefficient of Elasticity.

Another way of exhibiting the relation subsisting among the various quantities we have been discussing, is to combine equations (I), (II), and (III) in such a way, as to express the stress and strain in terms of loads and dimensions.

$$\text{Thus, } E = \frac{P}{A} \div \frac{l}{L};$$

$$\text{Or, } PL = AE. \quad \text{ (IV)}$$

Ultimate Strength.—The ultimate strength of a material is the resultant stress which produces rupture. It is usually reckoned as so many lbs. or tons per square inch of the section of the material. It is always understood that the section taken is the original section of the bar, and not that at the instant and point of fracture.

Working Load.—The working stress on a member of any structure is the maximum stress to which it is subjected in actual practice.

Factors of Safety.—The ratio of the *ultimate strength* or limiting stress to the safe working load, is termed the *factor of safety* of the material. To determine the proper value of the "factor of safety," a number of considerations must be taken into account. A great deal depends on the quality of the materials and upon the manner in which different structures are stressed, such as the action and frequency of the working load.

1st. The load may be a constant *dead load*—i.e., one which is steady and produces no appreciable vibration.

2nd. The load may be a *live load*, such as a regiment of walking soldiers, or a rolling load, in the case of a moving railway train, passing over a bridge.

3rd. Where the quality of materials is variable or liable to change, the factor of safety must be larger, than for more uniform materials and for those which are less affected by exposure to atmospheric and other deteriorating influences.

4th. In structures where the whole load cannot be ascertained with accuracy, the factor of safety must be increased, to allow for the unknown stressing actions.

5th. In some structures, there is a liability to a sudden increase in the working load. Thus, in the case of a crane, where the weight may be allowed to descend rapidly and then be suddenly stopped, the maximum stress may be very much greater, than that due to the statical weight.

It is, therefore, necessary, that these special stresses be duly allowed for in the "factor of safety."

TABLE OF ULTIMATE STRENGTH AND WORKING STRESS OF MATERIALS WHEN IN TENSION, COMPRESSION, AND SHEARING.

Materials.	Ultimate Strength. Tons per sq. inch.			Working Stress. Tons per sq. inch.		
	Tension.	Compression.	Shearing.	Tension.	Compression.	Shearing.
Cast iron, ...	7.5	45	14	1.5	9	3
Wrought-iron bars, . .	25	20	20	5	3.5	4
Steel bars, ...	45	70	30	9	9	5
Copper bolts, ...	15	25	...	3	5	...
Brass sheet, ...	14	...	...	3	...	...

TABLE OF FACTORS OF SAFETY.*

Material.	FACTORS OF SAFETY FOR			
	A Dead Load.	A Live or Varying Load producing		In Structures subject to Varying Loads and Shocks.
		Stress of one kind only.	Equal Alternate Stresses of different kinds.	
Cast iron, and brittle metal and alloys, ...	4	6	10	15
Wrought iron and mild steel, . .	3	5	8	12
Cast steel, ...	3	5	8	15
Copper and other soft metals and alloys, ...	5	6	9	15
Timber, ...	7	10	15	20
Masonry and brickwork, . .	20	30	...	...

* These numbers were taken from Prof. Unwin's *Elements of Machine Design*, &c. They are all larger and safer than those given in Prof. Rankine's *Rules and Tables*.—A.J.

To find the Factor of Safety for a Mixed Load.—“Given the proportions of *live* and *dead* load on a structure, to find the factor of safety for a mixed load; multiply the factor of safety for a dead load, by a number proportional to the dead part of the load, and the factor of safety for a live load by the number proportional to the live part of the load; add together the products, and divide by the sum of the multipliers.”—(*Rankine.*)

EXAMPLE.—In an iron bridge, suppose

$$\text{Dead load : live load} :: 5 : 4.$$

Then, from the above table, we get $(3 \times 5) + (5 \times 4) = 35$; and $35 \div (5 + 4) = 4$, as the factor of safety for mixed loads.

Work done in Stretching a Bar.—Resilience.—If a load of *gradually* increasing amount be applied to a bar so as to stretch it, the amount of actual stretch, or elongation of the bar will, with the limitations already specified, be directly proportional to the load producing it. A diagram might, therefore, be drawn to represent graphically the work done in stretching the bar, as explained in Lecture II. of Volume I. The area of the diagram would represent the work done. The load will increase uniformly from 0 to P . The mean value of the force doing the work is, therefore, $\frac{1}{2} P$, and the stretch or displacement is l . Hence, we have for the work done:—

$$W = \frac{1}{2} P l.$$

But from equations (I) and (III)—

$$P = f A, \quad \text{and } l = \frac{f L}{E}.$$

$$\left. \begin{array}{l} \text{Hence,} \quad W = \frac{f^2}{E} \times \frac{A L}{2} \\ \text{Or,} \quad W = \frac{f^2}{E} \times \frac{1}{2} \text{ volume of the bar.} \end{array} \right\} \dots (V)$$

The work done is therefore proportional to the volume of the bar, or to its weight.

When the bar is loaded to its elastic limit, or *proof stress*, as it is sometimes called, then the *work done* in stretching it is termed the Resilience of the bar, and the ratio $\frac{f^2}{E}$ is its Modulus or Coefficient of Resilience.

Example II.—What is the resilience of a material? If a wrought-iron tie bar, 5 feet long and 3 inches in diameter, has a limit of elasticity of 15 tons per square inch, and a modulus of elasticity of 30,000,000 lbs. per square inch, what is its resilience?

(Take $\pi = \frac{22}{7}$.) (Adv. S. & A. Exam. 1893).

ANSWER.— $f = 15 \times 2240$ lbs., $E = 30,000,000$ lbs. per square inch, $A = \frac{1}{4} \times \frac{22}{7} \times 3^2$ square inches, and $L = 5$ feet.

$$\therefore \text{Resilience} = \frac{(15 \times 2240)^2}{30,000,000} \times \frac{\frac{11}{14} \times 3^2 \times 5}{2} = 665.28 \text{ ft.-lbs.}$$

Sudden Pull, or Live Load.—We have just seen that a *constant* force of $\frac{1}{2}P$ lbs. acting through a distance of l feet will do the same amount of work in stretching a bar as would a load gradually increasing from zero to P lbs.; therefore, the strain produced by a *sudden pull* of $\frac{1}{2}P$ lbs. is the same as that due to P lbs. applied gradually. It follows, therefore, that if P be applied *suddenly*, but without initial velocity, the strain will be doubled, and the work done will be :—

$$W = P \times 2l = 2Pl \text{ ft.-lbs.}$$

Or, in words, the work done on the bar by a suddenly applied or *live load* P , is *four* times that done by a gradually applied or *dead load* of the same amount.

Shrunk Rings.—In the construction of built-up guns, the process consists in shrinking on a series of concentric rings, each ring gripping the next inner one with a certain pre-determined tension.

The reason for this arrangement will be better understood when we come to deal with the strength of thick cylinders. The principles set forth in the preceding sections enable us to calculate the dimensions of rings to give a certain grip.

Let D = The external diameter of an inner ring.

„ d = The internal diameter of the next outer ring.

„ f = The required tension.

When the outer ring is shrunk on, its diameter is then D . The inner fibres of this ring are then stretched by an amount $\pi(D-d)$; and by definition, we have :—

$$\text{Strain} = \frac{\pi(D-d)}{\pi d} = \frac{D-d}{d}.$$

If E denote the modulus of elasticity of the material of the ring, then :—

$$E = \frac{\text{stress}}{\text{strain}} = \frac{f}{\frac{D-d}{d}}$$

Hence,
$$d = D \left(\frac{E}{E + f} \right).$$

EXAMPLE III.—The external diameter of an inner ring is 20 inches. Work out the diameter which the outer ring must have in order to grip the inner one with an initial tension of 8 tons per sq. inch. Take the modulus of elasticity as 30,000,000.

ANSWER.—Here $D = 20$ inches, and $f = 8 \times 2240 = 17,920$ lbs. per sq. inch.

$$\therefore d = 20 \times \frac{30,000,000}{30,017,920} = 19.98 \text{ inches.}$$

Strength of Thin Cylinders.—By *thin* cylinders are meant cylindrical vessels whose thickness is small compared with their diameter. The resistance which such vessels offer to forces tending to burst them, both longitudinally and circumferentially, is easily deduced as follows:—Consider a cylindrical ring, whose breadth is b inches, thickness t inches, and internal diameter is d inches. Let p denote the intensity of the internal pressure, in lbs. per sq. inch, tending to burst the ring, and f the induced stress within the material of the ring, also in lbs. per sq. inch.

Then the magnitude of the total internal force tending to tear asunder the ring at the ends of a diameter is $p d b$ lbs. And the resistance which the ring offers to this bursting force is $2 t b f$ lbs.

These being equal, we have :—

$$2 t b f = p d b \quad \therefore f = \frac{p d}{2 t}. \quad \dots \quad \text{(VI)}$$

This result shows that the stress, in a circumferential direction, is independent of the length of the cylinder.

Whatever be the form of the ends of the cylinder—whether they be flat or hemispherical—the total force tending to cause rupture circumferentially is $p \frac{\pi}{4} d^2$ lbs.; resisting this force, we have a ring of material whose total sectional area is $\pi d t$ sq. inches.

Let f_1 be the longitudinal stress due to the longitudinal bursting force; then the total resistance is $\pi d t f_1$ lbs.

And
$$\pi d t f_1 = p \frac{\pi}{4} d^2$$

Hence,
$$f_1 = \frac{p d}{4 t} \dots \dots \dots \text{(VII)}$$

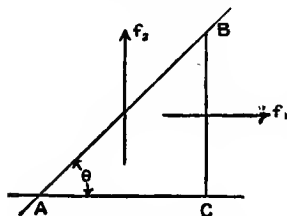
From this we see that:—

$$f_1 = \frac{1}{2} f.$$

So that in a cylindrical boiler, which comes within the category of thin cylinders, the stress in a longitudinal direction is only one-half of the stress circumferentially.

Helical Seams.—If we made a boiler of rings, joined together circumferentially, then, so long as the strength of those joints was greater than one-half that of the solid plate, the boiler would still be as strong as one without joints, because the solid plate longitudinally would still be weaker than the circumferential joints. When, instead of solid rings, these are made up of pieces joined together longitudinally, it is obvious that the strength of the boiler is determined entirely by that of its longitudinal joints, unless the circumferential joints are less than half as strong.

As a compromise, it has been proposed to have, instead of circumferential and longitudinal joints, one continuous seam running spirally, called a helical joint.



ILLUSTRATING STRESS ON
HELICAL SEAMS.

Let the accompanying figure represent a portion of such a boiler flattened out. AB is the helical seam, which, when flattened out, becomes a straight line, making the angle θ with the longitudinal direction. The longitudinal and circumferential stresses are represented by f_1 and f_2 respectively. The intensities of those stresses on AB being denoted by f_1' and f_2' , we have:—

$$f_1' \times AB = f_1 \times BC; \quad \text{and } f_2' \times AB = f_2 \times AC.$$

$$\therefore f_1' = f_1 \sin \theta; \quad \text{and } f_2' = f_2 \cos \theta.$$

Resolving f_1' and f_2' normally to AB , we have, for the total normal stress:—

$$f_n = f_1' \sin \theta + f_2' \cos \theta$$

$$,, = f_1 \sin^2 \theta + f_2 \cos^2 \theta.$$

But,

$$f_1 = \frac{1}{2} f_2$$

∴

$$f_n = \frac{1}{2} f_2 \sin^2 \theta + f_2 \cos^2 \theta.$$

Or,

$$\frac{f_n}{f_2} = 1 - \frac{1}{2} \sin^2 \theta.$$

Let,

$$n = \frac{B C}{A C};$$

Then,

$$\sin^2 \theta = \frac{B C^2}{A B^2} = \frac{B C^2}{B C^2 + A C^2} = \frac{n^2}{n^2 + 1}$$

Hence,

$$\frac{f_n}{f_2} = 1 - \frac{1}{2} \cdot \frac{n^2}{n^2 + 1} = \frac{n^2 + 2}{2n^2 + 2} \quad \dots \quad \text{(VIII)}$$

For example, if $n = 1$, i.e., $\theta = 45^\circ$,

$$\text{Then, } \frac{f_n}{f_2} = \frac{3}{4}.$$

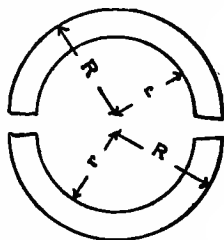
That is to say, that the normal stress on a spirally-running joint, making an angle of 45° with the axis of the boiler, would be three-fourths of that on a longitudinal joint. With joints of equal efficiency, therefore, the helical seam would be 33·3 per cent. stronger than the longitudinal one.

Strength of Thick Cylinders.—When the thickness of a cylindrical vessel, subjected to internal pressure, is *not* small in comparison with its internal diameter, the problem requires to be treated differently.

A complete determination of the strength of thick cylinders of all proportions is not an easy matter; and as for an *accurate* solution of the problem, the thing is simply impossible.

For moderate proportions of cylinders, such as are used in hydraulic appliances, the following demonstration yields results fairly substantiated by practice.

If such a cylinder were to give way under internal pressure, the plane of rupture would evidently contain the axis of the cylinder; whilst the rupture itself would appear as shown in the accompanying figure. From this figure it is clear that the circumferential *stretch* is the same from the inner to the outer surface. Now, remembering the definition of strain previously given, it is obvious that in this case, the strain in any cylindrical



ILLUSTRATING STRAIN IN THICK CYLINDERS.

layer within the material will be inversely as its radius; and since, within the elastic limit, the stress is proportional to the strain, it follows that the stress on any layer is also inversely as the radius of that layer:—

Let f = Stress at inner surface at distance r , from axis of cylinder.

„ x = Radius of any layer within cylinder thickness.

„ dx = The thickness of elementary layer.

Then the stress on the material at radius x , will be $f \frac{r}{x}$, and the total resistance per unit length of the elementary hoop is $2 f r \cdot \frac{dx}{x}$.

The total resistance of the cylinder is, therefore:—

$$\begin{aligned} &= 2 f r \int_r^R \frac{dx}{x} \\ &= 2 f r \log_e \frac{R}{r}. \end{aligned}$$

But the total bursting force within the cylinder, per unit of length, is $2 p r$, where p is the difference of the internal and external pressures.

Hence, equating these expressions for equal and opposite forces, we have:—

$$\begin{aligned} 2 p r &= 2 f r \log_e \frac{R}{r}. \\ \frac{p}{f} &= \log_e \frac{R}{r}. \quad \dots \dots \dots (IX) \end{aligned}$$

The logarithms here required are hyperbolic.

Equation (IX) is not in a convenient form for application, because the ratio $\frac{R}{r}$ involves the quantity which is required; but a very simple and useful formula, quite accurate enough for most practical purposes, may be obtained as follows:—

For values of $\frac{R}{r}$ less than 2, $\log_e \frac{R}{r} = \frac{2(R - r)}{R + r}$, very approximately.

Making this substitution in (IX), we have:—

$$\begin{aligned} \frac{p}{f} &= \frac{2(R - r)}{R + r}, \\ \frac{R}{r} &= \frac{2f + p}{2f - p}. \quad \dots \dots \dots (IXa) \end{aligned}$$

Whence,

In practical calculations, the quantity usually required is the thickness of the cylinder; calling this t , we get:—

$$t = R - r$$

$$t = \left(\frac{2f + p}{2f - p} \right) r - r$$

$$t = \frac{2pr}{2f - p}$$

$$\text{Or,} \quad t = \frac{pd}{2f - p} \dots \dots \dots \text{(X)}$$

Where d = internal diameter of cylinder.

EXAMPLE IV.—The internal diameter of an hydraulic cylinder is 8 inches, and the ultimate tensile strength of the material of which it is made is 16,000 lbs. per sq. inch. What thickness of metal would be required in the sides of such a cylinder if the metal be not stressed beyond one-sixth of its ultimate strength, the water being under a pressure of 2000 lbs. per sq. inch? Prove the formula which you employ. (Hons. S. & A. Exam., 1889.)

ANSWER.—Here $f = \frac{1}{6} \times 16,000$; $p = 2000$; and $d = 8$ ". Substituting these values in equation (X), we have:—

$$t = \frac{2000 \times 8}{2 \times \frac{1}{6} \times 16,000 - 2000} = 4.8 \text{ inches.}$$

If formula (IX) be used, then:—

$$\log_e \frac{R}{r} = \frac{p}{f} = \frac{2000}{\frac{1}{6} \times 16,000} = 0.75.$$

$$\text{Or,} \quad \log_{10} \frac{R}{r} = 0.75 \times .4343 = .326.$$

$$\text{Therefore,} \quad \frac{R}{r} = 2.12.$$

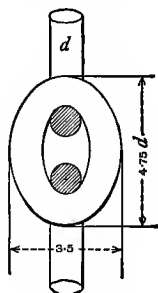
$$\text{And,} \quad t = R - r = 4.48 \text{ inches.}$$

Chains.*—Chains are used both as transmitters of energy, like belts or ropes, and as simple fastenings. But, as these two modes of using chains are more or less connected, it is found convenient to treat them generally, without taking into account their special

* The following two figures, the Admiralty Rules for proof stress of chains, and the two Tables were obtained by permission from *Design of Structures*, by S. Anglin, C.E., published by Charles Griffin & Co.

applications. Chains may be arranged under any one of the following headings:—1st, Round iron chains with open links; 2nd, round iron chains with stud links; 3rd, flat bar chains; 4th, gearing chains.

Chains are generally made from round wrought-iron bars of the best quality. The links are of different shapes, the elliptical form being the most common. They may be classed under two sorts—(a) The close or open link, which is commonly used for cranes and like purposes; (b) the stayed or stud-link, where a



OPEN-LINK.



STUD-LINK.

stud or stay (usually made of cast iron) is fitted across the shorter diameter of each link. These studs most effectually keep the sides apart, and prevent any link jamming a neighbouring one. They add materially to the strength of the chain, for they are in compression whilst the sides of the links are in tension. Stud-links are used for large cables, and for other purposes where the chain may be subjected to heavy stresses.

When a load or tension is applied to any chain, each link is subjected to a bending action in addition to a tensile stress. This bending action is greatest at the extremities of the longer diameter of the link. Hence, on this reasoning, the link should be made stronger at its ends, but the question of the best section is complicated by the uncertainty as to the strength of the link at the *weld*. The diminution of strength due to "welds" in bars has been found to average 20 per cent. The strength of a stud-link may be taken as equal to double the strength of a rod of wrought iron, of the same diameter and quality of material as that of which the chain is composed; whereas, the strength of an open-link chain is only about 70 per cent. of this amount, even with perfect welding. It will easily be seen, that chains in passing over pulleys are subjected to other bending stresses than those enumerated above. Hence, the links of

chains to be used for this purpose should be made as *short* as possible, in order to increase the flexibility of the chain, and diminish this bending action.

Fatigue of Chains.*—Chains which have been in use for some time and subjected to many sudden jerks (such as the lifting chains for cranes and slings), suffer from "fatigue," the material becomes crystalline, or short in the grain, and consequently brittle and unsafe. The best precaution to adopt to restore the equilibrium of the material and re-establish its strength, is to periodically anneal the chains by drawing them very slowly through a fire, thus allowing them to become heated to a dull red, and then to cool them slowly in a heap of ashes. This method is carried out at Woolwich Arsenal by the War Department and by other high-class works.

Ultimate Stress, Proof Stress, and Working Stress on Chains.—There are three kinds of stresses to be distinguished when applied to chains—(1) The ultimate tensile stress, (2) the proof stress, and (3) the working stress. These are approximately related to each other as follows:—

The proof stress may be taken as one-half of the ultimate strength, and the working stress is often stated at about one-half the proof stress.

The Admiralty rules for the proof stress of studded-chain cables and close or open-link crane chains are as follows:—

Let d = diameter of the iron forming the chain;

For studded chain cables—

Proof load in tons = $18 d^2 = 11\frac{1}{2}$ tons per sq. in. of section.

For close-link crane chains—

Proof load in tons = $12 d^2 = 7.7$ tons per sq. in. of section.

If the working load be taken at one-half the proof load, we get—

For studded chains—

Greatest working load = $9 d^2 = 5.75$ tons per sq. in. of section.

For close-link chains—

Greatest working load = $6 d^2 = 3.85$ tons per sq. in. of section.

* The author was recently called upon to make a series of comparative tests between a $\frac{1}{2}$ -inch sling chain, which had been in constant use during several years for lifting logs of wood without ever having been annealed, and a new well-annealed wrought-iron sling chain of the same size. The old chain broke with sudden snaps at 6.22 tons mean stress, and only 3.5 per cent. elongation, showing a brittle crystalline structure. The new well-annealed $\frac{1}{2}$ -inch chain only gave way at 7.8 tons with 18.5 per cent. elongation, showing a fine fibrous ductile structure.

Weight of Chains.—The weight of chains in lbs. per foot may be expressed from the equation, $w = 9 d^2$, where d = diameter of iron in inches.

The following tables are given by Professor Unwin, M.Inst.C.E., in his *Elements of Machine Design*; also, in *Design of Structures*, by Mr. S. Anglin, C.E., Wh.Sc. The breaking strengths were calculated from the Woolwich experiments.

TABLE OF STRENGTH AND WEIGHT OF CLOSE-LINK CRANE CHAINS, AND SIZE OF EQUIVALENT HEMP CABLE.

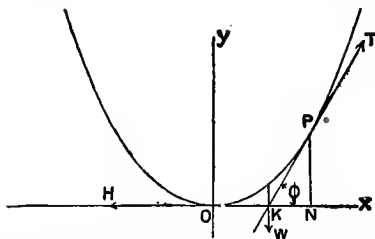
Diameter of iron d in inches.	Weight of chain per fathom.	Breaking strength in tons.	Testing load in tons.	Girth of equivalent rope in inches.	Weight of rope in lbs. per fathom.
$\frac{1}{4}$	3.5	1.9	0.75	2	$1\frac{1}{2}$
$\frac{5}{16}$	6.0	3.0	1.10	$2\frac{1}{2}$	$1\frac{1}{2}$
$\frac{3}{8}$	8.5	4.3	1.6	$3\frac{1}{2}$	$2\frac{1}{2}$
$\frac{7}{16}$	11.0	5.9	2.3	4	$3\frac{1}{2}$
$\frac{1}{2}$	14.0	7.7	3.0	$4\frac{3}{4}$	6
$\frac{9}{16}$	18.0	9.7	3.8	$5\frac{1}{2}$	7
$\frac{5}{8}$	24.0	12.0	4.6	$6\frac{1}{2}$	$8\frac{1}{2}$
$\frac{11}{16}$	28.0	14.6	5.6	7	$10\frac{1}{2}$
$\frac{3}{4}$	31.5	17.3	6.8	$7\frac{1}{2}$	12
$\frac{13}{16}$	37.0	20.4	7.9	$8\frac{1}{2}$	15
$\frac{7}{8}$	44.0	23.1	9.1	9	$17\frac{1}{2}$
$\frac{15}{16}$	50.0	26.1	10.5	$9\frac{1}{2}$	$19\frac{1}{2}$
1	56.0	29.3	12.0	10	22
$1\frac{1}{8}$	71.0	36.3	15.3	$11\frac{1}{2}$	$27\frac{1}{2}$
$1\frac{1}{4}$	87.5	44.1	18.8	$12\frac{1}{2}$	$34\frac{1}{2}$
$1\frac{3}{8}$	105.8	52.8	22.6	$13\frac{3}{4}$	$41\frac{1}{2}$
$1\frac{1}{2}$	126.0	62.3	27.0	15	$49\frac{1}{2}$

TABLE OF STRENGTH AND WEIGHT OF STUDDED-LINK CABLE.

Diameter of Iron d in inches.	Weight in Lbs. per Fathom.	Breaking Strength in Tons.	Tensile Load in Tons.	Girth of Equivalent Rope in inches.	Weight of Rope in Lbs. per Fathom.
$\frac{5}{8}$	24	9.5	7	$6\frac{1}{2}$	9
$\frac{11}{8}$	28	11.4	$8\frac{1}{2}$	$7\frac{1}{2}$	12
$\frac{3}{4}$	32	13.5	$10\frac{1}{2}$	8	14
$\frac{7}{8}$	44	20.4	$13\frac{1}{2}$	$9\frac{1}{2}$	$19\frac{1}{2}$
1	58	24.3	18	$10\frac{1}{2}$	$22\frac{1}{2}$
$1\frac{1}{8}$	72	29.5	$22\frac{3}{4}$	12	$30\frac{3}{4}$
$1\frac{1}{4}$	90	38.5	$28\frac{1}{2}$	$13\frac{1}{2}$	$39\frac{1}{2}$
$1\frac{3}{8}$	110	48.5	34	15	$48\frac{1}{2}$
$1\frac{1}{2}$	125	59.5	$40\frac{1}{2}$	16	55
$1\frac{5}{8}$	145	66.5	$47\frac{1}{2}$	17	62
$1\frac{3}{4}$	170	74.1	$55\frac{1}{8}$	18	$68\frac{1}{2}$
$1\frac{7}{8}$	195	92.9	$63\frac{1}{4}$	20	86
2	230	99.5	72	22	104
$2\frac{1}{8}$	256	112	$81\frac{1}{4}$	24	124
$2\frac{1}{4}$	285	126	$91\frac{1}{8}$	26	145

Strength of Suspended Chains and Wires.—When a uniformly heavy chain or wire is suspended between two points, to find the equation to the curve in which it hangs, and the tension at a point:—1st. Let T be the tension at any point P ; and H , that at the lowest point O . If W be the weight of the part OP of the chain, it is evident that OP will be in equilibrium under the action of three forces—the tensions at O and P and its own weight W , acting through its centre of gravity. These three forces, therefore, must pass through some point K , in Ox , such that KP will be a tangent to the curve at the point P . Let the curve be referred to the co-ordinate axes, Ox , Oy , and let

ON = x, and NP = y, OP = s. Also let w be the weight of



ILLUSTRATING THE STRESSES IN SUSPENDED CHAINS AND WIRES.*

a unit length of chain, and for H write $m w$. Resolving vertically and horizontally, we get:—

$$T \sin \phi = W = s w. \quad T \cos \phi = H = m w.$$

$$\text{Hence, } \tan \phi = \frac{s}{m}; \text{ or, } \frac{dy}{dx} = \frac{s}{m}. \quad \dots \dots \dots (1)$$

$$\text{But, } \frac{dy}{ds} = \sin \phi = \frac{1}{\operatorname{cosec} \phi} = \frac{1}{\sqrt{\cot^2 \phi + 1}} = \frac{s}{\sqrt{m^2 + s^2}}$$

$$\therefore dy = \frac{s \cdot ds}{\sqrt{m^2 + s^2}}.$$

Integrating this expression, we have:—

$$y = \sqrt{m^2 + s^2} + C.$$

2nd. To find the value of the constant C, we know that $s = 0$, when $y = 0$.

$$\therefore 0 = m + C,$$

$$\text{Or, } C = -m.$$

$$\text{So that, } y + m = \sqrt{m^2 + s^2}.$$

$$\text{and, therefore, } s = \sqrt{(y + m)^2 - m^2}.$$

Substituting this value of s in (1), and inverting:—

$$\frac{dx}{dy} = \frac{m}{\sqrt{(y + m)^2 - m^2}}.$$

Multiplying each side by dy , and integrating, we get:—

$$x = m \cdot \log_e \{ (y + m) + \sqrt{(y + m)^2 - m^2} \} + C.$$

$$\text{When } x = 0, y = 0, \text{ then:—} C = -m \cdot \log_e m.$$

$$\text{Hence, } x = m \cdot \log_e \left\{ \frac{y + m + \sqrt{(y + m)^2 - m^2}}{m} \right\} \quad \dots \quad (2)$$

* For a neater solution of this problem on the *Common Catenary* with the origin below the vertex of the curve, see three pages further on.

Equation (2) is sometimes useful in the solution of problems. In order to get the equation to the curve in which the chain hangs, or, in other words, the relation between x and y , we write (2) as follows:—

$$\frac{x}{m} = \log_e \left\{ \left(\frac{y+m}{m} \right) + \sqrt{\left(\frac{y+m}{m} \right)^2 - 1} \right\}$$

That is,
$$e^{\frac{x}{m}} = \left(\frac{y+m}{m} \right) + \sqrt{\left(\frac{y+m}{m} \right)^2 - 1}; \quad \dots \dots (3)$$

Where e is the base of the Napierian or hyperbolic logarithms.

Now, since:—

$$\left\{ \left(\frac{y+m}{m} \right) + \sqrt{\left(\frac{y+m}{m} \right)^2 - 1} \right\} \cdot \left\{ \left(\frac{y+m}{m} \right) - \sqrt{\left(\frac{y+m}{m} \right)^2 - 1} \right\} = 1.$$

Therefore—

$$\left\{ \left(\frac{y+m}{m} \right) - \sqrt{\left(\frac{y+m}{m} \right)^2 - 1} \right\} = \frac{1}{\left\{ \left(\frac{y+m}{m} \right) + \sqrt{\left(\frac{y+m}{m} \right)^2 - 1} \right\}} = \frac{1}{e^{\frac{x}{m}}}$$

Hence,
$$e^{-\frac{x}{m}} = \left(\frac{y+m}{m} \right) - \sqrt{\left(\frac{y+m}{m} \right)^2 - 1}. \quad \dots \dots (4)$$

Now, adding (3) and (4) and reducing, we have finally:—

$$\left. \begin{aligned} y &= \frac{m}{2} \left\{ e^{\frac{x}{m}} + e^{-\frac{x}{m}} \right\} - m \\ \text{Or, } y &= \frac{m}{2} \left\{ e^{\frac{x}{m}} + e^{-\frac{x}{m}} - 2 \right\} \end{aligned} \right\} \dots \dots \dots (XI)$$

The curve whose equation is (XI) is called a *catenary*.*

3rd. To find T , the tension at any point, we have:—

$$T = \frac{s w}{\sin \phi} = s w \div \frac{s}{\sqrt{s^2 + m^2}}$$

Or,
$$T = w \cdot \sqrt{s^2 + m^2}.$$

But,
$$\sqrt{s^2 + m^2} = y + m.$$

$\therefore T = w(y + m). \quad \dots \dots \dots (XII)$

When the curve is very flat, as in the case of telegraph wires, then $s = x$ approximately, and (1) becomes $\frac{dy}{dx} = \frac{x}{m}$.

* See footnote on previous page.

Multiplying by dx and integrating, we get:—

$$y = \frac{x^2}{2m} \cdot \cdot \cdot \cdot \cdot \cdot \quad (\text{XIII})$$

This requires no correction, because x and y vanish together.

EXAMPLE V.—A telegraph wire, which weighs $\frac{1}{10}$ of a lb. per yard, is stretched between poles on level ground, so that the greatest dip of the wire is 3 feet. Find approximately the distance between the poles when the tension at the lowest point of the wire is 140 lbs. (Hons. S. and A. Exam., 1891.)

ANSWER.—Here, $H = 140$, $y = 3$ ft., and $w = \frac{1}{30}$; and since:—

$$H = mw$$

$$\therefore 140 = m \times \frac{1}{30}.$$

$$\text{Or,} \quad m = 4200.$$

Putting these values of y and m in equation (XIII), it becomes:—

$$3 = \frac{x^2}{2 \times 4200}.$$

$$\therefore x = \sqrt{3 \times 2 \times 4200} = 158.7 \text{ ft.}$$

Distance between poles:—

$$= 2x = 2 \times 158.7 = 317.4 \text{ ft.}$$

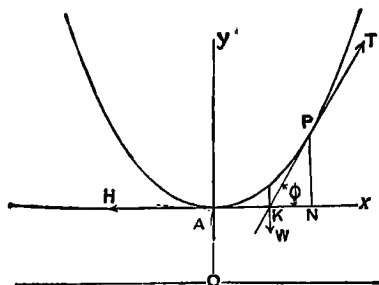
If the more exact equation (2) be used, then:—

$$2x = 2 \times 4200 \times \log_e \left\{ \frac{4203 + \sqrt{4203^2 - 4200^2}}{4200} \right\}$$

$$.. = 8400 \times \log_e \frac{218}{210}$$

$$.. = 8400 \times 0.0374 = 314.16 \text{ ft.}$$

The Catenary Curve with Origin at a Definite Distance below its Vertex — (*By W. M. Madden, M.I.E.E.*).— Referring to pp. 16 to 19, where a uniformly heavy chain or wire is suspended between two points, let T be the tension at any point P , and H that at the lowest point A . If W be the weight of the part AP of the chain, it is evident that AP will be in equilibrium under the action of three forces—viz., the tensions at A and P , and its own weight W , acting through its centre of gravity.



THE CATENARY CURVE WITH ORIGIN AT A DISTANCE m BELOW ITS VERTEX, AS APPLIED TO SUSPENDED CHAINS AND WIRES.

Also, let w be the weight of a unit length of chain; s , the length of span AP ; and mw , the tension H .

Then, we get directly from the triangle of forces:—

$$T \sin \phi = W = sw; \quad T \cos \phi = H = mw;$$

whence, $s = m \tan \phi \quad \therefore ds = m \sec^2 \phi d\phi$

$$dx = \cos \phi ds = m \sec \phi d\phi$$

$$dy = \sin \phi ds = m \frac{\sin \phi d\phi}{\cos^2 \phi}$$

Omitting constant of integration, so that $y = m$ when $\phi = 0$

$$y = m \sec \phi$$

This determines the origin O at a distance m below A ; and, since $H = mw$, the tension at A is equal to the weight of a length m of the chain.

Integrating the above equation for x —

$$\text{We get,} \quad x = m \int \frac{d\phi}{\cos \phi} = m \int \frac{\cos \phi d\phi}{\cos^2 \phi} = m \int \frac{d \sin \phi}{1 - \sin^2 \phi}$$

$$\text{Or,} \quad x = \frac{m}{2} \left\{ \int \frac{d \sin \phi}{1 + \sin \phi} + \int \frac{d \sin \phi}{1 - \sin \phi} \right\}$$

$$\therefore \quad x = \frac{m}{2} \{ \log_e (1 + \sin \phi) - \log_e (1 - \sin \phi) \}$$

$$\therefore \quad x = \frac{m}{2} \log_e \frac{1 + \sin \phi}{1 - \sin \phi}$$

Here, no constant is required ; since $x = 0$ when $\phi = 0$

$$\text{Consequently, } e^{\frac{2x}{m}} = \frac{1 + \sin \phi}{1 - \sin \phi}, \text{ and } e^{-\frac{2x}{m}} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\therefore e^{\frac{2x}{m}} + e^{-\frac{2x}{m}} + 2 = \frac{2\{(1 + \sin^2 \phi) + (1 - \sin^2 \phi)\}}{1 - \sin^2 \phi} = \frac{4}{\cos^2 \phi} = 4 \sec^2 \phi$$

$$\text{Or,} \quad e^{\frac{x}{m}} + e^{-\frac{x}{m}} = 2 \sec \phi = \frac{2y}{m}$$

$$\text{Hence,} \quad y = \frac{m}{2} \left\{ e^{\frac{x}{m}} + e^{-\frac{x}{m}} \right\}$$

The curve whose equation is of this form is called a *catenary*.

Since $T = mw \sec \phi$, and $y = m \sec \phi$, $T = wy$. Or, the tension at any point on its curve is equal to the weight of a chain whose length is the vertical distance of the point from a horizontal line drawn at distance m below the vertex.

$$\text{Again,} \quad s = m \tan \phi = m \sqrt{\sec^2 \phi - 1} = \sqrt{y^2 - m^2} = \frac{m}{2} \sqrt{\frac{4y^2}{m^2} - 4}$$

$$\text{That is,} \quad \frac{2s}{m} = \sqrt{\frac{4y^2}{m^2} - 4} = \sqrt{\left\{ e^{\frac{x}{m}} + e^{-\frac{x}{m}} \right\}^2 - 4} = e^{\frac{x}{m}} - e^{-\frac{x}{m}}$$

$$\therefore \quad s = \frac{m}{2} \left\{ e^{\frac{x}{m}} - e^{-\frac{x}{m}} \right\}$$

Which gives s in terms of x .

LECTURE I.—QUESTIONS.

1. What do you understand by the terms *strain*, *stress*, and *modulus of elasticity*? A tie rod 100 feet long, and of 2 square inches sectional area, is stretched three-quarters of an inch under a tension of 32,000 lbs. What is the intensity of the stress, the strain, and the modulus of elasticity under these circumstances? *Ans.* 16,000 lbs. per square inch; 0·000625; 25,600,000.

2. A ship is moored by two cables of 90 feet and 100 feet in length respectively. The first cable stretches $2\frac{1}{2}$ inches, and the second stretches 3 inches, under the pull of the ship; find the strain of each cable. *Ans.* 0·0243; 0·0025.

3. Define the term *Resilience*. Show that the work done on a material by a *live load* is four times that done by an equal *dead load*. A wrought-iron tie rod 20 feet long and $\frac{1}{2}$ square inch cross-sectional area bears a dead load of 5,000 lbs. Find the work done on stretching the rod by this load. What live load would produce an instantaneous elongation of another $\frac{1}{16}$ inch? Take $E = 30,000,000$. *Ans.* 33·3 ft.-lbs.; 3,125 lbs.

4. A rod of iron 25 feet long and 2 square inches cross-sectional area checks a weight of 80 lbs., which falls from a height of 20 feet before beginning to strain it. Find the greatest stress and strain produced. Take $E = 25,000,000$. *Ans.* 40,000 lbs. per square inch; ·0016.

5. If the modulus of elasticity of a piece of steel in lbs. per square inch is 32,000,000, how much would a bar $\frac{3}{4}$ of an inch in diameter and 25 inches long extend under a load of 10 tons? If its limit of elasticity is 21 tons per square inch, what is its resilience?

6. What is the resilience of a bar? A bar of steel is $\frac{7}{8}$ inch in diameter, and 30 inches in length, and is under a tensile pull of 10 tons, what is the work stored up in the bar, the modulus of elasticity being 32,000,000 lbs. per square inch?

7. Built-up guns are made of concentric rings, the outer hoops, or rings, being shrunk or forced upon inner tubes with a regulated tension. Supposing the external diameter of the inner tube to be 12 inches, and that the substance of its covering hoop is to have given to it an initial grip of 4 tons per square inch of its sectional area; the exterior diameter of this second hoop is 18 inches, and is to be covered with a third hoop, having an initial grip of 8 tons per square inch of its sectional area; will you work out in arithmetic the difference of dimensions that will afford the above conditions?

8. Prove that when a thin spherical shell is exposed to the bursting pressure of gas or liquid the stress in the material is half as great as that within the curved surface of a thin cylindrical shell exposed to the like pressure, each shell being of the same thickness and diameter.

9. A long thin pipe of given internal radius is subjected to fluid pressure; find the tension of the material of the pipe. If the internal radius of the pipe is 6 inches, and the thickness of the pipe 0·5 inch, what fluid pressure per square inch would increase the radius of the pipe by 0·001 inch? The modulus of elasticity being 20,000,000, and the elasticity of the material being supposed to continue perfect. *Ans.* 277·7 lbs. per square inch.

10. A steel hydraulic cylinder, 10 feet long and 6 inches in diameter, acts as a brake on a lift. It has a movable piston fitted with a spring valve, the cylinder being full of liquid when the lift is at its highest position, and the piston and rod at the end of the stroke inside the cylinder. It was found that when the lift began to descend the internal pressure was 1,000 lbs. per square inch, which gradually rose to 2,000 lbs. when the piston had travelled 9 feet. Treating the cylinder as a thin one, what would be the law of variation of thickness at different points? Prove the formula.

11. A uniformly heavy chain is suspended from two given points: find the equation to the curve in which it hangs, and the tension at any point of the curve.

12. Prove that the tendency of a thin cylindric pipe to burst laterally (neglecting the strength of flanges, &c.) is twice as great as to burst endwise.

A wrought-iron pipe is 2 feet diameter, $\frac{1}{2}$ inch thick, its working stress is 5 tons to the square inch, but strength of plate is diminished 30 per cent. because of riveted joint. What is the working pressure? What head of water does this correspond to?

13. Prove the law for the tensile stress produced in a thick cylinder by internal fluid pressure. Describe how we attempt by chilling to give maximum strength.

14. A steel tube 5 inches internal and 7 inches external diameter has steel strip wound on it to the external diameter of 12 inches under a constant winding tensile stress of 15 tons per square inch. What is the stress at any place in the solid metal or the winding?

15. If a thin vessel is subjected to fluid pressure, p , inside (in excess of the outside pressure), prove that the total bursting force at any plane section is pA if A is the area of the whole section. How do we calculate the tensile stress in the section a of the metal? Find the rule for the stress in a thin spherical vessel. Does the rule apply to a thick vessel? Give reasons for your answer.

16. State clearly how we arrive at a rule for the proper thickness of a pipe, proving any formula used by you. In fixing the proper value of the stress that the material (say cast-iron) will stand, why do we not use the results of experiment on cast-iron test pieces in tension?

(B. of E. Adv., 1901.)

17. Knowing the axial and lateral strains in a tie bar of homogeneous material when subjected to a tensile force, show how we calculate the modulus of rigidity of the material. (B. of E. H., Part I., 1901.)

18. A tube 3 inches internal and 8 inches external diameter is subjected to a collapsing pressure of 5 tons per square inch; show by curves the radial and circular stresses everywhere. Prove your formulæ. The limits of elasticity are not supposed to be exceeded. (B. of E. H., Part I., 1901.)

19. A horizontal circular tube of steel is 7 feet diameter, $\frac{1}{2}$ inch thick, 100 feet long supported at the ends, its total load distributed uniformly all over being 30 tons, what are the greatest stresses in the metal? The tube is filled with compressed air, what must its pressure be if there is just no compressive stress in the metal? State what is now the nature of the stress in the metal at the place where it is greatest.

(B. of E. H., Part I., 1902.)

N.B.—See Appendices B and C for other questions and answers. This note refers to all the Lectures.

LECTURE I.—C. & G. EXAM. QUESTIONS.

1. A steam engine has a piston 18 inches in diameter, and the greatest difference of steam pressure between the two sides of the piston when the engine is at work is 120 lbs. per square inch : what must the piston-rod diameter be in the body of the rod if the greatest intensity of stress per square inch is not to exceed 2,500 lbs. ? (C. & G., 1900, O., Sec. B.)

2. The links of a chain are made out of $1\frac{1}{2}$ -inch round bar iron, having a tenacity in pure tension of 22.5 tons per square inch : what load could be safely lifted with such a chain if the stress is not to exceed $\frac{1}{4}$ th of the breaking stress of the chain ? You may assume that only three-quarters of the full tenacity would be developed in the material before rupture, when worked up into the form of a link. (C. & G., 1900, O., Sec. B.)

3. How much would a steel tie-bar 3 inches in diameter and 25 feet 6 inches long extend under a total load of 33 tons ? The modulus of elasticity of the steel is 12,500 tons per square inch. (C. & G., 1900, O., Sec. B.)

4. A cylinder 10 inches in diameter has a cover fixed on by $1\frac{1}{8}$ -inch studs. The internal fluid pressure is 200 lbs. per square inch above the atmospheric pressure. How many such studs would you employ if the tensile stress per square inch at the bottom of the threads is not to exceed 2,500 lbs. ? (The diameter at the bottom of the thread of a bolt = 0.9 diameter of bolt - .095 inch.) (C. & G., 1901, O., Sect. B.)

N.B.—See Appendices B and C for other questions and answers.

LECTURE I.—A.M.INST.C.E. EXAM. QUESTIONS.

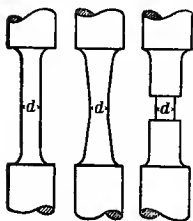
1. Show that a chain supported at the ends and carrying a load which is uniform per foot of the span hangs in a parabolic curve. Hence find approximately the pull in a telegraph wire when the span, the dip, and the weight of the wire are given. (I.C.E., Oct., 1897.)

2. Distinguish between stress and strain. A round bar 20 feet long and 2 inches in diameter is stretched $\frac{3}{8}$ inch by a load of 7 tons, applied along the axis. Find the intensity of stress on a cross-section and the coefficient of elasticity of the material. (I.C.E., Oct., 1897.)

3. The greatest stress permissible being 5,000 lbs. per square inch, find the shortest rod of 1 square inch section which can be safely used to check a weight of 50 lbs. falling 3 feet. Take $E = 12,000$ tons per square inch.

(I.C.E., Oct., 1897.)

4. Compare, in general terms, the three rods shown in the sketch as regards their suitability to withstand tensional stresses produced by impacts of given energy, dealing both with the magnitude and distribution of the stresses. (I.C.E., Feb., 1898.)



5. Prove that the shape of a hanging chain loaded uniformly horizontally is parabolic. The points of support are 50 feet and 70 feet above the lowest point in the chain, and the horizontal distance between the points of support is 300 feet. The load is 3 tons per foot horizontal, state the pull in the chain at the points of support and the middle.

(I.C.E., Oct., 1898.)

6. Describe the particular distributions of load under which an equilibrated suspension chain would hang in the following curves:—(a) a parabolic arc, (b) the common catenary, (c) the curve of sines.

(I.C.E., Oct., 1898.)

7. Explain the meaning of the following terms, giving some verbal illustrations: (a) modulus of elasticity; (b) elastic limit; (c) permanent set; (d) plastic elongation; (e) ultimate elongation; (f) contraction of area. (I.C.E., Feb., 1899.)

8. By how much would a wrought-iron tie-bar be elongated under a direct pull of 30 tons, if its length were 30 feet, and its cross-section 5 inches by 1 inch? (Assume $E = 12,000$ tons.) (I.C.E., Feb., 1899.)

9. A standard test-bar of cast iron, 1 inch broad and 2 inches deep, laid upon supports 3 feet apart, is found to deflect just $\frac{1}{8}$ inch under a central load of 25 cwts. Calculate the modulus, E , and also the apparent tensile stress in the extreme fibre. (I.C.E., Feb., 1899.)

10. Explain the terms (a) stress, (b) strain, (c) elasticity, and state the law connecting (a) and (b) when the elasticity of a material is perfect. A steel tie-rod, 1 inch diameter 20 feet long, stretches $\frac{1}{8}$ inch under a pull of 5 tons: find the modulus of elasticity. (I.C.E., Oct., 1899.)

11. Prove the ordinary formula which gives the stress on the material of a cylindrical boiler-shell of given diameter and thickness under a given internal fluid pressure, explaining carefully the conditions under which it is approximately correct and stating the reasons why it cannot generally be used for an external pressure. (I.C.E., Oct., 1899.) (See my "Text-Book on Steam and Steam Engines" for further treatment.)

12. Find the curve in which a string, whose weight is neglected, uniformly loaded horizontally, would hang. (I.C.E., Oct., 1899.)

13. If the load on the platform of a suspension bridge be uniformly distributed show that the chains hang in a parabola, the weight of the chains and suspending rods being omitted from the calculation. Find the pull at any point of the chain, and assuming the chains attached to the top of the piers, find the moment tending to overturn them. Describe the arrangements adopted in practice to prevent this.

(I.C.E., Feb., 1900.)

14. What is the modulus of elasticity (Young's modulus) and how is it measured? Mention some of the practical questions which depend upon its value in different materials. (I.C.E., Oct., 1900.)

15. Assuming the modulus to be 12,000 tons in iron wire, what would be the elongation of a wire originally 1000 feet in length, due to its own weight when suspended vertically? (I.C.E., Oct., 1900.)

16. An iron telegraph wire is stretched between a pair of supports 160 feet apart, and hangs with a sag of 2 feet in the middle. Assuming its weight to be uniformly distributed over the span, find the tensile stress per square inch at the centre. The weight of a cubic inch of iron may be taken at 0.27 lb. (I.C.E., Feb., 1901.)

17. Explain the meaning and uses of a factor of safety, stating what factor you would select for the following cases:—(a) A girder of mild steel carrying the dead weight of a wall. (b) A steel cross-girder in a railway bridge. (I.C.E., Feb., 1901.)

18. In a tensile test of a piece of flat wrought-iron bar, the following results were obtained:—

- (i.) Original dimensions of cross-section 2.010 inches by 0.505 inch.
- (ii.) Final dimension of cross-section at point of fracture 1.520 inch by 0.410 inch.
- (iii.) Gross load at limit of elasticity 34,500 lbs.
- (iv.) Gross load at fracture 53,100 lbs.
- (v.) Total extension on length of 10 inches = 1.46 inch.
- (vi.) Extension on 10" length under a gross load of 20,000 lbs. = $\frac{1}{1000}$ ".

Find from the above data:—(a) The modulus of elasticity of the material; (b) the limit of elasticity and tenacity per square inch; (c) the reduction of area per cent.; (d) the approximate work done per cubic inch of the material in fracturing the bar. Would you consider this good material?

(I.C.E., Oct., 1901.)

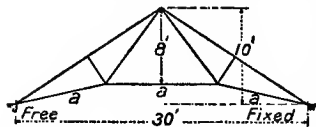
19. A foot-bridge 10 feet in width is carried over a river 100 feet in width by two cables of uniform section, with a dip of 10 feet in the centre. Find the greatest pull on the cables, their cross-sectional area, length and weight for the following data:—Maximum load on platform 120 lbs. per square foot. Working-stress in metal of cables 4 tons per square inch. Weight of cable material per cubic foot 484 lbs. (I.C.E., Oct., 1901.)

20. A short cylindrical block is subject to a simple push, P , in the direction of its axis. Find an expression for the normal and tangential forces on a plane whose normal is inclined to the axis of the block at an angle, θ ; and show that the tangential force has its maximum value when θ is 45° .

(I.C.E., Feb., 1902.)

21. A suspension bridge has a span of 200 feet, the dip of the chains is 64 feet, and the weight of the roadway carried 2,800 lbs. per foot run. Calculate the tension at the middle and at the two ends of each of the two chains. (I.C.E., Feb., 1902.)

22. In the roof truss given find the stresses in the different members due to a normal wind pressure of 27 lbs. per square foot, acting on the left-hand side of the roof if the principals are 8 feet apart. (I.C.E., Feb., 1902.)



23. Assuming that the stresses due to the dead loads on the above roof have been similarly determined, show how you would determine the sizes of the tie-bars (α) say. Assume some value for the stresses due to dead load,

and also assume that the severest stresses due to the wind are those obtained in the previous question. (I.C.E., Feb., 1902.)

24. A connecting-rod of an engine is 6 feet long and 4 inches in diameter at the centre. What factor of safety does it possess when subjected to a pure compressive force of $14\frac{1}{2}$ tons? The rod is steel. (I.C.E., Feb., 1902.)

25. A mild steel tie-rod is made of angle-bar 4 inches by 4 inches by $\frac{1}{2}$ inch and is 24 feet long. If it is subjected to a direct pull of 21 tons, what is the intensity of tensile stress per square inch, and how much will it elongate under the load. (I.C.E., Feb., 1902.)

26. A round wrought-iron rod is 25 feet long and 1 inch in diameter, find the tensile load which would, if suddenly applied, instantaneously elongate the bar by 0.08 inch. (I.C.E., Feb., 1902.)

27. Wrought iron expands $\frac{1}{10000}$ of its length for every 15° rise of temperature (Fahrenheit). Find the highest temperature at which a rivet should be closed, if the stress due to cooling is not to exceed 15 tons per square inch. Modulus of elasticity 28,000,000 lbs. per square inch. (I.C.E., Oct., 1902.)

28. A chain, with its two ends level, carries concentrated loads at 5 points along its length; show how to find graphically the position and amount of the resultant of the loads and the amount of the vertical reaction of the supports. (I.C.E., Feb., 1903.)

N.B.—See Appendices B and C for other questions and answers.

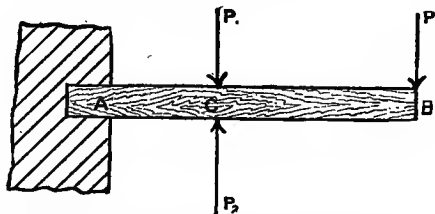
LECTURE II.

CONTENTS.—Strength of Beams and Girders—Definitions of Shearing Force and Bending Moment—Beam Fixed at one end and Loaded at the other—Beam Fixed at one end and Loaded Uniformly—Beam Supported at both ends and Loaded in the middle—Example I.—Beam Supported at ends and Loaded anywhere—Beam Supported at both ends and Loaded Uniformly—Examples II. and III.—Floating Beams—Traveling Loads—Two Loads Moving at a Fixed Distance apart—Example IV.—Distributed Travelling Load—Questions.

Strength of Beams and Girders.—The subject under this heading is one that naturally divides itself into two portions. (1) The determination of the resultant effects of the applied loads at any section of a beam or girder; and (2) the nature and amount of the resistance offered by the beam or girder to rupture at that section.

When the section under consideration is in the same plane as the load, the only effect the load has at that section is a tendency to *shear* the beam; but in the more general case, where the load acts at a distance from the given section, we have, in addition, a tendency to curve or bend the beam at the section. Hence the name *Bending Moment* is given to this latter effect.

In the accompanying figure, let A B represent a cantilever



ILLUSTRATING SHEARING AND BENDING ACTION.

or beam fixed at one end, with a load P applied at the free end; and let C be any section in the beam. At C let there be applied two equal and opposite forces P_1 , P_2 , of the same magnitude as P . The introduction of these forces does not affect the equilibrium of the system, as P_1 and P_2 balance each other. Hence, the effect of P at the section C is equivalent to that of a

couple $P P_2$, with a single force P_1 . A general proof of this important theorem is given in Vol. I., Lecture III., Prop. II. The couple constitutes the Bending Moment (B.M.), and the single force P_1 , the Shearing Force (S.F.) at the section C.

DEFINITION.—The Shearing Force at any section of a beam is the algebraic sum of all the forces acting on either side of that section.

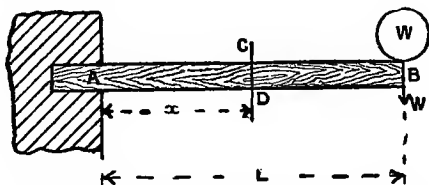
DEFINITION.—The Bending Moment at any section is the algebraic sum of the moments of all the forces acting on either side of that section.

Or, in symbols, if P denote any one of the forces acting on one side of a section, and at a distance x , from it; consider all the forces on the same side of the section as P , paying due regard to their sign—that is, if we reckon forces acting upwards as positive, we must regard those acting downwards as negative.

$$\begin{array}{lcl} \text{Then,} & \text{S.F.} = \Sigma P. & \\ \text{And,} & \text{B.M.} = \Sigma Px. & \end{array} \left. \vphantom{\begin{array}{l} \text{Then,} \\ \text{And,} \end{array}} \right\} \dots \dots \dots \text{(I)}$$

Beam Fixed at one end and Loaded at the other.—Let CD be a cross-section anywhere within the length of the beam at a distance of x inches from the fixed end A . To find the S.F. and B.M. at CD , we observe that the only force acting to the right of the section is W lbs. Hence:—

$$\text{S.F.} = W \text{ lbs.} \dots \dots \dots \text{(II)}$$



BEAM FIXED AT ONE END, LOADED AT OTHER.

It is independent of x , and therefore the same for all such sections as CD .

The B.M. at CD is W multiplied by its distance from the section in inches. Hence:—

$$\text{B.M.} = W \times BD = W(L - x) \text{ inch-lbs.}^* \dots \dots \text{(III)}$$

This equation is true whatever may be the position of W on the beam, so long as L denotes its distance in inches from the fixed end, and CD is between W and the support.

*Where the older term "inch-lbs." has been used in connection with the Bending Moment at any section of a loaded beam, then the more modern term of "lb.-inches" may be substituted.

In this case, the diagram of the S.F. is a straight line parallel to the base and at a distance of W lbs. from it. Since (III) is the equation of a straight line, the B.M. is therefore a quantity increasing uniformly from zero, where $x = L$, to $W L$ inch-lbs., where $x = 0$, as shown by the accompanying figure.

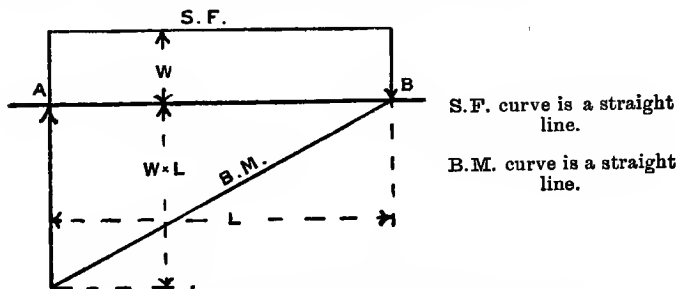
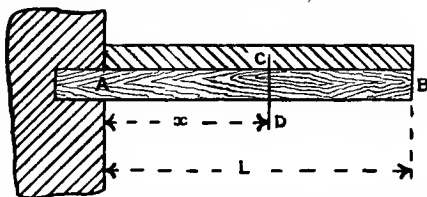


DIAGRAM OF S.F. AND B.M. FOR BEAM FIXED AT ONE END AND LOADED AT THE OTHER.

Beam Fixed at one end, and Loaded Uniformly.—Let the load on the beam be w lbs. per inch-run, it is required to find the shearing force and bending moment at any section CD , at x inches from the fixed end. As before, consider the part of the beam to the right of CD . The only force is the weight of that portion of the load carried by BD , so that:—

$$\text{S.F.} = w \times BD = w (L - x) \text{ lbs.} \quad \dots \quad (\text{IV})$$



BEAM FIXED AT ONE END AND LOADED UNIFORMLY.

The moment of the portion of the load on BD with respect to C is the same as if it were all concentrated at the middle point of BD . Hence:—

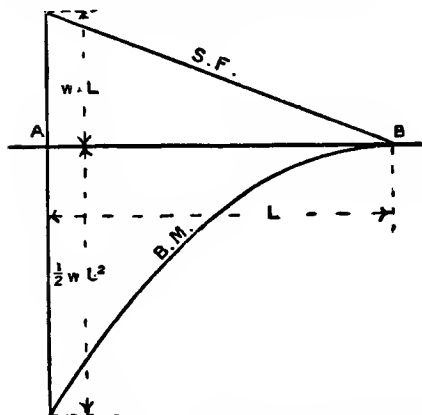
$$\text{B.M.} = w \times BD \times \frac{1}{2} BD = \frac{1}{2} w \times BD^2 = \frac{1}{2} w (L - x)^2 \text{ inch-lbs.} \quad (\text{V})$$

Equations (IV) and (V) show us that both the S.F. and B.M. vanish when $x = L$; and when $x = 0$, we get:—

$$\text{S.F.} = w L \text{ lbs.} \quad \dots \dots \dots (\text{IV}_a)$$

And, $\text{B.M.} = \frac{1}{2} w L^2 \text{ inch-lbs.} \quad \dots \dots \dots (\text{V}_a)$

The diagrams of S.F. and B.M. for this case take the forms shown in the accompanying figure.



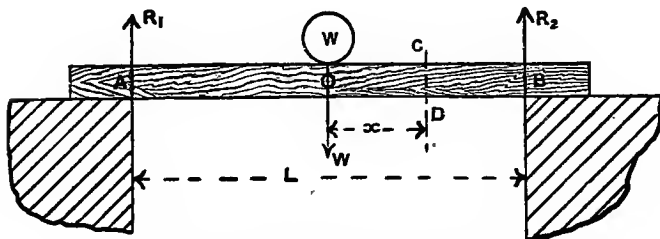
S.F. curve is a straight line.

B.M. curve is a parabola with vertex at B.

DIAGRAM OF S.F. AND B.M. FOR BEAM FIXED AT ONE END AND LOADED UNIFORMLY.

Beam Supported at both ends, and Loaded at the Middle.—In this case we measure x from the middle point of the beam. Since W is equidistant from A and B , the reactions at those points, R_1 and R_2 , are equal to each other, and since their sum is W , we have:—

$$R_1 = R_2 = \frac{1}{2} W \text{ lbs.}$$



BEAM SUPPORTED AT BOTH ENDS AND LOADED AT MIDDLE.

The only force to the right of CD is R_2 , and its leverage is BD .

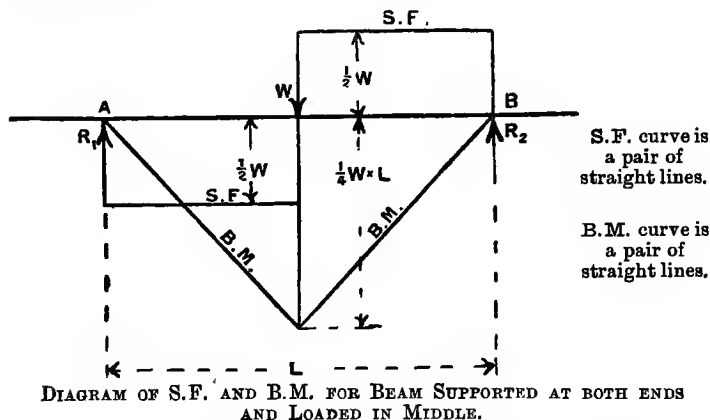
Hence, $S.F. = R_2 = \frac{1}{2} W$ lbs. (VI)

And, $B.M. = R_2 \times BD = \frac{1}{2} W (\frac{1}{2} L - x)$ inch-lbs. . (VII)

Here, the B.M. vanishes when $x = \frac{1}{2} L$, and increases uniformly from this until $x = 0$, when it attains its maximum value, $\frac{1}{4} W L$ inch-lbs.

Or, Maximum B.M. $= \frac{1}{4} W L$ inch-lbs. . . . (VII_a)

The following figure shows the diagrams of S.F. and B.M. for this case:—



EXAMPLE I.—In a beam of length L , supported at both ends and loaded at the middle with a load W , show that the bending moment is greatest at the centre of the beam and equal to $\frac{1}{4} W L$. Then determine graphically the bending moment and shearing force at a point 6 ft. from one support in a beam of 25 ft. span loaded with 5 tons at its centre. (Adv. S. & A. Exam., 1890.)

ANSWER.—We have already seen from equation (VII) that for a beam loaded as in this example, the B.M. at any distance x , from its middle point, is:—

$$B.M. = \frac{1}{2} W (\frac{1}{2} L - x).$$

This is obviously greatest when $x = 0$ —that is, at the centre.

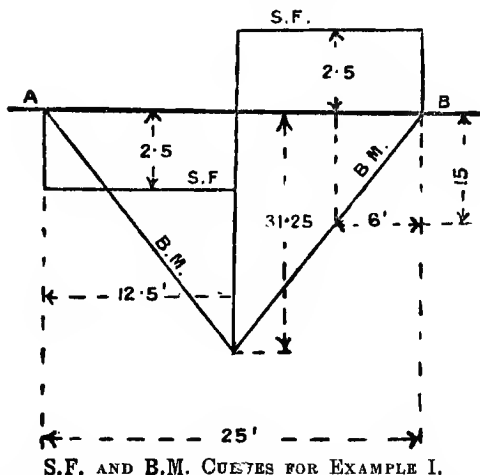
Then:—

$$\text{Maximum B.M.} = \frac{1}{4} W L; \text{ and } S.F. = \frac{1}{2} W.$$

For the values of W and L given in the example, we get :—

$$\text{Maximum B.M.} = \frac{1}{4} \times 5 \times 25 = 31.25 \text{ ft.-tons.}$$

$$\text{And,} \quad \text{S.F.} = \frac{1}{2} \times 5 = 2.5 \text{ tons.}$$

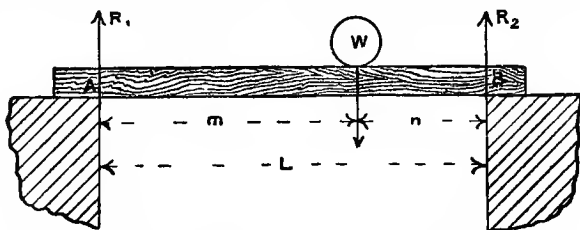


The accompanying figure shows the diagrams of B.M. and S.F. as constructed from these data.

At 6 feet from one end the B.M. measures 15 ft.-tons. This is easily verified by means of the formula for B.M., because $x = 12.5 - 6 = 6.5$.

$$\therefore \text{B.M.} = \frac{1}{2} \times 5 \times (12.5 - 6.5) = 15 \text{ ft.-tons.}$$

Beam Supported at both ends, and Loaded Anywhere.—With a single concentrated load, the maximum bending moment will



BEAM SUPPORTED AT BOTH ENDS, AND LOADED ANYWHERE.

always occur immediately under the load, whether it be at the middle of the beam or not.

For the B.M. at any section at a distance x , from one end is $R \times x$, and this is greatest when x is largest; that is, when the section is under the load.

To find the reactions at the supports, we take moments about A and B, and get $R_2 \times L = W \times m$.

$\therefore R_2 = \frac{m}{L} W$ lbs. and $R_1 = \frac{n}{L} W$ lbs. These are the values of the S.F. to the right and left of W respectively.

$$\left. \begin{aligned} \text{S.F. (to right)} &= \frac{m}{L} W \text{ lbs.} \\ \text{S.F. (to left)} &= \frac{n}{L} W \text{ lbs.} \end{aligned} \right\} \quad (\text{VIII})$$

Multiplying the first of these equations by n , or the latter by m , we get:—

$$\text{Maximum B.M.} = \left(\frac{m}{L} \frac{n}{L} W \right) \text{ inch-lbs.} \quad \dots \quad (\text{IX})$$

We can now construct the diagrams of S.F. and B.M.:—

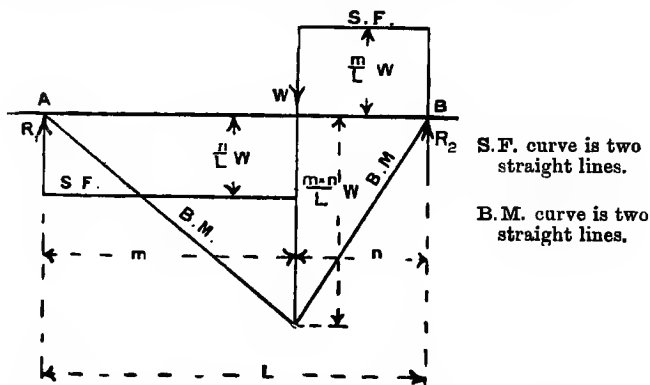
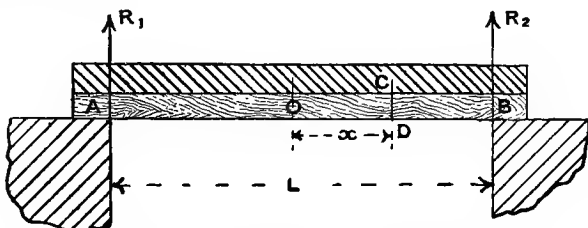


DIAGRAM OF S.F. AND B.M. FOR SINGLE LOAD IN ANY POSITION.

Beam Supported at both ends and Loaded Uniformly.—As before, let the weight per inch-run be denoted by w , then the total load carried by the beam will be $w L$ lbs., and the reactions

R_1 and R_2 , will each be $\frac{1}{2} w L$ lbs. Taking the forces to the right of the section C D.



BEAM SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

We get, S.F. = $R_2 - w \times BD = \frac{1}{2} w L - w (\frac{1}{2} L - x) = w x$ lbs. (X)

And, B.M. = $R_2 \times BD - w \cdot BD \times \frac{1}{2} BD$

$$,, = \frac{1}{2} w L \times BD - \frac{1}{2} w \cdot BD^2$$

$$,, = \frac{1}{2} w \cdot BD (L - BD)$$

$$,, = \frac{1}{2} w (\frac{1}{2} L - x) (\frac{1}{2} L + x).$$

$$\therefore \text{B.M.} = \frac{1}{2} w (\frac{1}{4} L^2 - x^2) \text{ inch-lbs.} \quad \dots \quad (XI)$$

The limiting values of S.F. and B.M. are :—

When, $x = \frac{1}{2} L$; then, S.F. = $\frac{1}{2} w L$ lbs.; and, B.M. = 0. (X_a)

When, $x = 0$; then, S.F. = 0; and :—

$$\text{Maximum, B.M.} = \frac{1}{8} w L^2 \text{ inch-lbs.} \quad \dots \quad (XI_a)$$

Plotting our diagrams of S.F. and B.M., we get the figure shown on next page.

When a beam carries more than one load, or is loaded in more ways than one, the simplest and safest way is to consider each load separately, without regard to the others, and then combine the separate effects so as to obtain the resultant action, as in Example II.

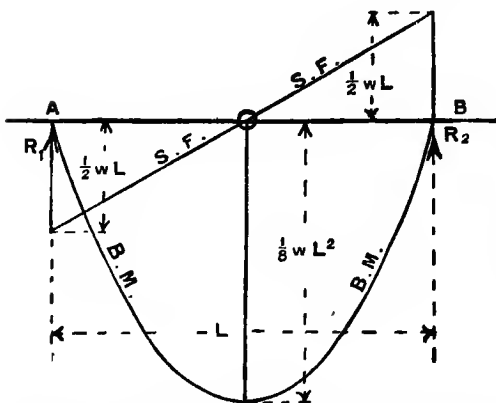
EXAMPLE II.—Draw the bending moment and shearing force diagrams for a beam 12 feet long, supported at both ends, and loaded with weights of 4 and 6 tons at distances of 3 and 8 feet respectively, from one end of the beam. Explain fully the mode of arriving at these diagrams. (Adv. S. & A. Exam., 1887.)

ANSWER.—Measuring distances from the left end of the beam, and considering each load separately, we have, for the 4 tons, to the left of the load :—

$$\text{S.F.}_1 = \frac{n}{L} W = \frac{9}{12} \times 4 = 3 \text{ tons.}$$

And, to the right of it:—

$$\text{S.F.}_1 = \frac{m}{L} W = \frac{3}{12} \times 4 = 1 \text{ ton.}$$



S.F. curve is a straight line.

B.M. curve is a parabola with vertex below the middle of the beam.

DIAGRAM OF S.F. AND B.M. FOR A BEAM SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

The maximum B.M.₁ due to this load is:—

$$\text{B.M.}_1 = \frac{m \times n}{L} W = \frac{3 \times 9}{12} \times 4 = 9 \text{ ft.-tons.}$$

It occurs immediately under the load.

Next taking the 6-tons load, we have to the left of it:—

$$\text{S.F.}_2 = \frac{n}{L} W = \frac{4}{12} \times 6 = 2 \text{ tons ;}$$

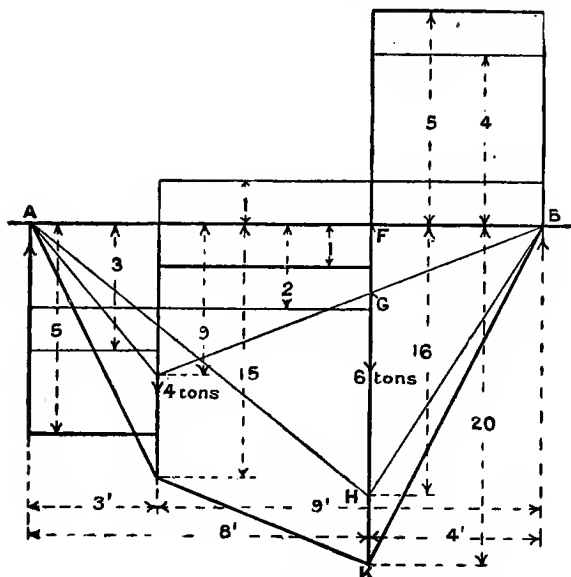
And to the right of it:—

$$\text{S.F.}_2 = \frac{m}{L} W = \frac{8}{12} \times 6 = 4 \text{ tons.}$$

The maximum B.M.₂ due to the 6 tons is:—

$$\text{B.M.}_2 = \frac{m \times n}{L} W = \frac{8 \times 4}{12} \times 6 = 16 \text{ ft.-tons.}$$

Plotting these results, we get the accompanying figure:—



S.F. AND B.M. CURVES FOR EXAMPLE II.

The thin lines show the actions of the separate loads, and the full lines their combined results, obtained by taking the algebraic sum of the former.

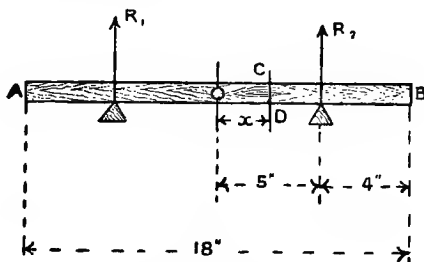
The student should here carefully observe the necessity of attending to the *sign* of the shearing force. Thus, between the weights we have a shearing force of 2 tons, which, on account of its sign, is drawn below the base line; also a shearing force of 1 ton drawn above the base line. The resultant shearing force between the loads is therefore the difference of these, and is drawn on the same side of the base line as the greater of its components.

The bending moments everywhere along the beam are of the same sign; therefore, to obtain the combined bending moment diagram, we have simply to add the ordinates of each separate diagram. Thus, to get the total bending moment at the section under the 6 tons load, we add FG (viz., that due to the 4 tons at that point) to FH (that due to the 6 tons). The result FK is therefore the total B.M. at that point.

It is quite sufficient to do this for the sections under each load, and then to join each of the points so obtained with each other and with the ends of the beam by straight lines. If drawn to scale, the B.M. at any other point can then be obtained by measuring the corresponding ordinate.

EXAMPLE III.—A horizontal uniform bar, 18 inches long, is laid over two supports, each 4 inches from its ends. Find two points at which the bending moments are zero. (Hons. S. & A. Exam.)

ANSWER.—Let w be the weight in lbs. per inch-run of the bar. Then the total weight of the bar will be $18w$ lbs., and the reactions will each be $9w$ lbs.



ILLUSTRATING EXAMPLE III.

Taking moments to the right of the section C D at a distance x inches from the centre of the bar, we get:—

$$\begin{aligned}
 \text{B.M.} &= R_2(5 - x) - w \cdot BD \times \frac{1}{2} BD \\
 ,, &= 9w(5 - x) - \frac{1}{2}w \cdot BD^2 \\
 ,, &= 9w(5 - x) - \frac{1}{2}w(9 - x)^2 \\
 ,, &= \frac{1}{2}w(9 - x^2) \text{ inch-lbs.}
 \end{aligned}$$

The B.M. will be zero when $9 - x^2 = 0$; i.e., when $x = \pm 3$ inches.

Hence, the required points are 3 inches on each side of the centre, or 2 inches inside of the supports.

Floating Beams.—When a *solid* body, such as a piece of wood of *uniform density*, floats in *still* water its weight and its buoyancy, or the resultant upward pressure of the water on the body, will at all points balance each other. There are consequently no shearing or bending stresses on the body, and each part is in equilibrium independently of the other parts.

But whenever those conditions are departed from, such as (1) when the floating body carries weights; (2) when it is not of uniform density, due to want of homogeneity in its material, if solid, or to its being hollow, or of a boat form; or (3) when it crosses waves, then bending and shearing stresses are set up.

Consider the case of a uniform beam of wood of rectangular section floating in still water. The beam will displace an amount of water exactly equal to its own weight. This is true, not only for the beam as a whole, but also for every individual segment of the beam. Any segment of the beam will displace just as much, and no more, water than it would do if floating by itself. The beam, therefore, is as free from stress as it would be if it were lying on a perfectly flat surface.

Suppose now that a weight W , be placed on the middle of a floating beam. This will cause the beam to sink to a greater depth and displace an *extra* volume of water. The *weight* of this *extra* displacement is exactly equal to W . What, now, is the condition of the beam as regards straining forces? Evidently, we need only consider the weight W , and the extra displacement, due to its being carried by the beam; because the upward reaction of the displacement due to the beam's own weight, is still at all points balanced by the downward weight of the beam. In other words, the condition of the beam, so far as its own weight and displacement are concerned, is in no way affected by the addition of the load.

To give definiteness to our ideas, let W be expressed in lbs., and let L denote the length of the beam in inches.

Then the forces we have to consider are:—

(1) W lbs. concentrated at the middle of the beam and acting downwards.

(2) The displacement of W lbs. of water uniformly distributed along the whole length of the beam and acting upwards, with an intensity of $\frac{W}{L}$ lbs. per inch of length.

The case is, therefore, analogous to that of a beam uniformly loaded and supported at its centre; or what is virtually the same thing, two beams of length equal to $\frac{1}{2} L$, fixed at one end and loaded uniformly. For, in order to obtain the shearing force and the bending moment at any section of the beam, x inches to either side of W , we have simply to substitute $\frac{W}{L}$ for w , and $\frac{1}{2} L$ for L in equations (IV) and (V), and we get:—

$$\text{S.F.} = \frac{W}{L} (\frac{1}{2} L - x) \text{ lbs.} \quad \dots \quad \text{(XII)}$$

And,
$$\text{B.M.} = \frac{W}{2L} \left(\frac{1}{2}L - x\right)^2 \text{ inch-lbs.} \quad \dots \quad (\text{XIII})$$

Under W the shearing force and bending moment are each a maximum. Their values may be found by making $x = 0$.

Then the
$$\text{Maximum S.F.} = \frac{1}{2} W \text{ lbs.} \quad \dots \quad (\text{XII}_a)$$

And the
$$\text{Maximum B.M.} = \frac{1}{8} W L \text{ inch-lbs.} \quad \dots \quad (\text{XIII}_a)$$

The diagrams of S.F. and B.M. for this case are constructed in identically the same way as for a beam fixed at one end and carrying a uniform load, but taking $\frac{1}{2}L$ as a base line instead of L .

Suppose that, instead of one weight in the middle, the beam is loaded with two weights, one at each end, and each equal to W lbs., it is easy to see that the condition now is that of a beam uniformly loaded with $\frac{2W}{L}$ lbs. per inch-run and supported at each end. We have, therefore, only to apply formulæ (X) and (XI), substituting $\frac{2W}{L}$ for w , when we get:—

$$\text{S.F.} = \frac{2W}{L} \cdot x \text{ lbs.} \quad \dots \quad (\text{XIV})$$

And,
$$\text{B.M.} = \frac{W}{L} \cdot \left(\frac{1}{4}L^2 - x^2\right) \text{ inch-lbs.} \quad \dots \quad (\text{XV})$$

Here the shearing force is a maximum when $x = \frac{1}{2}L$, and the bending moment a maximum when $x = 0$.

Or,
$$\text{Maximum S.F.} = W \text{ lbs.} \quad \dots \quad (\text{XIV}_a)$$

And,
$$\text{Maximum B.M.} = \frac{1}{4} W L \text{ inch-lbs.} \quad \dots \quad (\text{XV}_a)$$

The diagrams of S.F. and B.M. are, therefore, in every way similar to those for a uniformly loaded beam supported at the ends.

Travelling Loads.—The simplest case of a movable load is that, wherein we are given a weight, say a heavy cylindrical body, rolling along a beam, to find the equations of maximum S.F. and B.M. for any position of the load, and exhibit these results in a diagram.

Referring to formulæ (VIII) and (IX), and the diagrams already deduced for a fixed load in *any* position on a beam, we

have for the maximum S.F. to the immediate right of the load :—

$$\text{S.F.} = \frac{m}{L} W \text{ lbs.}$$

And, to the immediate left of the load :—

$$\text{S.F.} = - \frac{n}{L} W \text{ lbs.}$$

For the maximum B.M., which occurs immediately under the load :—

$$\text{B.M.} = - \frac{m n}{L} W \text{ inch-lbs.}$$

Putting $m = x$ so that $n = L - x$, we obtain, when the load is x inches from the left end of the beam :—

$$\left. \begin{aligned} \text{The Maximum S.F. (just to right of the section)} &= \frac{W}{L} x \\ \text{,, ,, (just to left of the section)} &= \frac{W}{L} (x-L) \end{aligned} \right\} \text{(XVI)}$$

$$\text{And, Maximum B.M.} = \frac{W}{L} (L - x)x \text{ (XVII)}$$

To construct the diagram of S.F., we observe that its equation is that of a straight line, and that to the right of the section considered its value is zero when the load just starts from the left end of the beam, and increases uniformly as the load approaches the other end. That is :—

When, $x = 0$; then, S.F. = 0.

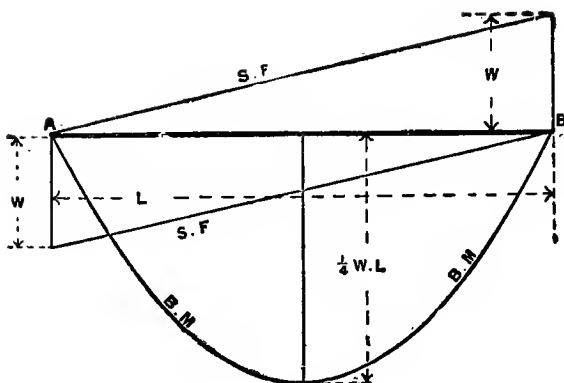
Also when, $x = L$; then, S.F. = W lbs.

There is also another line for the shear at all positions just to the left of the load. This line passes through B, and its ordinate is $-W$ at the end A.

The equation of the B.M. curve is that of a parabola, whose axis is vertical, and passes through the middle point of the beam, where, of course, the maximum value of B.M. occurs. To construct this diagram, we have :—

When, $x = 0$, or $x = L$; then B.M. = 0.

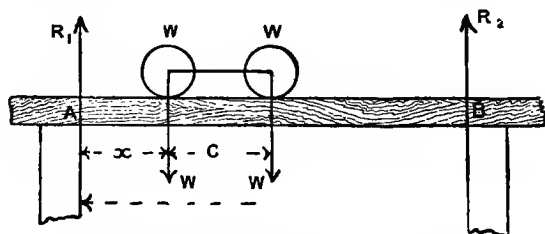
Also when, $x = \frac{1}{2} L$; then B.M. = $\frac{1}{4} W L$ inch-lbs.



DIAGRAMS OF MAXIMUM S.F. AND B.M. FOR ROLLING LOAD.

Two Loads moving at a fixed distance apart.—From the above simple case we may easily pass to a very important practical example of moving loads—viz., overhead travelling cranes.

Here the crane rests on a carriage with four wheels running on two rails carried by girders, the weight of the whole machine together with the load being equally distributed over the wheels. Hence, considering one girder only, our problem is reduced to that of two equal loads moving along the girder at a fixed distance apart.



ILLUSTRATING TRAVELLING CRANE PROBLEM.

In the figure let W be the weight resting on each wheel, and c be the distance between their centres. If the motion be supposed to be in the direction shown by the arrow, it is evident that until the carriage gets to the middle of the girder the maximum

shearing force and bending moment will occur under the leading wheel—that is, if we estimate the shearing force to the immediate right of the wheel. But as the same thing takes place in the reverse order when the carriage moves from the other end of the girder in the opposite direction, we shall take the section of the girder immediately under the following wheel and estimate the shearing force and bending moment for that position. This method of procedure will be found to lead to simpler equations than if we had taken the leading wheel as our point of reference.

Now, considering the forces acting to the left of the wheel, we easily see :—

$$\text{That,} \quad \text{S.F.} = R_1.$$

$$\text{And,} \quad \text{B.M.} = R_1 \times x.$$

To find R_1 we take moments about B, which gives us :—

$$R_1 \times L = W \{L - (x + c)\} + W(L - x)$$

$$,, \quad ,, = W \{2(L - x) - c\}.$$

$$\therefore \quad R_1 = \frac{W}{L} \{2(L - x) - c\}.$$

$$\text{Hence,} \quad \text{S.F.} = \frac{W}{L} \{2(L - x) - c\}. \quad . \quad . \quad . \quad (\text{XVIII})$$

$$\text{And,} \quad \text{B.M.} = \frac{W}{L} \{2(L - x) - c\}x. \quad . \quad . \quad . \quad (\text{XIX})$$

The equation for the S.F. is that of a straight line, and for the B.M. a parabola. To find the position and dimensions of those diagrams, we see that :—

$$\text{When} \quad x = 0, \text{ S.F.} = \frac{W}{L} (2L - c), \text{ and B.M.} = 0.$$

Again, both S.F. and B.M. will vanish when $2(L - x) - c = 0$; that is, when $x = L - \frac{c}{2}$.

To find the maximum ordinate of the B.M. curve, we have the condition that, when the B.M. is a maximum :—

$$\frac{d}{dx} (\text{B.M.}) = 0.$$

$$\text{That is,} \quad \frac{d}{dx} \{2(L - x) - c\}x = 0.$$

$$\text{Or,} \quad \frac{d}{dx} \{ (2L - c)x - 2x^2 \} = 0.$$

$$(2L - c) - 4x = 0.$$

$$\text{Hence,} \quad x = \frac{L}{2} - \frac{c}{4}.$$

The shearing force diagram will, therefore, consist of two straight lines parallel to each other, and the bending moment diagram will consist of two equal parabolas intersecting at the middle of the girder. The axes of these equal parabolas will be equidistant from the middle of the girder and $\frac{1}{2}c$ units apart.

The following numerical example will elucidate this important case much better than a bare examination of formulæ:—

EXAMPLE IV.—In a travelling crane of 40 feet span the load is supported on a carriage which runs upon two similar girders, the axles of the carriage being 8 feet apart, and a load of $2\frac{1}{2}$ tons coming upon each wheel. Obtain a diagram showing the maximum bending moment at every section of the girder, and give the numerical values at distances of 10, 15, and 20 feet from one end. (Hons. S. & A. Exam., 1880.)

ANSWER.—Applying our general formulæ, we have, for the bending moment at any distance x ft., from one end:—

$$\text{B.M.} = \frac{W}{L} [(2L - c) - 2x]x.$$

Here, $W = 2.5$ tons, $L = 40$ ft., and $c = 8$ ft.

$$\therefore \quad \text{B.M.} = \frac{2.5}{40} [80 - 8 - 2x]x.$$

$$\text{Or,} \quad \text{B.M.} = \frac{1}{8} (36 - x)x \text{ ft.-tons.}$$

For the numerical values asked for, we have:—

$$\text{When } x = 10 \text{ ft.; B.M.} = \frac{1}{8} (36 - 10) 10 = 32.5 \text{ ft.-tons.}$$

$$\text{When } x = 15 \text{ ft.; B.M.} = \frac{1}{8} (36 - 15) 15 = 39.375 \text{ ft.-tons.}$$

$$\text{When } x = 20 \text{ ft.; B.M.} = \frac{1}{8} (36 - 20) 20 = 40 \text{ ft.-tons.}$$

We have seen, that the B.M. attains its maximum value when:—

$$x = \frac{L}{2} - \frac{c}{4} = \frac{40}{2} - \frac{8}{4} = 18 \text{ ft.}$$

$$\text{Hence, the maximum B.M.} = \frac{1}{8} (36 - 18) 18 = 40.5 \text{ ft.-tons.}$$

The S.F. is not asked for in the question, but we here add it so as to make the example more complete.

The maximum S.F. occurs when $x = 0$, and when $x = L$, its value for this case then being:—

$$\text{Maximum S.F.} = \frac{2.5}{40}(80 - 8) = 4.5 \text{ tons.}$$

And, like the B.M., it is zero when $x = 40 - \frac{8}{2} = 36$ feet.

The following figure shows the S.F. and B.M. diagrams as required for this example.

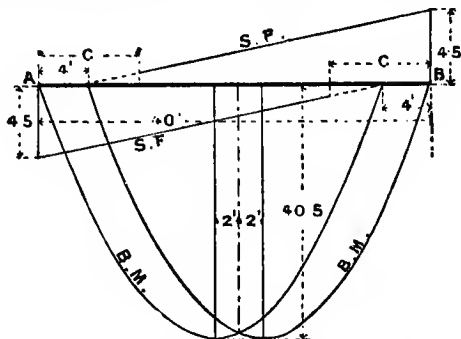
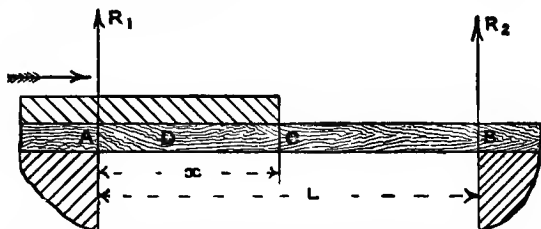


DIAGRAM OF MAXIMUM S.F. AND B.M. FOR A TRAVELLING CRANE.

Distributed Travelling Load.—The last case we shall consider is that in which a continuous load of uniform density, and long enough to completely cover it, comes on to a girder and moves off at the other end, such as a long train of uniform weight passing over a bridge.



ILLUSTRATING TRAVELLING LOAD OF UNIFORM INTENSITY.

In the figure, let w denote the load per unit of length. When the load is in the position shown, it is clear that the S.F., at all points to the right of C, will be equal to R_2 ; and that at any section D, to the left of C, the S.F. will be less than R_2 by the weight of the portion of the load covering CD. It at once

follows that the S.F. is greatest at C, the front of the load, and this is true for all positions of C.

Hence, $S.F. = R_2$.

Taking moments about A, we have:—

$$R_2 \times L = wx \times \frac{1}{2} x,$$

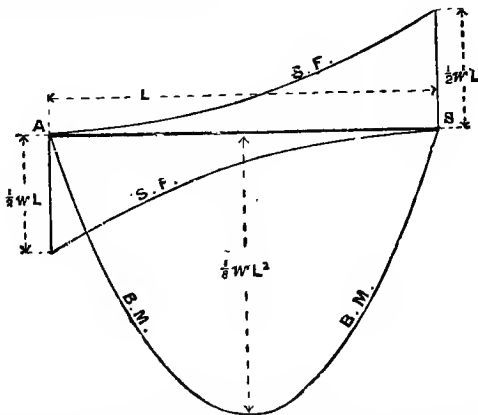
$$\therefore R_2 = \frac{wx^2}{2L}.$$

$$\text{That is, } S.F. = \frac{wx^2}{2L} \quad \dots \dots \dots (XX)$$

The shearing force is, therefore, proportional to the square of the length of the part of the load resting on the girder.

The curve of the maximum bending moment is very easily deduced in this case. We have only to remember that the B.M. at *any fixed* section in the girder will get greater and greater for every additional part of the load that comes upon it; so that when the girder is wholly covered by the load the B.M. at every position will then be a maximum. The B.M. diagram is therefore identical with that given for a beam loaded uniformly, whilst the S.F. diagram becomes a parabola instead of a straight line.

The following figure shows how the S.F. and B.M. diagrams are constructed for this case.



S.F. AND B.M. DIAGRAMS FOR TRAVELLING CONTINUOUS
LOAD OF UNIFORM INTENSITY.

LECTURE II.—QUESTIONS.

1. Define "*bending moment*" and "*shearing force*." A uniform beam weighing 15 cwts. rests on supports at its ends 20 feet apart. The beam is loaded with three weights of 4, 6, and 10 cwts. at distances of 2, 7, and 12 feet respectively from one of the supports. Find the B.M. and S.F. at a point 8 feet from the same support. *Ans.* B.M. = 98 ft.-cwts.; S.F. = 3 cwts.

2. A bar of pine 48 inches long rests on props at its extremities, and just supports 7 weights, of 14 lbs. each, hung at equal intervals of 6 inches along the bar. Find the value of a single weight, which, if hung at the centre of the bar, would stress it to the same extent.

3. A batten of fir, 6 feet in length and supported at its extremities, will just sustain a load of 520 lbs. when hung at the centre. If this weight be removed, and two weights, each equal to P lbs., be hung at distances of 2 and 4 feet along the bar, what is the greatest value which may be assigned to P ? *Ans.* 390 lbs.

4. A beam, 20 feet long, whose weight is neglected, is supported at both ends and loaded with 1 ton evenly distributed along its length. Find the bending moment at a distance of 7 feet from one end. *Ans.* 5,096 ft.-lbs.

5. A beam, whose weight may be neglected, rests on supports at its ends 15 feet apart. Weights of 10, 6, 5, and 12 cwts. rest on the beam at intervals of 3 feet apart, the weight of 10 cwts. being 3 feet from one support. Find the points where the maximum bending moment and shearing force occur, and obtain their values. Construct the diagrams of bending moments and shearing force for the whole beam. *Ans.* The max. B.M. = 66 ft.-cwts., and occurs at all points between the weights 6 and 5 cwts.; the max. S.F. = 17 cwts., and occurs at the point where the weight of 12 cwts. rests.

6. A uniform cantilever, or beam fixed at one end and free at the other, 10 feet long, weighs 6 cwts., and carries two loads, one of 2 cwts. at the free end, and the other 4 cwts. at its middle point. Construct the shearing force diagram for the whole cantilever, and find the shearing forces at points $2\frac{1}{2}$ feet and 6 feet from fixed end. *Ans.* 10.5 cwts.; 4.4 cwts.

7. A block of wood weighing 800 lbs., 20 feet long and 12 inches square, floats in water, and is loaded—

(1) By a weight of 200 lbs., placed at each extremity;

(2) By a weight of 400 lbs. at the centre.

Show what forces act on the beam, and draw the curves of shearing force and bending moment for each case.

8. A girder is supported at both ends, and has a clear span of 30 feet. Show by means of a curve the position and magnitude of the greatest bending moment produced by a load of 20 tons as it rolls from one end to the other of the girder. Obtain the numerical results for distances respectively of 10 and 15 feet from one end.

9. Prove an algebraic formula to show that, with a continuous load of uniform intensity passing over a beam AB such as when a long train passes over a bridge A to B, the maximum shearing stress to any point K of the beam occurs when the part AK is fully loaded while the part

KB is entirely unloaded, and that the magnitude of the stress is proportional to the square of the distance of K from the point A. A train of 1 ton per foot run, and upwards of 100 feet in length, passes over a bridge of 100 feet span; what would be the maximum shearing stresses at distances of 25 and 50 feet respectively from one end of the bridge? Show how to determine graphically the shearing stresses in the beam.

10. Prove the ordinary formula for the deflection of a beam of length l , supported at its ends and loaded in the middle with a load W . A beam of uniform rectangular section supported at the ends is 20 feet between the supports; what should be its depth in order that the deflection may not exceed $\frac{1}{8}$ inch under a maximum stress of 8,000 lbs. per square inch of section? $E = 28 \times 10^6$ lbs. per square inch.

11. A wall has to be carried over a gap of 16 feet, and it is proposed to use a rolled steel joist for the purpose. What size of joist would you adopt from the following data: (a) the weight of wall is equivalent to a uniformly distributed load of 2,600 lbs. per foot run of joist; (b) the greatest intensity of stress per square inch is not to exceed 5 tons? (C. & G., 1900, H., Sec. A.)

12. Show how to obtain, and sketch the diagrams of maximum possible bending moment and maximum possible shear for a uniform rolling load of a given amount per foot run, as it passes over a girder of given span.

(C. & G., 1900, H., Sec. A.)

13. A girder of 22 feet span is supported at the two ends, a load of 10 tons rests on a point 2 feet from the left end, and two other loads of 6 and 7 tons respectively, at distances of 7 and 13 feet respectively from the first load: find the bending moment in inch-tons under each load, and also the shearing force. (C. & G., 1900, O., Sec. B.)

14. A steel joist is used as a girder on a span of 17 feet 6 inches, and is freely supported at the two ends. It carries a uniformly distributed load of 18 cwts. per foot run, and two concentrated loads, one of 3 tons 3 feet from the left-hand support, and another of $1\frac{1}{2}$ tons 10 feet from the same end. Find the bending moments in inch-pounds under each of the concentrated loads, and also at the centre of the girder, and also the shearing forces in pounds at each of these points. Sketch the bending moment diagram for the whole girder. (C. & G., 1901, O., Sec. B.)

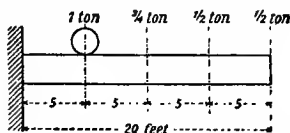
15. A girder, supported at the two ends, is 10 feet long, and is loaded uniformly with a load of $\frac{1}{2}$ ton per foot run, and also carries a weight of 3 tons placed 2 feet from one support. Sketch the curves of shearing force and bending moment, and find their numerical values at the centre of the span and at a section immediately under the concentrated load.

(C. & G., 1902, O., Sec. B.)

N.B.—See Appendices B and C for other questions and answers.

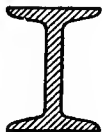
LECTURE II.—A M. INST. C.E. EXAM. QUESTIONS.

1. A cantilever 20 feet long carries the loads shown in the sketch. Draw



diagrams of bending moment and shearing force. (I.C.E., Oct., 1897.)

2. A rolled steel joist 40 feet in length, and having the section shown in the fig.—which may be taken as equivalent to depth 10 inches, breadth 5 inches, thickness throughout $\frac{1}{2}$ inch—is continuous over three supports, forming two spans of 20 feet each. What uniformly distributed load would produce a maximum stress of $5\frac{1}{2}$ tons per square inch? Sketch the diagrams of bending moments and shearing force. (I.C.E., Feb., 1898.)



3. A rectangular beam of timber, 12 inches square and 15 feet between supports, is subjected to a uniformly distributed load of 200 lbs. per foot run. Illustrate diagrammatically the distribution of shearing and longitudinal stresses over a transverse section 3 feet from one of the supports, and determine the greatest intensity of each stress. (I.C.E., Feb., 1898.)

4. A train, equivalent to a uniform rolling load of 30 cwts. per foot, has entered upon a single-line girder bridge of 100 feet span, and extends over 80 feet of the span (from the first abutment). The dead weight of the structure is a uniform load of 1 ton per foot. Calculate the vertical shearing force at the head of the train, due to dead and live loads, taking both girders together. (I.C.E., Oct., 1898.)

5. If the rolling load, in the last-mentioned case, consisted of any known system of loaded axles, how would you proceed to find, by calculation or by graphic construction, the maximum shearing forces at various points in the span, due to live load only? (I.C.E., Oct., 1898.)

6. In the beam A B C D E, the length (A E) of 24 feet is divided into four equal panels of 6 feet each by the points B, C, D. Draw the diagram of moments for the following conditions of loading, writing their values at each panel-point:—1st. Beam supported at A and E, loaded at D with a weight of 10 tons. 2nd. Beam supported at B and D, loaded with 10 tons at C, and with a weight of 2 tons at each end A and E. 3rd. Beam *encasté* from A to B, loaded with a weight of 2 tons at each of the points C, D, and E. (I.C.E., Oct., 1898.)

7. A floating balk of timber, 40 feet long, of straight prismatic form, carries a weight of 6 cwts. slung from its centre. Sketch the diagram of bending moments in the beam, and calculate the shearing force at a point 10 feet from the end. (I.C.E., Oct., 1898.)

8. A horizontal beam span $2a$ is supported at the ends and loaded uniformly with w lbs. per foot run; show that the shearing force and bending moment at any point distant x from the centre are given by the formulæ:—

$$F = wx \qquad M = \frac{1}{2} w (a^2 - x^2).$$

Show how these straining actions are represented graphically.

(I.C.E., Oct., 1899.)

9. A beam 24 feet span is supported horizontally at the ends, and loaded with weights of 3 and 5 tons concentrated at points distant 8 and 18 feet from one end. Construct roughly to scale diagrams showing the shearing force and bending moment at any point. (I.C.E., *Oct.*, 1899.)

10. Explain carefully the connection which exists between the shearing force and the bending moment at any point of a loaded beam, (1) when the load is continuous, (2) when it is concentrated at various points. Illustrate your remarks by reference to the special cases given in questions 6 and 7.

(I.C.E., *Oct.*, 1899.)

11. The platform of a bridge, 40 feet span, rests on a pair of girders below and is loaded uniformly with 2,000 lbs. per foot run. Construct to scale diagrams showing the shearing force and bending moment at any point of one of the girders, and find the maximum values of these straining actions. (I.C.E., *Feb.*, 1900.)

12. A four-wheeled truck weighing 8 tons traverses the bridge of the preceding question; 5 tons resting on the leading pair of wheels and 3 tons on the following pair, axles 8 feet apart. Find the maximum straining actions at the centre of one of the girders. (I.C.E., *Feb.*, 1900.)

13. A rectangular tank 10 feet square is carried by three beams below, each supporting an equal share of the whole weight of 14 tons. The section of each beam being rectangular, depth 8 inches, breadth 3 inches, find the maximum stress on the material. (I.C.E., *Feb.*, 1900.)

14. A girder of uniform section is continuous over the two centre piers of a bridge of three equal spans, the ends being simply supported by piers at precisely the same level. Find the points of contrary flexure, and construct diagrams showing the shearing force and bending moment at any point when uniformly loaded throughout. (I.C.E., *Feb.*, 1900.)

15. A rigid body of unit width is rectangular in vertical section, which is 30 feet long and 10 feet high. A force of 20 tons is applied vertically to its upper surface, one-third the length from one vertical face, on the centre of which a normal force of 10 tons acts. If the specific gravity of the block is $2\frac{1}{2}$, find the distribution of pressure on the base.

(I.C.E., *Feb.*, 1900.)

16. A beam, A B C D, whose length, A D, is 40 feet, is supported at the two points, A and C, which are 30 feet apart. The beam itself weighs 112 lbs. per lineal foot, and it carries a load of 1 ton at the overhanging end, D. Find the reaction at the support, C, and also the bending moment at a point, B, midway between A and C. (I.C.E., *Oct.*, 1900.)

17. A cantilever bridge (of the Kentucky type) consists of three equal spans, A B, B C, and C D, of 300 feet each, the girder being hinged at two points, F and G, which are situated in the side-spans at a distance of 200 feet from each end of the bridge; so that A F is a span of 200 feet carried at F upon the cantilever, F B, 100 feet in length, forming part of the girder, F B C G, which is supported at B and at C. Assume a uniform dead load of 3 tons per foot lineal throughout, and a rolling load equivalent to 2 tons per foot of train. Calculate the bending moment at the centre of each span—first, when the rolling load extends from B to C; and second, when it extends from A to B and from C to D.

(I.C.E., *Oct.*, 1900.)

18. A beam is laid horizontally upon two supports which are 12 feet apart, and projects at each end 6 feet beyond the support. A load of 2 tons is carried upon each of the projecting ends, and a load of 1 ton upon the centre of the span. Calculate the bending moment at the centre and also at each support; and sketch the diagram of moments neglecting the weight of the beam. (I.C.E., *Feb.*, 1901.)

19. If a straight prismatic balk of timber floats in a horizontal position with a weight of 4 cwts. hung to each end, what will be the bending moment at the centre of its length? (I.C.E., *Feb.*, 1901.)

20. Two piers in a cantilever bridge stand at a distance of 240 feet apart (between centres of supports) and the span is crossed by a girder 320 feet in length, whose ends project as cantilevers extending 40 feet beyond each support. A uniform load of 15 cwts. per foot is distributed over the whole length of this girder, while each cantilever carries 60 tons at the extreme end. Find the bending moment at each pier and at the centre of the span. (I.C.E., *Feb.*, 1901.)

21. Construct the diagram of moments for the case described in the previous question, and calculate the shearing force immediately to the right and to the left of each support. (I.C.E., *Feb.*, 1901.)

22. Determine by exact computation the position of the points of contrary flexure in the girder described in the two previous questions.

(I.C.E., *Feb.*, 1901.)

23. The total length, A E, of a floating pontoon is 80 feet, and the centre-line, A B C D E, is divided into four equal parts by the intermediate points, B, C, and D. The pontoon has a uniform weight per foot lineal and a uniform cross-section throughout, and floats without sensible curvature with a uniform draught of water. Regarding the vessel as a girder, construct the diagram of bending moments for the following loads:—(1) A single load of 240 tons at C. (2) A load of 80 tons at each of the points, B, C, and D. In the latter case find the moments at 10 feet intervals from A to C. (I.C.E., *Feb.*, 1901.)

24. A girder, crossing a span of 120 feet, carries a uniformly distributed load of 10 cwts. per foot and a central load of 20 tons. Find the vertical shearing force at a section 30 feet from either support. (I.C.E., *Feb.*, 1901.)

25. In a certain ship 450 feet in length it is found, that for the central 200-foot length, the weight of the displaced water corresponding to this length exceeds the weight of that portion of the ship by 900 tons, while in each of the 125-foot end sections their weight exceeds that of the water displaced by 450 tons. Find the maximum bending moment in the ship's structure (its total weight is 7,000 tons) when lying in still water.

(I.C.E., *Oct.*, 1901.)

26. A rolling load of $1\frac{1}{2}$ tons per foot run crosses a bridge of 120-foot span. Find the maximum bending moments and shearing forces which can occur at sections of the bridge distant 30, 60, and 90 feet respectively from one end. Sketch diagrams of maximum possible bending moments and shearing forces for the whole bridge produced by the above rolling load. The main girders of the bridge are of the bowstring type. Show how you could determine by the method of sections the maximum stresses in the diagonals and verticals produced by the above rolling load.

(I.C.E., *Oct.*, 1901.)

27. A continuous girder consists of two equal spans of 45 feet each. Determine the bending moments at the supports, the maximum intermediate bending moments and the reactions, if a uniformly distributed load of 2,000 lbs. per foot run covers both spans. Assume both ends of the girder are free. (I.C.E., *Oct.*, 1901.)

28. A beam supported free at the two ends, bends under its own weight; find the upward force at the centre which will just neutralise this bending action, expressing the answer in terms of the weight of the beam.

(I.C.E., *Oct.*, 1901.)

29. A beam of timber 15 feet long, 18 inches by 12 inches cross-section, floats in sea water, and is loaded at the centre with a weight just sufficient

to totally immerse it. Find the maximum values of the bending moment and shearing force on the beam, and state approximately how they vary. (Specific gravity of oak 0.73, of sea-water 1.026.) (I.C.E., Feb., 1902.)

30. A steel rail is 32 feet long and weighs 100 lbs. per yard. It rests on two supports, one at one end of the rail, the other at a point 10 feet from the other end of the rail. Find the position and amount of the maximum bending moment and shear when a weight of 200 lbs. hangs from the free end. Construct the bending moment and shearing force diagrams for this rail. (I.C.E., Feb., 1902.)

31. A girder has a span of 40 feet, and two rolling loads of 10 tons and 15 tons respectively 10 feet apart pass over it. Find the maximum bending moment which can occur at any section and the maximum shear, and construct diagrams of maximum possible bending moment and shear.

(I.C.E., Feb., 1902.)

32. In a bridge, 120 feet span, with eight bays, the main girders are connected together by trough flooring, on which a uniform live load of 2 tons per foot moves. Draw the positions of the moving load which would give a maximum stress in the braces. (I.C.E., Oct., 1902.)

33. Compare the loads which can be safely carried at the centre of a bar 12 feet long, 6 inches deep, and 2 inches broad, and of a girder of the same length and cubic contents whose overall depth is 14 inches and breadth of flange $4\frac{1}{2}$ inches, thickness of web $\frac{1}{2}$ inch. (I.C.E., Oct., 1902.)

34. A girder 70 feet long carries a uniform load of 2 tons per lineal foot from one end to the middle, and a load of 20 tons at 20 feet from each end; draw the bending-moment and shearing-force diagrams.

(I.C.E., Feb., 1903.)

35. In a plate girder the maximum intensity of stress at right angles to the vertical cross-section of the web is 5 tons per square inch, and the intensity of shearing stress there is 2 tons per square inch; find the position of the planes of the principal stresses at that point and their intensities.

(I.C.E., Feb., 1903.)

36. Would two similar girders of the same length carry more weight if placed alongside each other, or if one is placed over the other, and the lower flange of one is riveted to the upper flange of the other? Give reasons for your answer. (I.C.E., Feb., 1903.)

N.B.—See Appendices B and C for other questions and answers.

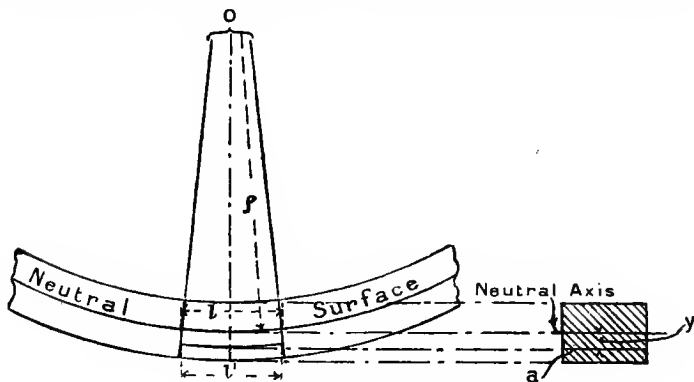
LECTURE III.

CONTENTS.—Resistance of Beams to Flexure—Examples I., II., III., and IV.—Thin Wrought-Iron Girders—Example V.—Curvature and Deflection of Beams—Example VI.—Uniform Beam on Three Supports—Uniform Beam fixed at one end and supported at the other—Beams fixed at both ends and loaded at centre—Beams fixed at both ends and loaded uniformly—Tables—Example VII.—Questions.

Resistance of Beams to Flexure.—In the previous Lecture we saw that the effect of loading a beam was to give rise to both shearing and bending.

From the theory of couples set forth in Vol. I. we know that nothing but a couple can balance a couple. The resistance which a beam offers to bending must be of this nature, and therefore a couple of equal magnitude to that of the applied load, but of opposite tendency. The tendency of the applied couple is to bend or curve the beam, whilst the tendency of the induced couple is to oppose this curving action.

When a beam is curved the longitudinal fibres on the convex side of it are stretched beyond their normal length, and consequently they are in tension. On the concave side the fibres



ILLUSTRATING FLEXURE OF BEAMS.

are shortened, and, therefore, they are in compression. Somewhere within the beam there must be a layer of fibres that are neither lengthened nor shortened, and are therefore unstressed.

This layer is termed the *neutral surface* of the beam, and the intersection of this surface with any cross-section of the beam is termed the *neutral axis* of that section. The neutral axis is of fundamental importance in the theory of beams, because it is the fulcrum about which both the bending and resisting couples act.

We shall now find the position of the neutral axis of any given section of a beam.

Let l be the length of a small portion of the neutral surface; l' that of a parallel layer of fibres on the stretched side of the beam, and at a distance y , from the neutral surface. If $l' = l$ when the beam is straight, it is evident that the amount of stretch in the fibres at distance, y , from the neutral surface will be $l' - l$, and the strain $\frac{l' - l}{l}$. Let ρ denote the radius of curvature of the neutral surface at the cross-section bisecting l . Then the radius of curvature corresponding to l' will be $= \rho + y$.

$$\text{Hence,} \quad \frac{\rho + y}{\rho} = \frac{l'}{l},$$

$$\text{Or,} \quad \frac{y}{\rho} = \frac{l' - l}{l}.$$

If f be the tensile stress at distance y , from the neutral axis, and E the modulus of elasticity of the material, we already know that:—

$$\frac{\text{stress}}{\text{strain}} = E,$$

$$\text{Or,} \quad \frac{f}{\frac{l' - l}{l}} = E.$$

Substituting $\frac{y}{\rho}$ for $\frac{l' - l}{l}$, and inverting, we get:—

$$\frac{1}{\rho} = \frac{f}{E y} \quad \dots \dots \dots (I)$$

If we had considered in the same way a layer of fibres at a distance y' , to the concave side of the neutral surface, and denoted the stress there as $-f'$ (the minus sign indicating compressive stress), we should have arrived at the equation:—

$$\frac{1}{\rho} = \frac{-f'}{E y'} \quad \dots \dots \dots (I_a)$$

Let a be a small element of the cross-sectional area at a distance y , then on the one side of the neutral axis we have for the total resistance to tension :—

$$\Sigma af = \frac{E}{\rho} \Sigma ay. \quad \dots \quad (II)$$

On the other side of the neutral axis the total resistance to compression is :—

$$- \Sigma af' = \frac{E}{\rho} \Sigma ay'. \quad \dots \quad (II_a)$$

But these forces constitute a couple, and are therefore equal. Hence, equating the right hand members, we have, neglecting the common factor, $\frac{E}{\rho}$:— $\Sigma ay = \Sigma ay'$.

The neutral axis, therefore, passes through the centre of area of each cross-section. If, however, E be not the same for Tensile and Compressive stresses, then the N.A. will not pass through the centre of the area, but will lie to the side having the greater value of E .

To obtain the magnitude of the resisting couple, we multiply the resistances, af and af' , by their respective distances, y and y' , from the neutral axis, and sum up these products for the whole section. Thus, from equation (II) the total moment of resistance on the convex side of the neutral axis is :—

$$\Sigma afy = \frac{E}{\rho} \Sigma ay^2;$$

And on the concave side :—

$$- \Sigma af'y' = \frac{E}{\rho} \Sigma ay'^2.$$

The sum of these results constitutes the total Resisting Moment, R.M., for the section.

$$\text{Hence,} \quad \text{R.M.} = \frac{E}{\rho} \Sigma ay^2 + \frac{E}{\rho} \Sigma ay'^2.$$

There is now no longer any need for distinguishing between y and y' , since the process of summation is the same all over the cross-section. We, therefore, finally get :—

$$\text{R.M.} = \frac{E}{\rho} \Sigma ay^2. \quad \dots \quad (III)$$

The quantity Σay^2 , being a purely geometrical function, depending only on the *form* of the section, is termed its

Moment of Inertia, and is usually denoted by the symbol I , and sometimes by the product $A k^2$ (see Lecture XII., Vol. I.). Table II., Lecture XII., gives the values of k^2 for most of the sections required in the following examples. These multiplied by A will give the required values of I .

Writing I for $\Sigma a y^2$, our equations become :—

$$\text{B.M.} = \text{R.M.} = \frac{E I}{\rho};$$

$$\text{Or, the curvature, } \frac{1}{\rho} = \frac{M}{E I} \quad \dots \dots \dots \text{(IV)}$$

Where M stands for either the B.M. or R.M.

Again, from equations (I) and (I_a), we get :—

$$\left. \begin{aligned} \frac{f}{y} &= \frac{E}{\rho} = \frac{M}{I} \\ \therefore M &= \frac{f}{y} I \\ \text{Or, } f &= \frac{M}{I} y \end{aligned} \right\} \dots \dots \dots \text{(V)}$$

Formulæ (IV) and (V) are the fundamental equations of the theory of the strength of beams and girders. In applying the latter equation, it must always be borne in mind that f stands for either the tensile or compressive stress at any distance y , above or below the neutral axis.

The greatest stress comes on the fibres farthest from the neutral axis, and is the principal effect to be considered in questions of strength. If this is amply provided for, the beam will be safe. Let y now denote the distance of the fibres farthest from the neutral axis :—

$$\text{Then, } f_{\max.} = \frac{M}{I} \times y = M \div \frac{I}{y}.$$

The ratio $\frac{I}{y}$ is usually denoted by Z , and is called the **Modulus of the Section**.*

* See Prof. Unwin's *Elements of Machine Design*, Pages 56 to 59, for a table of the moduli of sections of beams, where $Z = \frac{I}{y}$. Also, in Seaton & Rounthwaite's *Pocket Book of Marine Engineering Rules and Tables*, from which we extracted the tables at the end of this lecture, we also find $Z = \frac{I}{y}$ as the modulus of the section.

Hence, writing Z for $\frac{I}{y}$, we have:—

$$f_{max.} = \frac{M}{Z} \quad \left. \begin{array}{l} \\ \end{array} \right\} \dots \dots \dots (VI)$$

Or,

$$M = Z f_{max.}$$

In applying this equation the student must be careful to remember that in those cases where the section of the beam or girder is not symmetrical about the neutral axis, there will be two values of y to be taken into account, and therefore two values of Z . On the whole, we think it safer to adhere to the general formula (V) as being less likely to lead to confusion; at the same time, it is very convenient to use equation (VI) in taking out quantities in the drawing office by aid of tables since it reduces the work of calculation.

EXAMPLE I.—A floor joist, 12 inches deep and 3 inches broad, has a span of 15 feet, and carries a uniformly distributed load of 1 cwt. per foot-run. Find the greatest intensity of stress within the timber. (S. and A. Adv. Exam., 1891.)

ANSWER.—In problems involving the calculation of stress within the beam, the student will find it best to express all dimensions in *inches*, and, therefore, bending moments in *inch-lbs.* or *inch-tons* as the case may be.

In this problem the greatest stresses will occur at the middle of the joist where the bending moment attains its maximum value, which, in this case, is:—

$$\text{Max. B.M.} = \frac{1}{8} w L^2 \text{ inch-lbs.}$$

$$\text{Here, } w = \frac{112}{12} \text{ lbs.}$$

$$\text{And, } L = 15 \times 12 \text{ inches.}$$

$$\therefore \text{B.M.} = \frac{1}{8} \times \left(\frac{112}{12} \right) \times (15 \times 12)^2 \text{ inch-lbs.}$$

$$\text{Or, } \text{B.M.} = 14 \times 15 \times 15 \times 12 \quad ,,$$

The value of I for a rectangular section is:—

$$I = \frac{1}{12} (\text{breadth}) \times (\text{depth})^3,$$

$$\text{Or, } I = \frac{1}{12} \times 3 \times 12^3 = 3 \times 12 \times 12.$$

The greatest stress at the middle section of the joist will occur in the fibres farthest away from the neutral axis. Hence, $y = 6$ inches.

Applying equation (V) we have:—

$$f = \frac{\text{B.M.}}{I} y,$$

$$,, = \frac{14 \times 15 \times 15 \times 12}{3 \times 12 \times 12} \times 6 = 525 \text{ lbs. per sq. in.}$$

EXAMPLE II.—A uniform beam of oak, 10 feet long, 15 inches deep and 10 inches wide, sustains, in addition to its own weight, a load of 5,000 lbs. placed at the centre. Find the greatest bending moment and the greatest stress in the fibres.

The specific gravity of oak is 0.934. (S. and A. Adv. Exam., 1894.)

ANSWER.—Here the greatest bending moment takes place at the centre of the beam and is made up of two parts: (1) that due to the beam's own weight which is uniformly distributed along its length; and (2) that due to the 5,000 lbs. concentrated at its middle.

$$\text{For (1),} \quad \text{B.M.}_1 = \frac{1}{8} w L^2 \text{ inch-lbs.}$$

$$\text{And for (2),} \quad \text{B.M.}_2 = \frac{1}{4} W L \quad ,,$$

$$\therefore \text{Total,} \quad \text{B.M.} = \frac{1}{8} w L^2 + \frac{1}{4} W L \text{ inch-lbs.}$$

Taking the weight of a cubic inch of water as 0.036 lb., then a cubic inch of oak will weigh $0.934 \times 0.036 = 0.0336$ lb.

$$\therefore \quad w = 0.0336 \times 15 \times 10 = 5.04 \text{ lbs.}$$

$$\text{And,} \quad \text{B.M.} = \frac{1}{8} \times 5.04 \times (10 \times 12)^2 + \frac{1}{4} \times 5,000 \times (10 \times 12)$$

$$,, = 9,072 + 150,000 = 159,072 \text{ inch-lbs.}$$

$$\text{Here,} \quad I = \frac{1}{12} \times 10 \times 15^3 = \frac{1}{2} \times 5 \times 5 \times 15 \times 15$$

$$\text{And,} \quad y = \frac{1}{2} \times 15 \text{ inches.}$$

$$\therefore \quad f = \frac{\text{B.M.}}{I} y = \frac{159,072}{\frac{1}{2} \times 5 \times 5 \times 15 \times 15} \times \frac{1}{2} \times 15$$

$$,, = 424.1 \text{ lbs. per sq. inch.}$$

EXAMPLE III.—A round steel spindle 10 inches long, and held at one end, revolves at the rate of 150 revolutions per minute round a vertical axis, to which the axis of the spindle is parallel

and from which it is 2 feet distant. The spindle has a uniformly distributed load, the whole revolving weight being 30 lbs. What should be the diameter of the spindle when the safe working stress of the material in tension or compression is taken at 25,000 lbs. per square inch? (S. & A. Hons. Exam., 1891.)

ANSWER.—The spindle in this problem may be likened to a beam fixed at one end and carrying a uniformly distributed load. The load being not the revolving weight of 30 lbs., but the centrifugal force of that weight due to its being whirled round at the rate of 150 revolutions per minute.

$$\text{Velocity of spindle,} = \frac{150 \times 2\pi \times 2}{60} = 10\pi \text{ ft. per sec.}$$

$$\text{Centrifugal force,} = \frac{30 \times (10 \times \pi)^2}{32 \times 2} = \frac{1500 \times \pi^2}{32} \text{ lbs.}$$

This force, multiplied by half the length of the spindle, gives us the bending moment at the fixed end of the spindle :—

$$\text{That is,} \quad \text{B.M.} = \frac{1500 \times \pi^2}{32} \times \frac{10}{2} \text{ inch-lbs.}$$

If D be the diameter of the spindle in inches, then from Lecture XII., Vol. I., we get :—

$$\text{The Moment of Inertia, } I = \frac{\pi}{64} D^4$$

$$\text{And the Modulus of Section, } Z = \frac{I}{\frac{D}{2}} = \frac{\pi}{32} D^3.$$

Now, $fZ = \text{B.M.}$; and, $f = 25,000$ lbs. per sq. inch.

$$\therefore 25,000 \times \frac{\pi}{32} D^3 = \frac{1500 \pi^2}{32} \times \frac{10}{2}$$

$$\text{Or,} \quad D^3 = 0.3 \times \pi = 0.94248$$

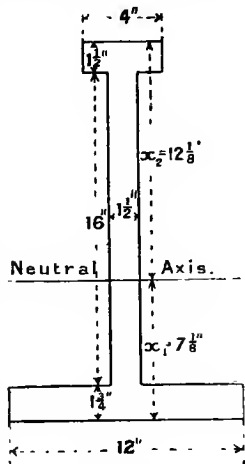
$$\text{Hence,} \quad D = \sqrt[3]{0.94248} = 0.98 \text{ inch.}$$

EXAMPLE IV.—The section of a cast-iron girder, and the maximum safe tensile and compressive stresses being given, explain how to determine its moment of resistance to bending. The dimensions of the section of a cast-iron girder are the

following:—Top flange, 4 by $1\frac{1}{2}$ inches; bottom flange, 12 by $1\frac{3}{4}$ inches; web, 16 by $1\frac{1}{2}$ inches. Determine the moment of resistance, the greatest permissible tensile and compressive stresses being $2\frac{1}{2}$ and $7\frac{1}{2}$ tons per square inch respectively. If the girder be 20 feet long, and is supported at its two ends, find the greatest safe load which it will carry when uniformly distributed along its length. (S. and A. Hons. Exam., 1895.)

ANSWER.—As this is an excellent example for showing the student how the R.M. of a girder section is calculated, we shall go into the matter in detail. Let the accompanying figure represent the cross-section in question.

We have first to find the position of the neutral axis N A, by writing down the sectional areas of the parts composing the figure, and taking moments about the lower edge.



SECTION OF GIRDER.

$$\text{Area of top flange} = 4 \times 1\frac{1}{2} = 6 \text{ sq. in.}$$

$$,, \text{ bottom } ,, = 12 \times 1\frac{3}{4} = 21 \text{ ,,}$$

$$,, \text{ web } = 16 \times 1\frac{1}{2} = 24 \text{ ,,}$$

$$\therefore \text{Total area of section} = 51 \text{ ,,}$$

Then, since N A passes through the centre of area of the section, we have:—

$$51 \times x_1 = 6 \times 18\frac{1}{2} + 24 \times 9\frac{3}{4} + 21 \times \frac{7}{8} = 363\frac{3}{8}.$$

$$\therefore x_1 = \frac{363\frac{3}{8}}{51} = 7\frac{1}{8}.$$

$$\text{And } x_2 = 19\frac{1}{4} - 7\frac{1}{8} = 12\frac{1}{8}.$$

We calculate the value of I, the moment of inertia of the section about the neutral axis, by finding that for each of the parts into which the section is divided and taking their sum. As the neutral axis does not pass through the centre of any of

those parts, we shall have to employ Prop. I. of Lecture XII., Vol. I., to which we again refer the student.

Remembering that the moment of inertia of a rectangular area about an axis through its centre of gravity is:—

$\frac{1}{12} (\text{breadth}) \times (\text{depth})^3$, we have:—

$$\text{For top flange, } I_t = \frac{1}{12} \times 4 \times (1\frac{1}{2})^3 + 6 \times (11\frac{3}{8})^2.$$

$$\text{,, ,, ,,} = 1.125 + 776.343 = 777.468.$$

$$\text{For bottom flange, } I_b = \frac{1}{12} \times 12 \times (1\frac{3}{4})^3 + 21 \times (6\frac{1}{4})^2.$$

$$\text{,, ,, ,,} = 5.359 + 820.312 = 825.671.$$

$$\text{For web, } I_w = \frac{1}{12} \times 1\frac{1}{2} \times (16)^3 + 24 \times (2\frac{5}{8})^2.$$

$$\text{,, ,,} = 512.0 + 165.375 = 677.375.$$

$$\therefore \text{ For whole section, } I = 777.468 + 825.671 + 677.375 = 2280.5.*$$

To illustrate what we said about the moduli of unsymmetrical sections, we shall find both moduli for this example:—

$$\text{For tension, } Z_t = \frac{I}{x_1} = \frac{2280.5}{7.125} = 320.0.$$

$$\text{For compression, } Z_c = \frac{I}{x_2} = \frac{2280.5}{12.125} = 188.0.$$

The question gives as the greatest permissible values for:—

$$\text{Tensile stress, } f_t = 2.5 \text{ tons per sq. inch.}$$

$$\text{Compressive stress, } f_c = 7.5 \text{ ,, ,,}$$

Since, $R.M. = Z f_{max.}$, we must take the lower of the two values of R.M. in fixing the load to be carried by the girder. These are:—

$$Z_t \times f_t = 320 \times 2.5 = 800 \text{ inch-tons.}$$

$$\text{And, } Z_c \times f_c = 188 \times 7.5 = 1410 \text{ inch-tons.}$$

$$\therefore \text{ B.M. = R.M. = 800 inch-tons.}$$

* Another and rather shorter method of finding I for this form of section is to (1) produce the sides of top and bottom flanges to meet the neutral axis $N A$, (2) calculate the moments of inertia of the two full rectangles thus formed, (3) subtract from their sum the moments of inertia of the four rectangular areas which are in excess of the section of the beam. All these moments may be found by the formula $I = \frac{1}{12} B \times D^3$, which will only require to be used four times as the blank rectangles on each side of the web are equal in pairs.

The girder will, therefore, safely carry a uniformly distributed load, given by the equation:—

$$\frac{1}{8} w L^2 = 800.$$

$$\therefore W = w L = \frac{8 \times 800}{20 \times 12} = 26\frac{2}{3} \text{ tons.}$$

This will make the maximum compressive stress

$$f_c \text{ max.} = \frac{800}{188} = 4.255 \text{ tons per sq. inch,}$$

instead of 7.5 as given; showing that the girder is not well designed.

In a properly proportioned section we should have:—

$$Z_t \times f_t = Z_c \times f_c$$

Thin Wrought-Iron Girders.—In the case of wrought-iron girders where the flanges are thin compared with their distance apart, and where the bending resistance of the web is disregarded as a provision against the shearing force acting at the section, the formulæ for the moment of resistance are very simple.

Let A_t = Area of flange in tension.

„ A_c = Area of flange in compression.

„ H = Distance between centres of flanges.

„ f_t = Mean stress in tension flange.

„ f_c = Mean stress in compression flange.

Distance between centre of tension flange and the neutral axis is:—

$$y_t = \left(\frac{A_c}{A_t + A_c} \right) H;$$

and the distance between centre of compression flange and the neutral axis is:—

$$y_c = \left(\frac{A_t}{A_t + A_c} \right) H.$$

The moment of inertia of the flanges, with respect to the neutral axis, is:—

$$I = A_t \left(\frac{A_c}{A_t + A_c} \right)^2 H^2 + A_c \left(\frac{A_t}{A_t + A_c} \right)^2 H^2$$

$$\text{Or, } I = \{A_t \times A_o^2 + A_c \times A_t^2\} \times \left(\frac{H}{A_t + A_c}\right)^2$$

$$\text{Since, } f = \frac{M}{I} y.$$

$$\therefore f_t = \frac{M}{\{A_t \times A_o^2 + A_c \times A_t^2\} \left(\frac{H}{A_t + A_c}\right)^2} \times \left(\frac{A_c}{A_t + A_c}\right) H$$

$$\text{Hence, } \left. \begin{aligned} f_t &= \frac{M}{A_t \times H} \\ \text{Similarly, } f_c &= \frac{M}{A_c \times H} \end{aligned} \right\} \dots \dots \dots \text{(VII)}$$

EXAMPLE V.—A wrought-iron riveted girder of I section has a top flange of 9 square inches in sectional area, and a bottom flange of 8 square inches. The distance between the centres of gravity of the flanges is 12 inches, and the ends of the beam rest on abutments, 16 feet apart. The girder being loaded uniformly with a load equal to 1 ton per lineal foot (including the weight of the beam). What would be the mean stress per square inch on the metal in each flange at the dangerous section? The resistance of the web to bending is neglected. (S. & A. Hons. Exam., 1892.)

ANSWER.—By “dangerous section” is here meant the middle section of the girder, where the maximum bending moment occurs. (See equation (XI_a) of Lecture II., Vol. II.)

$$\text{Max. B.M.} = \frac{1}{8} \left(\frac{1}{12}\right) \times (16 \times 12)^2 = 32 \times 12 \text{ inch-tons.}$$

Hence, mean stress in tension flange,

$$f_t = \frac{32 \times 12}{8 \times 12} = 4 \text{ tons per square inch.}$$

And, mean stress in compression flange,

$$f_c = \frac{32 \times 12}{9 \times 12} = 3.55 \text{ tons per square inch.}$$

Curvature and Deflection of Beams.—When we speak of the curvature or the deflection of a beam we mean that of its neutral surface.

If the beam is fixed at one end, we take the origin of co-ordinates at that end; but if supported or fixed symmetrically at both ends, we take it at the middle.

Let the co-ordinates of the neutral surface curve be denoted, as usual, by x and y , then the deflection of the beam at any distance x , from the origin will be measured by y and the tangent of its inclination to the horizon by $\frac{dy}{dx}$.

The equation of the curve into which the beam is bent will be :—

$$y = \varphi(x),$$

Where $\varphi(x)$ is a function of x to be determined for each particular case.

In treatises on the analytical geometry of plane curves it is shown that the general expression for radius of curvature is :—

$$\rho = \frac{\left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}.$$

Although of great importance, in the theory of beams $\frac{dy}{dx}$ is always such a small fraction, that its square becomes a perfectly negligible quantity in comparison with unity. We may, therefore, safely disregard the value of $\left(\frac{dy}{dx} \right)^2$ in the above formula, and write $\frac{d^2y}{dx^2} = \frac{1}{\rho}$.

But by equation (IV) of this Lecture we know that :—

$$\frac{1}{\rho} = \frac{M}{EI} \quad \therefore \quad \frac{d^2y}{dx^2} = \frac{M}{EI} \quad \dots \dots \dots \text{(VIII)}$$

In what follows we shall assume that the beam or girder is of uniform section so that I is constant, the more general cases where I varies being rather beyond the scope of this treatise.

We shall begin by working out the following example, which will form a good introduction to this rather mathematical part of our subject.

EXAMPLE VI.—Investigate a formula for calculating the amount of deflection of a beam supported at its ends and loaded uniformly. Find the deflection in a beam of timber of uniform rectangular section, 6 inches wide and 12 inches deep, the beam

being supported at its ends in a horizontal position on two walls 12 feet apart. There is to be taken into account a single concentrated load of 4,000 lbs. at the centre, and a uniformly distributed load of 2,500 lbs., the modulus of elasticity being 1,750,000 lbs. per square inch. (S. & A. Hons. Exam., 1891.)

ANSWER.—Taking the middle of the beam as the origin of co-ordinates, we have already proved (see equation (XI) in Lecture II.) that the bending moment at x inches from this point, in the case of a beam L inches between supports, and loaded uniformly with w lbs. per inch-run, is:—

$$\text{B.M.} = \frac{1}{2} w \left(\frac{1}{4} L^2 - x^2 \right) \text{ inch-lbs.}$$

Substituting this in formula (VIII) we get:—

$$\frac{d^2 y}{dx^2} = \frac{w}{2EI} \left(\frac{1}{4} L^2 - x^2 \right).$$

Now, multiplying both sides by dx , and integrating, we have:—

$$\frac{dy}{dx} = \frac{w}{2EI} \int \left(\frac{1}{4} L^2 - x^2 \right) dx.$$

$$\text{Or,} \quad \frac{dy}{dx} = \frac{w}{2EI} \left(\frac{1}{4} L^2 x - \frac{1}{3} x^3 \right).$$

This needs no correction because $\frac{dy}{dx} = 0$, when $x = 0$.

Integrating a second time, we get:—

$$y = \frac{w}{2EI} \int \left(\frac{1}{4} L^2 x - \frac{1}{3} x^3 \right) dx.$$

$$\text{Or,} \quad y = \frac{w}{8EI} \left(\frac{1}{2} L^2 x^2 - \frac{1}{3} x^4 \right). \quad \dots \quad \text{(IX)}$$

This also requires no correction, as x and y vanish together. Now, let Δ_1 denote the deflection of the beam for the distributed load:—

$$\Delta_1 = y, \text{ when } x = \frac{1}{2} L.$$

$$\text{Or,} \quad \Delta_1 = \frac{w}{8EI} \left\{ \frac{1}{2} L^2 \left(\frac{1}{2} L \right)^2 - \frac{1}{3} \left(\frac{1}{2} L \right)^4 \right\}.$$

$$\therefore \quad \Delta_1 = \frac{5wL^4}{384EI} \text{ inches.} \quad \dots \quad \text{(X)}$$

In the case of a beam carrying a single load of W lbs. at its

middle point, the bending moment due to that load at x inches from the middle point (Equation (VII), Lecture II.) is :—

$$\text{B.M.} = \frac{1}{2} W \left(\frac{1}{2} L - x \right) \text{ inch-lbs.}$$

$$\therefore \frac{d^2 y}{dx^2} = \frac{W}{2EI} \left(\frac{1}{2} L - x \right).$$

The first integration of this equation gives :—

$$\frac{dy}{dx} = \frac{W}{2EI} \left(\frac{1}{2} Lx - \frac{1}{2} x^2 \right) = \frac{W}{4EI} (Lx - x^2).$$

And the second integration :—

$$y = \frac{W}{4EI} \left(\frac{1}{2} Lx^2 - \frac{1}{3} x^3 \right). \quad \dots \dots (XI)$$

If Δ_2 be the total deflection in this case, then Δ_2 is the value of y when $x = \frac{1}{2} L$.

$$\therefore \Delta_2 = \frac{W}{4EI} \left\{ \frac{1}{2} L \left(\frac{1}{2} L \right)^2 - \frac{1}{3} \left(\frac{1}{2} L \right)^3 \right\}.$$

$$\text{Or,} \quad \Delta_2 = \frac{WL^3}{48EI} \text{ inches.} \quad \dots \dots (XII)$$

If the beam carry both loads at the same time, as given in the question, then the total deflection due to the two loads will be :—

$$\Delta = \Delta_1 + \Delta_2.$$

$$\therefore \Delta = \frac{5wL^4}{384EI} + \frac{WL^3}{48EI}$$

$$\text{Or,} \quad \Delta = \frac{L^3}{48EI} \left(\frac{5}{8} wL + W \right).$$

The numerical data given are :—

$$L = 12 \times 12 \text{ inches.}$$

$$I = \frac{1}{12} b d^3 = \frac{1}{12} \times 6 \times 12^3 = 6 \times 12^2.$$

$$W = 4,000 \text{ lbs.}$$

$$wL = 2,500.$$

$$\text{And,} \quad E = 1,750,000.$$

$$\therefore \Delta = \frac{(12 \times 12)^3}{48 \times 1,750,000 \times 6 \times 12^2} \left(\frac{5}{8} \times 2,500 + 4,000 \right)$$

$$\Delta = 0.2288 \text{ inches.}$$

Uniform Beam on Three Supports.—Suppose we are given a uniform beam resting on three supports all on the same level, to find the pressure on the middle support.

It is clear that if the middle support were taken away, the weight of the beam would cause it to bend down at the middle [as found above by equation (X)] through a distance

$$\Delta_1 = \frac{5 w L^4}{384 E I} \text{ inches.}$$

We have also seen by equation (XII) that a single concentrated load of W lbs. applied at the middle of the beam would produce an amount of deflection, $\Delta_2 = \frac{W L^3}{48 E I}$ inches.

This gives us the upward deflection caused by the reaction of the central support if we put its value, P , instead of W in the equation.

The total deflection will be zero if all three supports are on the same level.

$$\text{Then,} \quad \frac{P L^3}{48 E I} = \frac{5 w L^4}{384 E I}.$$

$$\text{Or,} \quad P = \frac{5}{8} w L.$$

The pressure on the middle support is thus seen to be $\frac{5}{8}$ of the weight of the beam; whilst the end supports each carry $\frac{3}{16}$ of the weight.

Uniform Beam Fixed at one End and Supported at the other.—If w be the weight of the beam in lbs. per inch-run and L its length in inches, then, we already know that at x inches from the fixed end, the

$$\text{B.M.} = \frac{1}{2} w (L - x)^2 \text{ inch-lbs.}$$

Putting this value of the B.M. in equation (VIII), we get:—

$$\frac{d^2 y}{dx^2} = \frac{w}{2 E I} (L - x)^2 = \frac{w}{2 E I} (L^2 - 2 L x + x^2).$$

$$\therefore \quad \frac{dy}{dx} = \frac{w}{2 E I} \int (L^2 - 2 L x + x^2) dx$$

$$,, = \frac{w}{2 E I} \{L^2 x - L x^2 + \frac{1}{3} x^3\}.$$

$$\text{And,} \quad y = \frac{w}{2 E I} \int \{L^2 x - L x^2 + \frac{1}{3} x^3\} dx$$

$$y = \frac{w}{2 E I} \left\{ \frac{1}{2} L^2 x^2 - \frac{1}{3} L x^3 + \frac{1}{12} x^4 \right\}. \quad \dots \quad (\text{XIII})$$

This last equation gives the droop of the beam at any distance x , from the fixed end. At the free end let Δ_1 be the value of y when $x = L$.

Then,
$$\Delta_1 = \frac{w L^4}{8 E I} \text{ inches.} \quad \dots \quad \text{(XIV)}$$

Let P be the upward pressure in lbs., between the beam and a support placed under its free end. The bending moment, due to P , at x inches from the fixed end is $P(L - x)$ inch-lbs. Hence, the curvature produced by P will be:—

$$\frac{d^2 y}{dx^2} = \frac{P}{E I} (L - x).$$

$$\therefore \frac{dy}{dx} = \frac{P}{E I} \int (L - x) dx = \frac{P}{E I} (Lx - \frac{1}{2} x^2).$$

$$\text{And, } y = \frac{P}{E I} \int (Lx - \frac{1}{2} x^2) dx = \frac{P}{2 E I} (Lx^2 - \frac{1}{6} x^3). \quad \dots \quad \text{(XV)}$$

When $x = L$, let $y = \Delta_2$.

$$\therefore \Delta_2 = \frac{P L^3}{3 E I} \text{ inches.} \quad \dots \quad \text{(XVI)}$$

If $\Delta_2 = \Delta_1$, the supported end will be raised to the same level as the fixed end.

Then,
$$\frac{P L^3}{3 E I} = \frac{w L^4}{8 E I}.$$

Or,
$$P = \frac{3}{8} w L \text{ lbs.}$$

This result shows that the pressure on the prop is equal to $\frac{3}{8}$ of the weight of the beam.

It will be instructive for the student to observe that this result might easily have been inferred from the previous case of a beam resting on three props.

In that case the part of the beam immediately over the middle support is in exactly the same condition as the fixed end of the beam in this case; so that whatever is true of each half of the beam in the former case will here hold good for the whole beam. The pressure on the end supports is, therefore, identical in magnitude in each case; because $\frac{2}{3}$ of the weight of the whole beam is the same thing as $\frac{2}{3}$ of the weight of each half.

Beam Fixed at Both Ends and Loaded at the Centre.—When a beam is fixed, or built horizontally into a wall at both ends, the fixing causes a bending moment which is constant all over the

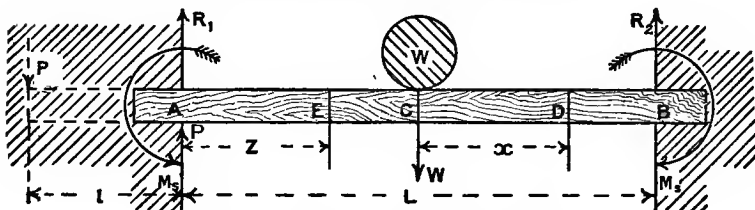
beam. For the reaction of the left support in keeping the beam horizontal is equivalent to a force P , acting downwards at some distance l , to the left of that support, and an upward force P , at the support. The bending moment at the support is then :—

$$M_s = P \times l.$$

And, at any other point, E , of the beam, at a distance, z (less than half the span from the support), the B.M. caused by this reaction at the support is :—

$$\text{B.M.} = P(z + l) - Pz = Pl = M_s.$$

Consequently, the fixing at the ends causes a constant B.M. all over the beam, equal to that at the supports, in addition to that caused by the load (*but in the opposite direction*).



BEAM FIXED AT ENDS AND LOADED AT CENTRE.

Taking our origin of co-ordinates at C , the centre, and the undeflected axis, or neutral line of the beam, as our axis of x , we have, at a section D , distant x from C :—

$$\text{B.M.} = R_2(\tfrac{1}{2}L - x) - M_s = \tfrac{1}{2}W(\tfrac{1}{2}L - x) - M_s$$

Hence, from eqn. (VIII),
$$\left\{ \frac{d^2y}{dx^2} = \frac{1}{EI} \left\{ \tfrac{1}{2}W(\tfrac{1}{2}L - x) - M_s \right\} \right.$$

$$\therefore \frac{dy}{dx} = \frac{1}{EI} \left\{ \tfrac{1}{4}W(Lx - x^2) - M_s x \right\}$$

The beam is horizontal at the centre and at the ends, therefore $\frac{dy}{dx}$ is zero when x is zero, and when $x = \tfrac{1}{2}L$.

$$\therefore 0 = \frac{1}{EI} \left\{ \tfrac{1}{4}W(\tfrac{1}{2}L^2 - \tfrac{1}{4}L^2) - \tfrac{1}{2}M_s L \right\}$$

Or,
$$M_s = \tfrac{1}{8}WL$$

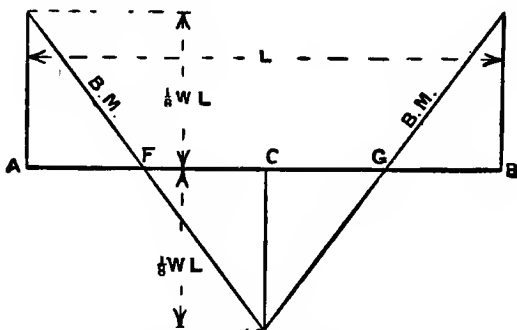
Inserting this value in the above equation for the B.M. we get :—

$$\text{B.M.} = \frac{1}{2} W \left(\frac{1}{2} L - x \right) - \frac{1}{8} W L$$

$$\text{Or,} \quad \text{B.M.} = \frac{1}{2} W \left(\frac{1}{4} L - x \right) \quad \dots \quad (\text{XVII})$$

$$\text{At the centre,} \quad \text{B.M.} = \frac{1}{2} W \cdot \frac{1}{4} L = \frac{1}{8} W L = M_s$$

$$\therefore \quad \text{Maximum B.M.} = M_s = \frac{1}{8} W L \quad \dots \quad (\text{XVIII})$$



B.M. DIAGRAM FOR BEAM FIXED AT ENDS AND LOADED AT CENTRE.

We thus see that, in this case the fixing of the ends reduces the maximum B.M. to half what it would be with free ends, and that this maximum B.M. occurs both at the centre and the ends.

The B.M. diagram is similar to what we had for a beam simply supported, but the base line is shifted half way down the diagram, so that it is crossed at F and G by the lines representing the B.M. It will be seen from equation (XVII) that the B.M. is zero where $x = \frac{1}{4} L$, and that it is positive on one side of this point, and negative on the other. This is one of the points where the B.M. curve cuts the base line, and it is called a *point of inflection*, because the beam is straight just at that point and the curvature changes sign. There is, of course, another point of inflection at the distance $\frac{1}{4} L$ on the other side of the centre.

In large girder bridges that part of the span between the two points of inflection is made separate from the remainder and rests on rollers at these points. This allows freedom of expansion without reducing the strength of the bridge.

Integrating the value of $\frac{dy}{dx}$ we get :—

$$y = \frac{1}{EI} \left\{ \frac{1}{2} W \left(\frac{1}{2} L x^2 - \frac{1}{3} x^3 \right) - \frac{1}{16} W L x^2 \right\}.$$

Therefore, at the ends, where $x = \frac{1}{2} L$:—

$$y = \frac{1}{EI} \left\{ \frac{1}{4} W \left(\frac{L^3}{8} - \frac{L^3}{24} \right) - \frac{1}{18} W \frac{L^3}{4} \right\} = \frac{W L^3}{192 EI}.$$

Hence, the difference of level between the centre and the ends is :—

$$\Delta = \frac{W L^3}{192 EI} \dots \dots \dots (XIX)$$

This is only one-fourth of the deflection when the beam simply rested on its supports (Equation XII), so that the beam is now four times as stiff.

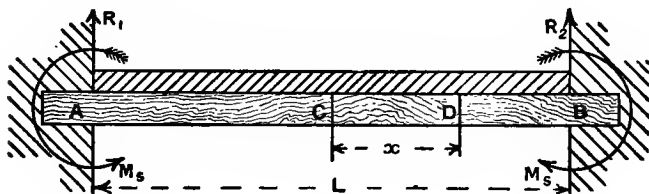
Beam Fixed at Both Ends and Loaded Uniformly.—Taking axes as before, the B.M. at any section, D, is :—

$$\text{B.M.} = R_2 \left(\frac{1}{2} L - x \right) - \frac{1}{2} w \left(\frac{1}{2} L - x \right)^2 - M_s.$$

$$\text{Or, B.M.} = \frac{1}{2} w L \left(\frac{1}{2} L - x \right) - \frac{1}{2} w \left(\frac{1}{4} L^2 - Lx + x^2 \right) - M_s.$$

$$\therefore \frac{d^2 y}{dx^2} = \frac{1}{EI} \left\{ \frac{1}{2} w \left(\frac{1}{4} L^2 - x^2 \right) - M_s \right\}.$$

$$\text{Or, } \frac{dy}{dx} = \frac{1}{EI} \left\{ \frac{1}{2} w \left(\frac{1}{4} L^2 x - \frac{1}{3} x^3 \right) - M_s x \right\}.$$



BEAM FIXED AT BOTH ENDS AND LOADED UNIFORMLY.

In this case also, $\frac{dy}{dx}$ is zero when x is zero, and when $x = \frac{1}{2} L$.

$$\therefore 0 = \frac{1}{2} w L^3 \left(\frac{1}{8} - \frac{1}{24} \right) - \frac{1}{2} M_s L.$$

$$\text{Or, } M_s = \frac{w L^2}{12} = \frac{W L}{12}.$$

$$\text{Hence, B.M.} = \frac{1}{2} w \left(\frac{1}{4} L^2 - x^2 \right) - \frac{1}{12} w L^2.$$

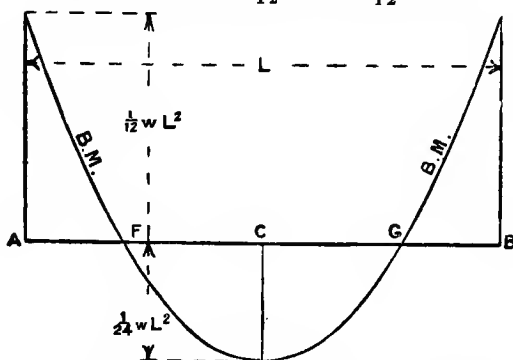
$$\text{Or, B.M.} = \frac{1}{2} w \left(\frac{1}{12} L^2 - x^2 \right) \dots \dots \dots (XX)$$

At the centre, where $x = 0$, the B.M. is :—

$$M_c = \frac{w L^2}{24} = \frac{W L}{24} = \frac{1}{2} M_s.$$

This is only half of that at the support. Hence, the greatest bending moment occurs at the support, and its value is:—

$$\text{Maximum B.M.} = \frac{1}{12} w L^2 = \frac{1}{12} W L. \quad \dots (XXI)$$



B.M. DIAGRAM FOR BEAM FIXED AT BOTH ENDS AND LOADED UNIFORMLY.

The points of inflection occur where $x^2 = \frac{1}{12} L^2$ or $x = \pm \frac{L}{2\sqrt{3}}$.

By integrating the above value of $\frac{dy}{dx}$:—

$$y = \frac{1}{EI} \left\{ \frac{1}{2} w \left(\frac{1}{8} L^2 x^2 - \frac{1}{12} x^4 \right) - \frac{1}{12} w L^2 \frac{x^2}{2} \right\} = \frac{w}{48 EI} (L^2 x^2 - 2 x^4).$$

Putting $x = \frac{L}{2}$ we obtain the amount by which the centre of the beam is deflected by the load, viz.:—

$$\Delta = \frac{w}{48 EI} \left(\frac{L^4}{4} - \frac{L^4}{8} \right) = \frac{w L^4}{384 EI} = \frac{W L^3}{384 EI} \quad \dots (XXII)$$

We thus see that, by fixing the ends horizontally for this manner of loading, the strength of the beam is increased in the ratio 3 : 2, and its stiffness in the ratio 5 : 1.

When the span of the beam is small, it may be designed wholly from considerations of strength; but when the span is great a beam may be strong enough, and yet not suitable, because it yields too much when the load is put on it. It then becomes necessary to take the stiffness into account by using one of the formulæ we have found for the deflection. The greatest deflection usually allowed in beams is 1 inch in 100 feet, or $\frac{1}{1200}$ of the span.

In the tables below we give a summary of these results, showing the relation between them.

TABLE III.—STRENGTH AND STIFFNESS OF BEAMS UNDER A TOTAL LOAD OF W LBS.

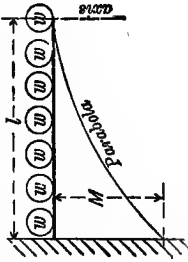
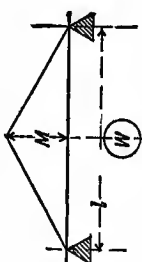
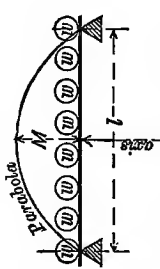
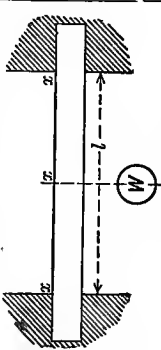
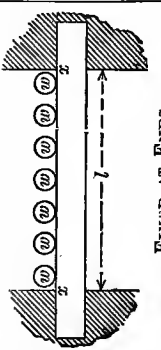
No.	MANNER OF SUPPORTING AND LOADING.	Maximum Bending Moment = M.	Relative Strength.	Deflection in Terms of W.	Deflection in Terms of M.	Deflection in Terms of Stress.	Relative Stiffness under same Load.
I.	 CANTILEVER LOADED AT END.	Wl	$\frac{1}{4}$	$\frac{1}{8} \cdot \frac{Wl^3}{EI}$	$\frac{1}{8} \cdot \frac{Ml^2}{EI}$	$\frac{1}{8} \cdot \frac{f l^2}{Ey}$	$\frac{1}{8}$
II.	 CANTILEVER LOADED UNIFORMLY.	$\frac{Wl}{2}$	$\frac{1}{8}$	$\frac{1}{8} \cdot \frac{Wl^3}{EI}$	$\frac{1}{4} \cdot \frac{Ml^2}{EI}$	$\frac{1}{4} \cdot \frac{f l^2}{Ey}$	$\frac{1}{8}$
III.	 SUPPORTED AT BOTH ENDS. LOADED AT CENTRE.	$\frac{Wl}{4}$	1	$\frac{1}{48} \cdot \frac{Wl^3}{EI}$	$\frac{1}{24} \cdot \frac{Ml^2}{EI}$	$\frac{1}{24} \cdot \frac{f l^2}{Ey}$	1

TABLE III.—STRENGTH AND STIFFNESS OF BEAMS UNDER A TOTAL LOAD OF W LBS. (*continued*).

No.	MANNER OF SUPPORTING AND LOADING.	Maximum Bending Moment = M	Relative Strength.	Deflection in Terms of W.	Deflection in Terms of M.	Deflection in Terms of Stress.	Relative Stiffness under same Load.
IV.	 SUPPORTED AT BOTH ENDS. LOADED UNIFORMLY.	$\frac{Wl}{8}$	2	$\frac{Wl^3}{384 \cdot EI}$	$\frac{Ml^2}{48 \cdot EI}$	$\frac{fl^2}{48 \cdot Ey}$	8
V.	 FIXED AT ENDS. LOADED AT CENTRE.	$\frac{Wl}{8}$	2	$\frac{Wl^3}{163 \cdot EI}$	$\frac{Ml^2}{48 \cdot EI}$	$\frac{fl^2}{48 \cdot Ey}$	4
VI.	 FIXED AT ENDS. LOADED UNIFORMLY.	$\frac{Wl}{12}$	3	$\frac{Wl^3}{163 \cdot EI}$	$\frac{Ml^2}{48 \cdot EI}$	$\frac{fl^2}{48 \cdot Ey}$	8

The quantities in the sixth column are obtained by substituting the value of the maximum B.M. given by the third column in the fifth, and for those in the seventh we have put the value of M (viz., $\frac{fI}{y}$) found in equation (V).

We also print for reference* a table of the strengths of materials and of the moduli of different sections.

* From Seaton & Rounthwaite's *Pocket Book of Marine Engineering Rules and Tables*, which may be consulted for other cases of beams; or Unwin's *Machine Design*, Part I.

Note.—Students who are specially interested in this subject, should procure a copy of the "British Standard Sections," issued by The Engineering Standards Committee, consisting of The Institution of Civil Engineers, Mechanical Engineers, &c. It comprises the following Lists of Standard Sections:—

List 1. Equal Angles.	List 5. Bulb Plates.
„ 2. Unequal Angles.	„ 6. Zed Bars.
„ 3. Bulb Angles.	„ 7. Channels.
„ 4. Bulb Tees.	„ 8. Beams.
List 9. Tee Bars.	

Printed and published by William Clowes & Sons, Ltd., Charing Cross, London, S.W., in February, 1903.

These sections are now universally approved of, and it is impossible, even if I got liberty, to include the whole Report and figures in this Volume.

The following books on "Theory of Structures" and on the "Strength and Elasticity of Materials" may be consulted with advantage when solving the questions to be found in Parts IV. and V., as well as Appendix B of this volume:—

Experimental Engineering, by Rollo C. Carpenter, C.E. (Chapman & Hall, London, 1895.)

Theory of Structures and Strength of Materials, by Henry T. Bovey, M.A., D.C.L. (John Wiley & Sons, New York.)

"The Practical Strength of Beams." Paper by B. Baker. (*Proc. Inst. C. E.*, vol. lxii., p. 251.)

Mechanical Engineering Materials, by E. C. R. Marks. (Technical Publishing Co., Manchester.)

Strength and Properties of Materials, by W. G. Kirkcaldy, London.

Strength and Determination of the Dimensions of Structures of Iron and Steel, by J. J. Weyrauch, translated by A. J. Du Bois. (New York, 1891.)

Strength of Materials and Structures, by Sir J. Anderson. (Longmans, Green & Co., 1892.)

Civil Engineering and Applied Mechanics (latest Editions), by Prof. Rankine. (Chas. Griffin & Co.)

Design of Structures, by S. Anglin, C.E. (Chas. Griffin & Co.)

Bridge Construction, by Prof. Fidler, M.Inst.C.E. (Chas. Griffin & Co.)

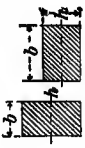
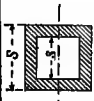
Strength of Materials, by Prof. Ewing, F.R.S., Cambridge.

STRENGTHS, &C., OF MATERIALS (SUMMARY).

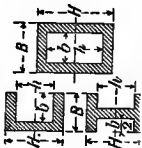
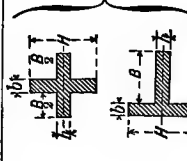
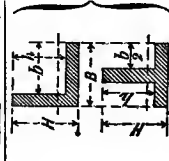
Material.	Ultimate Tensile Strength. lbs. per square inch.	Elastic Strength. lbs. per square inch.	Elongation per cent., when broken by Tensile Stress.
Cast-iron (ordinary good)	18,000	11,000	...
„ (Admiralty), .	{ not less than 20,160 }	...	...
Wrought-iron bars (ordinary good), . .	54,000	29,000	15 % in 8 ins.
Yorkshire plate—			
With grain, . .	54,000	26,000	20 % „
Across „ . .	49,000	...	14 % „
Staffordshire plate—			
With grain, . .	50,000	24,000	12 % „
Across „ . .	41,000	...	8 % „
Iron forgings—			
Large, ...	45,000	...	9 % „
Small, ...	50,000	...	13 % „
Steel castings (ordinary good), ...	67,000	35,000	10 % „
Steel castings (Admiralty) {	{ not less than 63,000 }	...	{ not less than 13½–18½ % in 2 ins.
„ (Lloyd's), . {	{ not exceeding 67,000 }	...	{ not less than 10 % in 8 ins.
Steel boiler plate—			
(Ordinary good), .	65,000	36,000	20 % „
(Admiralty) internal, {	{ not exceeding 60,480 }	...	20 % „
„ shell, .	60,480–67,200	{ not less than 31,360 }	...
(B. of T.) internal, .	58,240–67,200	...	...
„ shell, .	60,480–71,680	...	18 % in 10 ins.
Lloyd's, ...	58,240–67,200	...	{ not less than 20 % in 8 ins. 28 % to 24 % }
Steel forgings (Admiralty)	62,720–78,400	{ 34,500 to 43,120 }	{ 28 % to 24 % in 2 ins. }
Sheet copper, ...	30,000	5,600	35 % in 8 ins.
Copper wire (annealed),	40,000	...	...
Gun-metal (ordinary good), ...	27,000	6,500	10 % in 2 ins.
Gun-metal (Admiralty),	31,000	...	...
Phosphor bronze (cast),	35,000	19,000	12 % in 2 ins.
Manganese bronze „ .	55,000	...	10 % „
„ (rolled),	67,000	...	20 % „
Muntz metal, ...	50,000	30,000	30 % „
Naval brass, ...	54,000	24,000	25 % in 8 ins.

MOMENT OF INERTIA, MODULUS, &C., OF SOME SECTIONS.

The plane of bending is supposed perpendicular to plane of paper, and parallel to side of page.

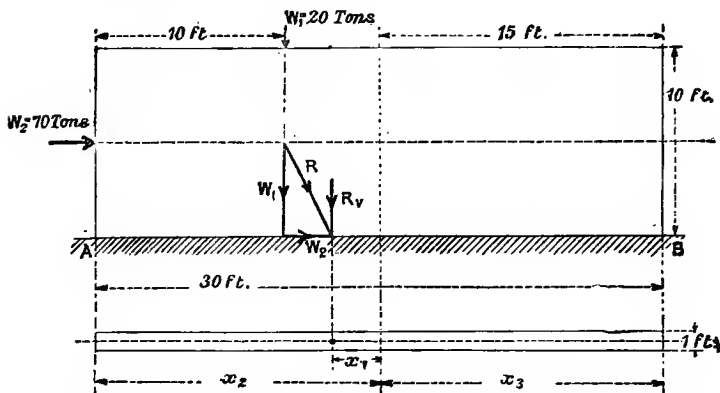
Form of Section.	Area of Section. A	Moment of Inertia of Section about Axis through Centre of Gravity. I	Square of radi. of gyration of Section. $\rho^2 = \frac{I}{A}$	"Modulus" of Section. $Z = \frac{I}{y}$
	b^2	$\frac{bh^3}{12}$	$\frac{h^2}{12}$	$\frac{bh^2}{6}$
	bh	$\frac{bh^3}{12}$	$\frac{h^2}{12}$	$\frac{bh^2}{6}$
	$S^2 - s^2$	$\frac{S^4 - s^4}{12}$	$\frac{S^2 + s^2}{12}$	$\frac{1}{6} \left(\frac{S^3 - s^3}{S} \right)$
	$b(H - h)$	$\frac{b(H^3 - h^3)}{12}$	$\frac{H^2 + hH + h^2}{12}$	$\frac{b(H^3 - h^3)}{6H}$
	$\cdot 7854d^2$	$\cdot 0491d^4$	$\frac{d^2}{16}$	$\cdot 0982d^3$
	$\cdot 7854(D^2 - d^2)$	$\cdot 0491(D^4 - d^4)$	$\frac{D^2 + d^2}{16}$	$\cdot 0982 \left(\frac{D^4 - d^4}{D} \right)$

MOMENT OF INERTIA, MODULUS, &c., OF SOME SECTIONS—Continued.

Form of Section.	Area of Section. A	Moment of Inertia of Section about Axis through Centre of Gravity. I	Square of radi- gyration of Section. $\rho^2 = \frac{I}{A}$	"Modulus" of Section. $Z = \frac{I}{y}$
	$BH - bh$	$\frac{BH^3 - bh^3}{12}$	$\frac{1}{12} (BH^3 - bh^3)$	$\frac{BH^3 - bh^3}{6H}$
	$Bh + \delta H$	$\frac{Bh^3 + \delta H^3}{12}$	...	$\frac{Bh^3 + \delta H^3}{6H}$
	$BH - bh$	$\frac{(BH^3 - bh^3) - 4BHbh(H - h)^2}{12(BH - bh)}$	...	$\frac{(BH^3 - bh^3) - 4BHbh(H - h)^2}{6(BH^2 + bh^2 - 2bHh)}$

EXAMPLE VII.—A rigid body of unit width is rectangular in vertical section, which is 30 feet long and 10 feet high. A force of 20 tons is applied vertically to its upper surface, one-third the length from one vertical face, on the centre of which a normal force of 10 tons acts. If the specific gravity of the block is $2\frac{1}{4}$, find the distribution of pressure on the base. (I.C.E., Feb., 1900.)

ANSWER.—



LINE DIAGRAM OF LOADED RIGID BODY, THE LOAD BEING NON-AXIAL.

In this question we may consider it as made up of two parts:—

(1) The weight of the body itself may be taken as giving a *uniform stress* all over the section or base.

(2) Compounding by the “parallelogram of forces” the two external forces W_1 and W_2 , so as to give a resultant R , which cuts the base at a point $2\frac{1}{2}$ feet from the centre of area. The vertical component R_v of the resultant is therefore the load acting vertically downwards, while the horizontal component tends to produce sliding of the body along the ground.

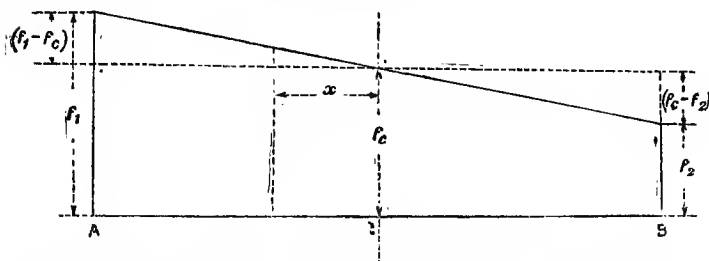
(1) *Uniform Stress*.—Since the weight of the body is acting along the axis of the body, it produces a uniform distribution of stress over the surface of a cross-section; the intensity of stress at all points of the surface is uniform and constant; and the resultant of the stress on the surface acts at a point called the *centre of stress*, which, in this part, coincides with the centre of area.

$$\text{Weight of body} = \frac{1 \times 10 \times 30 \times 62.5 \times 9}{2.240 \times 4} = 18.83 \text{ tons.}$$

$$\therefore \left. \begin{array}{l} \text{Uniform compressive stress} \\ \text{on base due to weight of} \\ \text{body only} \end{array} \right\} = \frac{\text{weight of body}}{\text{cross-sectional area of base}}$$

$$= \frac{18.83}{30} = .628 \text{ ton per sq. ft.}$$

(2) *Uniformly-varying Stress.*—Here the centre of stress of the cross-section does not coincide with the centre of area; then the distribution of stress over the surface is unequal, and it is assumed that the stress is a *uniformly-varying* one—that is, the intensity of stress at any point in the section varies directly as the distance of that point from a fixed line in the plane of the section.



SKETCH SHOWING STRESS ON A SECTION OR JOINT, THE LOAD BEING NON-AXIAL.

- Let A B = Trace of the base on a plane at right angles to it.
 „ C = End of a line through its centre of area.
 „ R_v = Component of the resultant R normal to the surface.
 „ x_1 = Distance of centre of stress from centre of area.
 „ x_2, x_3 = Distances from C of B and A respectively.
 „ A_s = Area of surface or base A B.
 „ f_1, f_2 = Extreme intensities of stress at A and B.
 „ f_c = Intensity of stress at centre of area of A B.
 „ I = Moment of inertia of the surface about an axis through C at right angles to the plane of the figure.

Then the stress represented by the rectangle may be considered as made up of two parts, viz. :—

(a) A uniform stress due to a load equal to R_v acting at the centre of area C, the intensity of which is $f_c = \frac{R_v}{A_s}$; and

(b) A uniformly-varying stress due to a bending moment (B M) = $R_v x_1$.

The intensity of this uniformly-varying stress on any line distant x from O is:—

$$\frac{(B M) x}{I} = \frac{R_v x_1 x}{I}.$$

Adding (a) and (b), we get:—

The intensity of stress at edge A,

$$f_1 = f_c + \frac{R_v x_1 x}{I} = \frac{R_v}{A_s} \left(1 + \frac{A_s x_1 x}{I} \right). \quad . \quad . \quad . \quad (1)$$

The intensity of stress at edge B,

$$f_2 = f_c - \frac{R_v x_1 x}{I} = \frac{R_v}{A_s} \left(1 - \frac{A_s x_1 x}{I} \right). \quad . \quad . \quad . \quad (2)$$

Since the section is symmetrical, $x_2 = x_3$; and equations (1) and (2) become:—

$$f_1 = \frac{R_v}{A_s} \left(1 + \frac{A_s x_1 x_2}{I} \right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$f_2 = \frac{R_v}{A_s} \left(1 - \frac{A_s x_1 x_2}{I} \right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In this question:—

$$A_s = 30 \text{ square feet,}$$

$$I = \frac{b d^3}{12} = \frac{1 \times (30)^3}{12} = \frac{27,000}{12} = 2,250 \text{ (feet)}^4 \text{ units,}$$

$$x_1 = 2\frac{1}{2} \text{ feet,} \quad x_2 = 15 \text{ feet.}$$

$$\therefore f_1 = \frac{R_v}{A_s} \left(1 + \frac{A_s x_1 x_2}{I} \right),$$

$$f_1 = \frac{20}{30} \left(1 + \frac{30 \times 5 \times 15}{2 \times 2,250} \right)$$

$$f_1 = \frac{2}{3} \left(1 + \frac{1}{2} \right) = 1 \text{ ton per square foot (compressive).}$$

$$f_2 = \frac{2}{3} \left(1 - \frac{1}{2} \right) = \frac{1}{3} \text{ ton per square foot (compressive).}$$

To obtain the maximum values of the intensities of the stress at the ends A and B, we must add the value of the uniform compressive stress due to the weight of the *body only*, to *each* of the above values, f_1 and f_2 .

$\therefore$ Maximum compressive stress at A

$$= 1 + 0.628 = 1.628 \text{ tons per square foot.}$$

Minimum compressive stress at B

$$= 0.3 + 0.628 = 0.961 \text{ ton per square foot,}$$

LECTURE III.—QUESTIONS.

1. A wrought-iron flanged girder is required to support a travelling load of 50 tons, the distance between the supports being 40 feet. The stress comes upon the girder at two points, the wheels on the traveller being 10 feet apart. What section of girder will be required to afford the necessary strength, presuming that the ultimate strength of the girder is six times that of the greatest stress to which it will be subjected?

2. Prove the law which governs the transverse strength of a beam of timber when supported at both ends and loaded at the centre. How are the constants required for applying this law arrived at?

3. A bar of wood, 7 feet long and 2 inches square, is supported at both ends, and is broken by a weight of 500 lbs. suspended at the centre. What weight in pounds will a rectangular bar of the same material, supported and loaded in like manner, sustain, when its length is 8 feet, its breadth $2\frac{1}{2}$ inches, and its depth 4 inches? *Ans.* 2187·5 lbs.

4. A rectangular beam of fir, of uniform section throughout, is supported horizontally on two walls 15 feet apart, and has to carry a load of $1\frac{1}{4}$ tons at 5 feet from one of the walls. The width of the beam is 5 inches; find its depth, taking the breaking load at four times the safe load. How much should the depth of the beam be increased, the breadth remaining constant, if the load were shifted from its original position to the centre of the beam, the breaking weight of a beam of fir 15 inches long, 1 inch broad, and 1 inch deep, supported at both ends and loaded in the middle, being taken at 360 lbs.? *Ans.* 8·9 inches; $\frac{1}{4}$ inch.

5. A solid rectangular girder, 3 inches deep and 2 inches broad, is supported at both ends on supports 5 feet apart. It is loaded with a uniformly distributed load, including its own weight, of 10 cwt. per foot run. What is the maximum intensity of stress at the outer fibres?

6. If two cast-iron beams—one circular in section and 2·73 inches in diameter, the other of rectangular section, 3 inches broad and 2 inches deep—be each supported at two points 20 inches apart, and loaded at the centre with a load of 2 tons; what will be the maximum intensity of stress produced in each case?

7. A beam of fir is built into a wall at one end, and projects 6 feet from the wall. The width of the beam is 4 inches; find its depth to bear safely a load of 1,200 lbs. uniformly distributed along its length. Assume that a bar of fir 1 foot long, 1 inch broad, and 1 inch deep, will break under a load of 125 lbs. when fixed at one end and loaded at the other end, and that the safe load is $\frac{1}{3}$ the breaking load. *Ans.* 6 inches.

8. What must be the breadth in inches of an oak cantilever or overhanging beam, 6 feet long and 9 inches deep, in order to carry a load of $\frac{1}{2}$ ton at its extremity, and how much must its breadth be increased in order that it may carry an additional load of $\frac{1}{2}$ ton uniformly distributed over its length? The actual stress is not to exceed $\frac{1}{4}$ of the breaking stress, and the breaking weight of an oak cantilever 6 inches long, 1 inch deep, and 1 inch broad, is 280 lbs. *Ans.* 2·37 inches; 1·18 inches.

9. A beam of fir supported at each end is inclined at an angle of 60° to the horizon, and is loaded at the centre of its length with a weight of 1 ton. The length of the beam is 10 feet, and its breadth is 2 inches, find the depth; the breaking load on the centre of a beam 1 foot long, 1 inch

broad, and 1 inch deep, and supported at the ends in a horizontal position, being 450 lbs. *Ans.* 3.527 inches.

10. A cast-iron cantilever or overhanging beam of T-section is 6 feet long, and 9 inches deep, the top flange being 6 inches wide. The beam has to carry, with safety, at its end a load of 1 ton, together with a distributed load of 1 ton over its length. Find the thickness of the top flange, the tensile breaking strength of cast-iron being 8 tons per square inch, and the admissible load for a safe stress being one-fourth the breaking load. *Ans.* 1 inch.

11. Find the greatest load that may be uniformly distributed on a cast-iron girder having top and bottom flanges united by a web of the following dimensions—width of upper flange 3 inches, of lower flange 9 inches, total depth 12 inches, thickness of each flange and of the web being 1 inch, distance between the points of support 10 feet—when the greatest admissible stress in the compression flange is 3 tons per square inch, and that in the tension flange is $1\frac{1}{2}$ tons per square inch. *Ans.* Maximum compressive stress = 2.5 tons; 8.8 tons.

12. Make a diagram of a flanged cast-iron girder to carry a load of 12 tons in the centre, the distance between the points of support being 20 feet. What should you make the depth of the beam, and what should be the sectional area of the top and bottom flanges respectively?

13. A rolled steel girder has a mean depth of 10 inches, the top and bottom flanges are each 6 inches wide, and the metal in the flanges and webs is $\frac{1}{2}$ inch in thickness throughout. If the breaking strength of the material be taken as 40 tons to the square inch of section for both tension and compression, then (using 4 as a factor for safety) what would be the maximum safe load uniformly distributed over such a girder, supposing it to be supported at each end, the supports being 12 feet apart? Also make a diagram showing the distribution of the shearing stress in the middle transverse section.

14. A rectangular beam of timber is supported at both ends, and loaded by a weight in the centre. Make the necessary calculations for measuring the strength of the beam to resist breaking. For a bar of larch 6 feet long by 2 inches square, supported as above, the breaking weight is 515 lbs.; taking this datum, you are required to solve the following question:—A cistern containing 2 tons of water rests on two cantilevers of larch, each 4 feet long and 5 inches in depth; find the breadth of each cantilever. *Ans.* 1.85 inches.

15. A cast-iron beam of rectangular section, and having its lowest side horizontal, is supported at both ends. What difference should you make in the upper outline according as the load is evenly distributed or collected in the centre?

16. A beam will safely carry a stationary load of 5 tons with a deflection of 2 inches, from what height may a weight of 200 lbs. be let drop upon the same beam without deflecting it to a greater extent? *Ans.* 54 inches.

17. A steady load of 10 tons, suspended at the centre of a beam, deflects it through $\frac{5}{8}$ inch. From what height would a weight of 300 lbs. require to fall in order to produce a like deflection when dropping on the beam? *Ans.* 22.7 inches.

18. A cylindrical iron beam is 15 inches in its external diameter, and the metal is $1\frac{1}{2}$ inches in thickness. The beam is fixed at the two ends, and is 35 feet between the supports; find the greatest load uniformly distributed that the beam will bear, the greatest safe stress on the metal being 9,000 lbs. per square inch.

19. Compare the resistance to bending of a wrought-iron I section beam

when the beam is placed like this I, and like this —. The flanges of the beam are each 6 inches wide and 1 inch thick, and the web is $\frac{1}{2}$ inch thick and measures 8 inches between the flanges.

20. A horizontal bar of round iron, 1 inch diameter, 6 feet long, hinged at the ends, is subjected to equal and opposite pushing forces of 1,000 lbs. at its ends, and a load of 10 lbs. is hung at the middle so that it is both a beam and a strut. Find the greatest stress anywhere. $E = 29 \times 10^6$ lbs. per square inch.

21. Draw the bending moment diagrams, and state the maximum bending moments for the six standard cases of loading and supporting a beam of the same length, same load. (1) Fixed at one end, loaded at the other. (2) Fixed at one end, loaded uniformly. (3) Supported at the ends, loaded in the middle. (4) Supported at the ends, loaded uniformly. (5) Fixed at the ends, loaded in the middle. (6) Fixed at the ends, loaded uniformly.

22. A uniform beam is fixed at its ends, which are 20 ft. apart. A load of 5 tons in the middle; loads of 2 tons each at 5 ft. from the ends. Find the diagram of bending moment and prove your rule. State what the maximum bending moment is, and where are the points of inflexion.

23. A rectangular beam, loaded in the middle, supported at the ends; find the shear stress at any point in any section. Find the deflection at the middle, and distinguish between the parts due to ordinary bending and to shear.

24. What occurs at the cross-section of a horizontal beam, carrying vertical loads? Where is the neutral line? What is the value of the stress at any place? What is meant by *bending moment*? Describe any model which illustrates, however roughly, what occurs at a section of the beam.

25. A symmetrically loaded beam of uniform section; given the diagram of bending moment when supported at the ends, what is the easy rule for finding the diagram when the beam is fixed at the ends? Prove the rule to be correct. (B. of E. H., Part I., 1900.)

26. A beam of timber 2 feet long, 3 inches square, supported at the ends and loaded at the middle, breaks with a load of 7,500 lbs. What load may be expected to break a beam of the same material, fixed at one end and loaded at the other, length 10 feet, breadth 5 inches, depth 9 inches. If the specimen beam deflected 0.034 inch for a load of 1,000 lbs., what would be the deflection of the second beam for a load of 200 lbs.? (B. of E., Adv., 1901.)

27. Suppose the vertical loads and supporting forces of a horizontal beam to be known, show how we find (1) the shearing force at a section, (2) the position of the neutral line, (3) the compressive stress at any part of the section, and (4) the curvature of the beam. Prove your statements. (B. of E. H., Part I., 1901.)

28. What are the functions of the top and bottom booms and of the diagonal pieces of a railway girder? Why are the booms usually larger in section towards the middle of the girder, and the diagonal pieces larger towards the ends of the girder? (B. of E. Adv., 1902.)

29. A rolled joist 12 inches deep, with flanges 4 inches wide and 1 inch thick, and a web $\frac{1}{2}$ inch thick, carries on a 14-foot span a distributed load of 1 ton per foot run, and a single concentrated load in the centre of $2\frac{1}{2}$ tons. Determine:—(a) The maximum bending moment and shearing force; (b) the maximum tensile and compressive stresses per square inch at the centre section; (c) the maximum shearing stress per square inch at a section close to the supports. (C. & G., 1901, H., Sec. A.)

30. Obtain a formula for the maximum deflection in a girder of uniform cross-section, freely supported at the two ends and loaded with a uniformly distributed load. Apply your formula to find how much a cast-iron bar of rectangular cross-section (2 inches deep and 1 inch thick) would deflect in the centre of a 3-foot span under a uniformly distributed load sufficient to produce a maximum stress per square inch of 8 tons. ($E = 12,500,000$ lbs. per square inch.) (C. & G., 1901, H., Sec. A.)

31. If a beam, 12 feet long and 5 inches square section, be just strong enough to carry 1 ton at its centre, what weight may be placed at the centre of a beam of the same material 20 feet long, 6 inches broad, and 11 inches deep? (C. & G., 1902, O., Sec. B.)

32. State precisely the assumptions made in dealing with beams, and obtain an expression for the moment of resistance of a beam of known cross-section in terms of the dimensions of the section and of the stress induced. Explain in what respects the assumptions fail to hold when the stress exceeds the elastic limit. (C. & G., 1902, H., Sec. A.)

33. A cast-iron test bar is 1 inch wide and 2 inches deep, and is tested on 3-feet centres. The elastic deflection per ton load at the centre was found to be .23 inch, and the bar broke when the load was $1\frac{1}{2}$ tons. Calculate the values of the modulus of rupture and of the coefficient of elasticity.

(C. & G., 1902, H., Sec. A.)

N.B.—See Appendices B and C for other questions and answers.

LECTURE III.—A.M. INST. C.E. EXAM. QUESTIONS.

1. Assuming the relation $d^2y/dx^2 = M/EI$, find the slope at the ends and deflection at the centre of a beam of span l supported at the ends and loaded with W at the centre. (I.C.E., Oct., 1897.)

2. Show the relations between the curves of loads, shearing force, and bending moment for a beam. Illustrate your answer by descriptions and sketches of the forms of these curves in the case of a beam supported at each end, and carrying a distributed load, varying in intensity at a uniform rate from zero at one end to w per foot at the other. (I.C.E., Feb., 1898.)

3. A suspension bridge has a span of 800 feet, a dip (or sag) of cord of 60 feet, and carries by means of four cables, a total fixed load of 250 tons uniformly distributed along the length of the platform. Assuming the hanging rods to be very numerous, determine the tension in each cable at the lowest point and at the piers. Assuming the cables to be attached to saddles resting upon rollers on the tops of the piers and the anchor cables to make an angle of 45° with the vertical, determine the maximum stress in the anchor cables and the total pressure on each of the piers.

(I.C.E., Feb., 1898.)

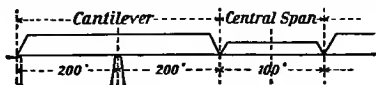
4. Prove the formulæ $\frac{M}{I} = \frac{f}{r}$ and $\frac{M}{IE} = \frac{1}{r}$, with respect to beams, and explain the assumptions on which they are based. (I.C.E., Feb., 1898.)

5. Deduce a formula for the maximum bending moment in a beam built into a wall at one end, supported at the other, and carrying a uniformly distributed load—assuming that when unloaded the beam touches but does not rest upon the outer support. You may assume any formulæ relating to cantilevers. (I.C.E., Feb., 1898.)



6. In a rolled steel beam the section (symmetrical about the neutral axis) is such that the moment of inertia works out at 72 inch-units. The beam is 8 inches deep, and is laid across an opening of 10 feet, and carries a distributed load of 9 tons. Find the maximum fibre stress—also the central deflection, taking E at 13,000 tons. (I.C.E., Oct., 1898.)

7. Suppose that, in the cantilever bridge which is here diagrammatically sketched, the dead weight of the central span is 1 ton per foot, and of the cantilevers 3 tons per foot; and that in each case the load is uniformly distributed. If the rolling load is taken at 2 tons per foot, what disposi-



tion of that rolling load would produce a reverse (sagging) moment in the cantilever? At what part of the bridge would the reverse bending moment take place? How far could it extend? What would be its greatest magnitude? (I.C.E., Oct., 1898.)

8. As between two girder-bridges of the same span, and tested under the same load, what would you expect to be their relative deflections at the centre of the span—(a) When the two bridges are designed with the same sectional area of flange, but with different depths? (b) When the two bridges are designed with different depths of girder, but with the same working stress per square inch of flange section? (In each case assume the girders to have parallel flanges.) (I.C.E., Oct., 1898.)

9. A beam of uniform section is used as a cantilever fixed at one end, A, and covered with a uniform load over the free length, A B. Give a hand sketch of the curve of deflection; prove that the curve is parabolic of the fourth order, and find the intersection of the two tangents drawn to the curve at A and B. (I.C.E., Oct., 1898.)

10. A beam of uniform section is supported at the two ends, 30 feet apart, and is found to bend 6 inches downwards in the middle under its own weight. Determine the slope of the beam at the points of support. Find also what the slope would be if the same deflection of 6 inches were produced by the imposition of a central load instead of a uniform load.

(I.C.E., Oct., 1898.)

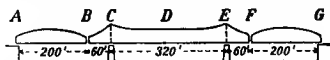
11. A horizontal beam of uniform section, whose moment of inertia is I , and whose total length is $2l$, is supported at the centre, one end being anchored down to a fixed abutment. Neglecting the weight of the beam, suppose it to be loaded at the other end with a single weight, W , and write an expression for the vertical deflection of that end below its unstrained position. (I.C.E., Oct., 1898.)

12. A tee-bar, 6 inches wide by 3 inches in height and $\frac{1}{2}$ inch thick, is used as a beam, with the top table of the T downwards. Find the position of the neutral axis, and the moment of resistance when the maximum compressive stress is 4 tons per square inch. (I.C.E., Feb., 1899.)

13. The chains of a suspension bridge, of 360 feet span, have a dip of 30 feet, and lie in a parabolic curve. Each of the two side spans has a width of 180 feet, the chains forming a parabolic arc similar to one-half of the central span. Under a total uniform load of 2 tons per lineal foot of roadway calculate the vertical and horizontal forces acting upon piers and abutments; also the tensile stress in the chain at the centre and at each end of the main span. (I.C.E., Feb., 1899.)

14. In the suspension bridge above described, suppose the saddles to be fixed to the tops of the towers, or that the roller-bearings refuse to move, and that a load of 2 tons per foot covers the central span, while the load on each side-span is $1\frac{1}{2}$ tons per foot. Find the direction and the magnitude of the resultant force upon the top of each tower, and show how you would trace the line of pressure down the tower. (I.C.E., Feb., 1899.)

15. The cantilever bridge, whose outline is sketched below, is virtually hinged at B and at F. The dead load is $2\frac{1}{2}$ tons per foot throughout the

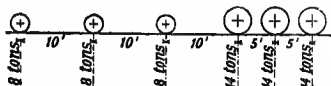


whole length; the live load, 2 tons per foot. Assume the live load to cover all the spans, and find the bending moments at the pier C and at the centre D of the middle span. (I.C.E., Feb., 1899.)

16. In the structure above described, assume next that the live load extends from C to E, and lastly that it covers the two spans A C and E G; and for each of these cases find the bending moments at C and at D.

(I.C.E., Feb., 1899.)

17. Suppose a train to be made up as here sketched, a six-coupled tank-engine of 42 tons weight being followed by a train of wagons whose axles are at uniform distances of 10 feet, and with 8 tons on each axle. In



traversing a girder bridge of 50 feet span, suppose the trailing axle of the engine to have reached the centre of the span; and find the greatest bending moment at any point in the girder due to the rolling load in that position. (I.C.E., *Feb.*, 1899.)

18. When the bridge referred to in the previous question is traversed by the train therein described, find the greatest shearing force that takes effect at the right abutment during the transit, and also the greatest shearing force at the centre of the span. (I.C.E., *Feb.*, 1899.)

19. An iron wire hangs across an opening of 700 feet between the towers of a suspension bridge, with a dip of 50 feet. The length of the wire is therefore, approximately, 709½ feet. Calculate the elongation of the wire under the stress due to its own weight. (Assume the same modulus, and take the mean stress as being very nearly the arithmetical mean between the stresses at the centre and at each end.) (I.C.E., *Feb.*, 1899.)

20. What will be the ratio between the deflection of a girder of uniform I section, and one of uniform strength, both girders being designed for the same span, with the same uniform depth, and with the same maximum working stress per square inch of flange section under a given central load? (I.C.E., *Feb.*, 1899.)

21. What will be the relative deflections of the two girders above referred to if, in another case, they are both designed with the same maximum working stress under a given distributed load?

(I.C.E., *Feb.*, 1899.)

22. A continuous girder, built for crossing two equal spans, has a uniform section whose moment of inertia is I , while its uniform weight per foot lineal is w . The girder is launched across the spans from one end, and when its centre comes nearly over the central pier, the leading end will droop downwards under its unsupported weight. Write the expression for the extreme deflection of the leading end, the length of each span being denoted by L . (I.C.E., *Feb.*, 1899.)

23. Timber beams of a given rectangular section are employed to carry a uniform distributed load over two adjacent spans of equal width with a central support between the two spans. If we decide to make the beams continuous over the two spans, what advantage shall we obtain in respect of (a) strength and (b) rigidity, as compared with two discontinuous beams of the same section? (I.C.E., *Feb.*, 1899.)

24. If the beams referred to in the last question are made continuous, how much of the total load will be borne upon the central support, and how much at the extreme ends of the beams? (I.C.E., *Feb.*, 1899.)

25. Explain the term "neutral axis" used in the theory of the bending of a beam, and find its position for a beam of given section when the limit of elasticity is not overpassed. For example, take a tube of rectangular section 20 feet deep, 10 feet wide, thickness of top side $\frac{1}{2}$ inch, of bottom side $1\frac{1}{2}$ inch, and of the vertical sides $\frac{1}{2}$ inch: find the neutral axis.

(I.C.E., *Oct.*, 1899.)

26. A beam of rectangular section, depth 8 inches, breadth 3 inches, 10 feet long, is supported horizontally at the ends and loaded with a weight

of 5 tons distributed uniformly. Employing the usual theory of transverse flexure: find the maximum stress produced at any point of a transverse section at the centre. (I.C.E., Oct., 1899.)

27. Distinguish between the kinds of elasticity called into play when a bar is (a) stretched, (b) twisted, and (c) bent. Find the deflection of the beam of the last question assuming a modulus of 12,000 in ton-inch units.

(I.C.E., Oct., 1899.)

28. A horizontal beam of uniform section originally straight rests on immovable supports to which it is rigidly fixed, so that its ends remain horizontally when the beam is loaded. Find the shearing force and bending moment at any point due to a uniformly distributed load, and state in what ratio (a) the strength (b) the stiffness is increased by fixing the ends. (I.C.E., Oct., 1899.)

29. Explain what are meant by the "booms" and "web" of a girder, and state the part played by each in supporting the load which it carries. If the booms be parallel and the web consist of a single triangulation of verticals combined with diagonals inclined at 45° , show how the diagonals must be arranged so as to be all in tension and find the stress on each, when the girder is loaded with 2 tons at each of the lower joints. Span = 120 feet. Number of divisions 8. (I.C.E., Feb., 1900.)

30. In the last question find the maximum stress, both tensile and compressive, on each diagonal during the passage of an additional travelling load of 5 tons. (I.C.E., Feb., 1900.)

31. Assuming that the deflection of a beam should not exceed a certain given fraction of the span, show that the least permissible ratio of depth to span can be found. What other considerations determine the ratio of depth to span in practice? (I.C.E., Feb., 1900.)

32. Show that the moment of resistance to bending in beams of a given type is proportional to the sectional area multiplied by the depth. Compare the strength of an I section, of depth 8 inches, breadth of each flange 3 inches, thickness of both flanges and web 1 inch, with that of a rectangular section of the same area and depth. (I.C.E., Feb., 1900.)

33. The span, A C, of an independent girder supported at each end is 120 feet, and the uniform depth from centre to centre of booms is 12 feet. The web consists of a single system of diagonal ties and vertical posts which divide the span into ten equal panels. A uniform dead load of 16 cwts. per foot is assumed to be equally divided between the upper and lower joints. Under this load, calculate the compressive stress in post No. 2, which is 24 feet from abutment, A. (I.C.E., Oct., 1900.)

34. The girder described in the last question carries a deck attached to the upper boom, and when a live load of 25 cwts. per foot (for each girder) extends along the deck from post No. 2 to the abutment, C, what will be the total compressive stress in post No. 2 due to dead and live load?

(I.C.E., Oct., 1900.)

35. In the railway bridge above described, the end-posts are vertical and the girders are supported beneath them upon rocker-bearings at A and roller-bearings at C, while the lower chords are united by wind-bracing. Under ordinary working conditions, what external forces can you discover that would impose stress of any kind upon the first bar of the lower chord at either end of the span? (I.C.E., Oct., 1900.)

36. At the centre of a plate-webbed cross girder the section is made up as follows:—Web-plate 17 inches deep and $\frac{1}{2}$ inch thick; upper and lower flanges each formed of a plate $7\frac{1}{2}$ inches $\times$ $\frac{1}{2}$ inch, united to the web by a pair of angle-bars $3\frac{1}{2}$ inches $\times$ $3\frac{1}{2}$ inches $\times$ $\frac{1}{2}$ inch. Calculate the moment of inertia, I, of the section. (I.C.E., Oct., 1900.)

37. If the bending moment at the centre of this cross girder is 60 foot-tons, what will be the maximum stress in the extreme fibres?

(I.C.E., Oct., 1900.)

38. Prove that the "curvature" of an elastic beam of uniform section is everywhere proportional to the bending moment. (I.C.E., Oct., 1900.)

39. In a girder of 150 feet span, supported at each end, the uniform depth from centre to centre of booms is 15 feet, and the sectional area of each boom is at all points adapted to a uniform working stress of 5 tons per square inch of gross section under the maximum load. What will be the deflection of the girder under the maximum load? (I.C.E., Oct., 1900.)

40. When a beam of uniform section is supported at each end and loaded at the centre, prove that the deflection-curve is a cubic parabola.

(I.C.E., Oct., 1900.)

41. A square bar of steel 3 inches by 3 inches, weighing about 30 lbs. per lineal foot, and 32 feet in length, is laid in a horizontal position and supported at each end. What will be the deflection of the bar under its own weight? Assume $E = 28,000,000$ lbs. (I.C.E., Oct., 1900.)

42. Laying the same bar across a central support so that each arm becomes a cantilever 16 feet long, by how much will the ends droop below the level of the central support? (I.C.E., Oct., 1900.)

43. Taking a similar bar of steel 36 feet in length, and laying it as a continuous beam across three equal openings, AB, BC, CD, of 12 feet each, find the bending moments at the two intermediate supports, B and C, and at the centre of the span, BC. Make also a sketch of the diagram of moments for the three spans. (I.C.E., Oct., 1900.)

44. A beam of cast iron, 1 inch broad and 2 inches deep, is tested upon supports 3 feet apart, and shows a deflection of $\frac{1}{4}$ inch under a central load of 1 ton. Calculate the modulus, E . (I.C.E., Feb., 1901.)

45. A rolled steel beam of uniform section is carried at the ends upon supports 20 feet apart. The moment of inertia of its section amounts to 300 inch-units; and the lower edge of the beam is 6 inches below the neutral axis. When a concentrated load of 8 tons is placed upon the centre, what will be the maximum tensile stress in the extreme fibres?

(I.C.E., Feb., 1901.)

46. In the case of the beam described in Question 45 under the central load of 8 tons, calculate the deflection at the middle of the span, assuming the modulus E to be 13,000 tons per square inch. (I.C.E., Feb., 1901.)

47. For the same beam calculate the deflection at the middle of the span under a uniformly distributed load of 16 tons. (I.C.E., Feb., 1901.)

48. If two precisely similar beams of rectangular section, one of cast-iron and the other of wrought-iron, were laid across the same span and loaded with the same load (within the elastic limit), what would be the relative deflections of the two beams? (I.C.E., Feb., 1901.)

49. A straight bar of steel 40 inches in length, 1 inch broad, and $\frac{1}{16}$ inch in thickness, is bent into the form of a bow, having an elastic deflection of 2 inches in the middle, and the ends are united by the bow-string. Taking the modulus, E , at 29,000,000 lbs., what will be the tension on the string? (I.C.E., Feb., 1901.)

50. A beam of timber, rectangular in transverse section, is 2 inches broad, 3 inches deep, and 4 feet in length, and rests upon supports at its ends. The breaking load at the centre is 2,000 lbs. What would have been the breaking load if the beam had been 4 inches deep, 2 inches broad, and 4 feet between the supports, but loaded at a distance of 1 foot from one end? (I.C.E., Oct., 1901.)

51. A rolled-steel joist 16 inches deep, with flanges 6 inches wide and

1 inch thick (the web being $\frac{3}{8}$ inch thick), is used to support a uniformly distributed load of 2 tons per foot run. If the span is 12 feet 6 inches, what is the maximum tensile stress in the metal of the lower flange?

(I.C.E., Oct., 1901.)

52. In designing a plate girder it is found that the shearing force acting on a particular section is 212 tons. If the mean depth of the girder at that point is 12 feet, find (a) the thickness of the web-plate, (b) the pitch of the rivets uniting the web-plate to the booms. Assume a working shearing-stress of 9,000 lbs. per square inch, and a diameter of $1\frac{1}{2}$ inch for the rivets. (I.C.E., Oct., 1901.)

53. A brick wall 18 inches thick and 30 feet in height has to be carried by a steel plate girder over a span of 30 feet. Assuming that the brick-work weighs 120 lbs. per cubic foot, and that the weight of the floors supported by the wall is equal to an additional distributed load of $\frac{3}{4}$ ton per foot run of the girder, determine suitable cross-sections for the girder at the centre and at the ends. (I.C.E., Feb., 1902.)

54. Find an expression for the maximum deflection of a uniform beam with its ends fixed in a horizontal direction under a uniformly distributed load of W pounds per foot run. If the beam is solid circular section $2\frac{1}{2}$ inches diameter, and the span is 10 feet, find (a) the maximum deflection, (b) the position of the points where there is no bending moment. Assume $E = 29,000,000$ lbs. per square inch. (I.C.E., Feb., 1902.)

55. A floor is supported by wooden joists spaced 12 inches apart from centre to centre. If the joists are 9 inches deep and 3 inches wide, and the clear span is 18 feet, what distributed load per square foot could the floor safely carry if the maximum longitudinal stress in the wood is not to exceed 1,200 lbs. per square inch. (I.C.E., Feb., 1902.)

56. In a test of a cast-iron beam on a span of 30 inches the following deflection results were obtained:—

Load in Centre of Span.	Deflection in Centre.
Lbs.	Inch.
200	0·010
400	0·018
600	0·028
800	0·037
1,000	0·046

Load in centre at rupture 3,800 lbs. If the cross section of the beam at the centre was 1·05 inches in breadth and 2·07 inches in depth, find the modulus of elasticity of the cast iron, and the maximum tensile stress when rupture occurred. (I.C.E., Feb., 1902.)

57. A beam 12 feet long is loaded at distances of 1, 5, and 10 feet from one end with weights of 4, 5, and 6 tons. It is supported at the middle and ends, the middle supporting force being 5 tons. Find the other supporting forces and the greatest bending moment. (I.C.E., Oct., 1902.)

58. In a bridge carrying two lines of railway of standard gauge, the cross-girders are 26 feet long, and the maximum load that can come upon each is 60 tons, distributed between the four rails; design the flanges of an intermediate cross-girder if its depth is 3 feet. (I.C.E., Oct., 1902.)

59. A suspension bridge, 120 feet span, is stiffened by two horizontal stiffening girders hinged at their centre; if the live load is 1 ton per foot run and it covers one-half of the bridge, find the horizontal component of the pull in the wire ropes of the bridge. (I.C.E., Oct., 1902.)

60. A mild steel bar, 5 feet long, 3 inches deep, and 1 inch broad, has a load of $\frac{1}{2}$ ton applied at its centre; what would be the deflection if the

modulus of elasticity is 13,000 tons per square inch? Start with the fact that the second differential coefficient of the deflection is equal to the bending moment divided by the product of the modulus of elasticity and the moment of inertia. (I.C.E., Oct., 1902.)

61. A wrought-iron beam 10 feet long, 6 inches deep, and 2 inches broad, has a weight of 5,000 lbs. dropped on it at its centre; what distance must this weight fall to produce a maximum stress of 5 tons per square inch in the beam? What load applied gradually at the centre would produce the same stress, and what would the deflection be under this load?

(I.C.E., Oct., 1902.)

62. A girder of Γ section, 12 feet long, flanges 4 inches broad by $\frac{3}{4}$ inch thick, depth centre to centre of flanges 6 inches, carries a load of 30 tons at the centre; draw a diagram showing the variation of the maximum normal stress in the flanges from the ends to the centre. What is the total shearing stress between the flanges and web from each end to the centre? (I.C.E., Oct., 1902.)

63. A beam of 80 feet span carries nine cross girders, including one at each end, 10 feet apart, which support a uniform load of 2 tons per foot run; draw the bending-moment diagram for the beam, and show that the ordinate at one-eighth the span is the shearing force to a certain scale in the first one-eighth span; that the difference between the ordinates at the second and first one-eighth of the span is the shearing force to the same scale in the second one-eighth, and so on. Give the scale of the bending-moment diagram, and state to what scale this shearing-force diagram would be drawn. (I.C.E., Oct., 1902.)

64. A cast-iron beam is the shape of an inverted T 9 inches deep overall, width of flange 6 inches, thickness of web and flange 1 inch; if its length is 12 feet, find what weight at the centre will cause a tensile stress of 1 ton per square inch in the flange. What would the maximum compressive stress then be? (I.C.E., Oct., 1902.)

65. A tank, which weighs $\frac{1}{2}$ ton and measures 10 feet $\times$ 6 feet $\times$ 3 feet, is filled with water and carried on three girders placed lengthwise, but so that each girder takes the same weight. If the depth of the girders is 6 inches, find the necessary breadth and thickness of the flanges.

(I.C.E., Feb., 1903.)

66. What is meant by resilience? A timber beam 30 feet long and 12 inches square in cross-section rests on a support at each end; if a load of 1 ton is placed in the centre of the beam, find the work done in deflecting it. (I.C.E., Feb., 1903.)

N.B.—See Appendices B and C for other questions and answers.

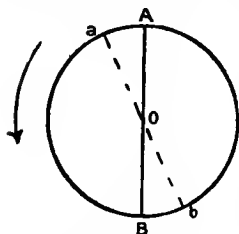
LECTURE IV.

CONTENTS.—Torsional Strength of Shafts—Examples I., II., and III.—Strength of Shafts subjected to Combined Twisting and Bending—Theorem—Examples IV. and V.—Stiffness of Shafts—Angle of Twist—Example VI.—Table of Powers Transmitted by Shafts—Questions.

Torsional Strength of Shafts.—In order to transmit energy through a shaft, the driving force must be applied at some distance from its centre. The driving force and its effective leverage, therefore, constitute what is termed a **Turning or Twisting Moment (T.M.)** which puts the shaft in a state of twist or torsion. The tendency of a purely torsional moment applied to a shaft is to cause the shaft to shear in planes normal to its axis, and this has to be met by the shearing resistance of the material, which resistance must, of course, be of the nature of a moment. The resistance the shaft offers to twisting we term its **Torsional Resistance (T.R.)**; and as this balances the turning moment, we have:—

$$\text{T.M.} = \text{T.R.}$$

We have now to find the value of T.R., as depending on the material and dimensions of the shaft, and shall confine ourselves to shafts of circular section—solid and hollow. Suppose the accompanying figure to represent an end view of a shaft; and suppose AB and ab to have been parallel diameters of two sections very near to each other when the shaft was at rest;



ILLUSTRATING STRAIN
IN A SHAFT.

then, when the shaft is at work transmitting energy, the diameters, AB and ab , will no longer be parallel, but will make an angle with each other, as shown. A longitudinal section, through the axis of a shaft, which is a plane when the shaft is at rest, thus becomes a screw surface when the shaft is working. We shall have occasion later to measure this angle of twist; but in the meantime we are mainly concerned with the distribution of shearing stress within the shaft.

Looking at the figure, we easily see that the *strain* in any ring of fibres must be proportional to the arc of this ring

which is included between the diameters AB and ab , when these are twisted out of parallelism by the turning moment. Within the elastic limit of the material, therefore, it follows that the shearing *stress* in any ring of fibres is proportional to the radius of that ring.

Therefore, let f = the greatest shearing stress, in lbs. per sq. inch, permissible in the material of the shaft.

D = the outside diameter,

d = the inside diameter of the shaft, both in inches.

And x = the radius of *any* ring of fibres within the material of the shaft.

Then the shaft must be so proportioned that f shall be the value of the stress in its outermost fibres which are $\frac{1}{2} D$ inches from the centre. Consequently, from what has already been said, we have :—

$$\text{Stress at } x = \frac{x}{\frac{1}{2} D} f = \frac{2x}{D} f.$$

Consider, now, the ring of fibres at x inches from the shaft centre, whose radial thickness is dx inches. The sectional area of this elementary ring will = $2\pi x dx$ sq. inches; and its resistance to shearing will be

$$2\pi x dx \times \frac{2x}{D} f \text{ lbs.} = \frac{4\pi f}{D} x^2 dx \text{ lbs.}$$

Now, the leverage at which this resisting ring of fibres acts, is x inches; therefore, its *moment* of resistance is $\frac{4\pi f}{D} x^2 dx \times x$, or $\frac{4\pi f}{D} x^3 dx$ inch-lbs.

Hence, summing up the moments of resistance of all such elementary rings which go to make up the shaft, we get:—

$$\begin{aligned} \text{T.R.} &= \frac{4\pi f}{D} \int_{\frac{1}{2}d}^{\frac{1}{2}D} x^3 dx, \\ &= \frac{4\pi f}{D} \left\{ \frac{(\frac{1}{2}D)^4}{4} - \frac{(\frac{1}{2}d)^4}{4} \right\} \\ &= \frac{\pi}{16} f \left(\frac{D^4 - d^4}{D} \right). \end{aligned}$$

Hence, for hollow shafts, we have :—

$$\text{T.R.} = \frac{\pi}{16} \left(\frac{D^4 - d^4}{D} \right) f \quad \dots \dots \dots \quad (\text{I})$$

For solid shafts, we make $d = 0$, and get :—

$$\text{T.R.} = \frac{\pi}{16} D^3 f \quad \dots \dots \dots \quad (\text{II})$$

It is instructive to compare the torsional resistances of solid and hollow shafts of the same weight and material. For this purpose let D_1 be the outer diameter of hollow shaft.

Then, if we neglect couplings, and consider the shafts to be of equal length, the weights will simply be proportional to their sectional areas; i.e. :—

$$\frac{\text{Weight of hollow shaft}}{\text{Weight of solid shaft}} = \frac{D_1^2 - d^2}{D^2}.$$

For equal weights, this ratio is unity; therefore we have the relation :—

$$D^2 = D_1^2 - d^2$$

Or,

$$D = \sqrt{D_1^2 - d^2}.$$

Now, we have from equations (I) and (II) :—

$$\begin{aligned} \frac{\text{T.R. of hollow shaft}}{\text{T.R. of solid shaft}} &= \frac{D_1^4 - d^4}{D_1 \times D^3} = \frac{D_1^2 + d^2}{D_1 \times D} \times \frac{D_1^2 - d^2}{D^2} \\ &= \frac{D_1^2 + d^2}{D_1 \times D} = \frac{D_1^2 + d^2}{D_1 \times \sqrt{D_1^2 - d^2}} \end{aligned}$$

It will simplify matters if we put $d = x \times D_1$, where x is a proper fraction, we then have :—

$$\frac{\text{T.R. of hollow shaft}}{\text{T.R. of solid shaft}} = \frac{1 + x^2}{\sqrt{1 - x^2}}.$$

For example, let $x = \frac{1}{2}$, then :—

$$\frac{1 + x^2}{\sqrt{1 - x^2}} = \frac{1 + \frac{1}{4}}{\sqrt{1 - \frac{1}{4}}} = \frac{5}{2\sqrt{3}} = 1.443.$$

This result shows that for the same length and weight, the hollow shaft having outer and inner diameters in the proportion of 2 to 1 will be 44.3 per cent. stronger than the solid one.

The turning moment driving a shaft may either be uniform or variable in amount. Shafts driven by means of gearing, and revolving at a uniform speed, are generally considered as cases

of uniform turning moment. As a typical example of variable turning moment, we have the case of the steam engine crank-shaft, where both the driving force of the steam on the piston and its effective leverage are continually varying throughout the stroke.

When the turning moment is uniform—that is, when the shaft revolves uniformly at n revolutions per minute, and transmits energy at the rate of so many H.P., this is all the data we require to know in order to estimate T. M. We have already seen (see Vol. I., Lect. III.) that the work done by a turning couple in one minute is equal to the magnitude of the turning couple multiplied by its angular displacement in the same time. Now our turning couple, or turning moment, as we call it, is T.M. inch-lbs., or $\frac{1}{12}$ T.M. foot-lbs., and the angular velocity of our shaft is $n \times 2\pi$ radians per minute.

Therefore, the

$$\text{Work done} = \frac{\text{T.M.}}{12} \times 2\pi n \text{ ft.-lbs. per minute}$$

$$\text{And the H.P.} = \frac{\frac{\text{T.M.}}{12} \times 2\pi n}{33,000} = \frac{n \times \text{T.M.}}{63,025}.$$

$$\therefore \text{T.M.} = 63,025 \cdot \frac{\text{H.P.}}{n}. \quad \dots \dots \dots \text{(III)}$$

EXAMPLE I.—Find the moment of resistance to torsion of a hollow shaft. Compare the strengths to resist torsion of a solid and hollow shaft of the same length and weight, the extreme diameter of the hollow shaft being double its internal diameter. A hollow shaft, the external and internal diameters of which are 20 inches and 8 inches respectively, runs at 70 revolutions per minute, with a surface stress of 6,000 lbs. per square inch; find the twisting moment and the horse-power transmitted. (S. & A. Hons. Exam., 1895.)

ANSWER.—The first two parts of this question have already been answered in the text.

With regard to the last part, we are asked to find the values of T.M. and H.P., being given:—

$$\begin{array}{ll} D_1 = 20 \text{ inches.} & f = 6,000 \text{ lbs. per sq. in.} \\ d = 8 \text{ inches.} & n = 70 \text{ revs. per min.} \end{array}$$

$$\text{Since T.R.} = \text{T.M.,}$$

$$,, = \frac{\pi}{16} \cdot \frac{D_1^4 - d^4}{D_1} \cdot f.$$

$$\therefore \quad \text{T.M.} = \frac{3.1416}{16} \times \frac{20^4 - 8^4}{20} \times 6000.$$

$$,, = 9,183,525 \text{ inch-lbs.}$$

$$\text{and} \quad \text{H.P.} = \frac{\text{T.M.} \times n}{63,025}$$

$$,, = \frac{9,183,525 \times 70}{63,025}$$

$$,, = 10,200.$$

EXAMPLE II.—If a steel shaft revolving at 60 revolutions per minute be required to transmit 220 horse-power, what should be its diameter so that the maximum stress produced in it may not exceed one-fifth of that at the elastic limit? The elastic limit in torsion is 18 tons per sq. inch. Prove any formula you may employ. (S. & A. Hons. Exam., 1894.)

ANSWER.—Combining formulæ (II) and (III) we have :—

$$\text{T.R.} = \text{T.M.},$$

$$\text{i.e.,} \quad \frac{\pi}{16} D^3 f = 63,025 \times \frac{\text{H.P.}}{n}.$$

$$\therefore \quad D = 68.5 \sqrt[3]{\frac{\text{H.P.}}{n f}}. \quad \dots \quad (\text{IV})$$

$$\text{Here, H.P.} = 220. \quad n = 60.$$

$$\text{And,} \quad f = \frac{1}{5} \times 18 \times 2240 = 8064 \text{ lbs. per sq. in.}$$

$$\therefore \quad D = 68.5 \times \sqrt[3]{\frac{220}{60 \times 8064}} = 5.27 \text{ inches.}$$

In cases where the turning moment exerted on a shaft varies, it is, of course, necessary that the shaft should be of strength sufficient to withstand safely the maximum value of T.M. So that in dealing with an example like that of the steam engine crank-shaft we take as the turning force the product of the maximum effective steam pressure on the piston into the piston area; and for the leverage we take the crank radius, although this is not quite accurate; because, if the crank be driven by means of a connecting-rod, the virtual leverage of the steam force at a certain point in the stroke exceeds that of the crank radius by an amount depending on the relative lengths of the crank and connecting-rod.

But on the other hand, the effective steam pressure on the piston is, as a rule, much below its maximum value when the piston reaches the point of greatest leverage. On the whole, therefore, it is quite accurate enough for all practical purposes to estimate the maximum turning moment in the way we have indicated.

Thus, Let p = Greatest effective steam pressure acting on the piston, in lbs. per sq. inch.

„ A = Area of piston, in sq. inches.

„ r = Crank-radius, in inches.

Then, max. T.M. = $p A r$ inch-lbs.

By *effective* steam pressure, we mean the *difference* between the pressures behind, and in front of, the piston.

EXAMPLE III.—Find the diameter of the crank-shaft for a horizontal engine which is to be worked with an effective mean steam pressure of 45 lbs. per square inch throughout the stroke, the diameter of the cylinder being 36 inches, the stroke 5 feet, and the working load being taken at $\frac{1}{8}$ of the breaking load. The shaft is to be of wrought iron, such that a 1-inch shaft will break with the torsion produced by 800 lbs. acting at the end of a 12-inch lever. (S. & A. Hons. Exam.)

ANSWER.—Let f_b be the breaking stress of the experimental shaft, then the working stress in the crank shaft, according to the question, will be $\frac{1}{8} f_b$.

To find the value of f_b we are given that when T.M. = 800×12 inch-lbs., and $D = 1$ ", fracture takes place. From these data, therefore, we deduce:—

$$f_b = \frac{800 \times 12}{\frac{\pi}{16} \times 1^3} = \frac{800 \times 12}{\frac{\pi}{16}} \text{ lbs.}$$

The area of a 36-inch piston = 1017.87 square inches, and r is 30 inches.

$$\therefore \text{Max. T.M.} = 45 \times 1017.87 \times 30 \text{ inch-lbs.}$$

$$\text{Also,} \quad \quad \quad = \frac{\pi}{16} D^3 f.$$

$$\therefore \quad \quad \quad D^3 = \frac{45 \times 1017.87 \times 30}{\frac{\pi}{16} f};$$

$$\text{but,} \quad f = \frac{1}{8} f_B = \frac{800 \times 2}{\frac{\pi}{16}}.$$

$$\text{Hence,} \quad D = \sqrt[3]{\frac{45 \times 1017.87 \times 30}{800 \times 2}},$$

$$,, = 9.5 \text{ inches, nearly.}$$

Strength of Shafts subjected to combined Twisting and Bending.—In Example III. the diameter of the shaft has been calculated as for a purely twisting moment. But in no case of a shaft being driven by a crank is the effect of the load quite so simple as this. Besides the turning moment, which we have already seen how to deal with, there is always in action a *bending* moment of greater or less magnitude depending on the engine arrangement. The worst case is that in which the crank is overhung. When this is so, the bending moment is caused by the load on the piston acting along a line (the centre line of the cylinder) at a certain distance from the shaft bearing nearest to the crank.

Let l = the distance between the centre line of the cylinder and the middle of the nearest shaft bearing, in inches; and
 p and A = (as before) the effective steam pressure and piston area respectively.

Then the magnitude of the bending moment which we have now to take into account is

$$\text{B.M.} = p A l \text{ inch-lbs.}$$

This bending moment is balanced by the moment of resistance of the shaft, which, as will be shown in Lecture III., is—

$$\text{R.M.} = \frac{\pi}{32} D^3 f_t;$$

Where, D = diameter of the shaft journal, in inches,

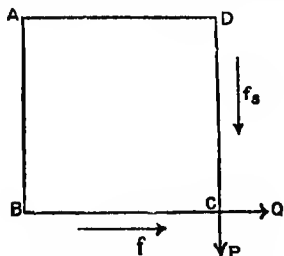
And, f_t = the tensile stress in the outer fibres of the journal, in lbs. per sq. inch.

Hence, we see that when a crank-shaft is being turned by the steam on the piston, it is subjected simultaneously to a shearing stress of intensity f_s , and a tensile stress of intensity f_t . The problem now before us is to combine these stresses so as to

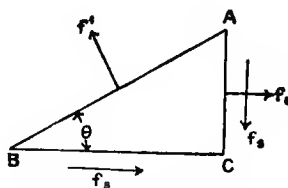
obtain what is termed the *Equivalent* tensile or shearing stress; but in order to render all the steps in the process clear and intelligible, we require to demonstrate the following theorem:—

Theorem.—A shearing stress on any plane produces a shearing stress of equal intensity on planes at right angles to it.

Let A B C D be a rectangular block of material whose thickness is 1 inch perpendicular to the plane of the paper. And let f_s be the intensity of the shearing stress over the face whose edge is



ILLUSTRATING SHEARING STRESS THEOREM.



ILLUSTRATING EQUIVALENT TENSILE STRESS.

C D. It is easy to see that the total shearing force on the face C D which tends to pull that face parallel to itself, must be accompanied by a similar effect on the face B C in order that the block may not be turned around A. To find the relation between those forces, take moments about A, and we get:—

$$P \times A D = Q \times A B.$$

$$\text{Or, } (f_s \cdot C D) \times B C = (f \cdot B C) \times C D.$$

$$\therefore f_s = f.$$

Hence, we see that the shearing stress induced in a shaft by the turning moment is accompanied by a shearing stress of equal intensity on planes at right angles to it; that is, parallel to the axis of the shaft.

In the right-hand figure let A C represent the edge of a small portion of a plane normal to the axis of the shaft, and B C that of another plane at right angles to A C. On the former of these planes there is a shearing stress of intensity f_s , due to the turning moment, and a direct tensile stress of intensity f_t , due to the bending moment acting on the shaft. By the theorem just proved, we also have on B C a shearing stress f_s . Let f' denote the intensity of a tensile stress, which, acting on a

plane A B inclined to A O and B C, would balance the stresses on these latter planes. As before, let the width of the three planes perpendicular to the plane of the paper be unity.

Resolving vertically and horizontally, we have :—

$$(f' \cdot A B) \cos \theta = (f_s \cdot A C),$$

$$\text{and} \quad (f' \cdot A B) \sin \theta = (f_s \cdot B C) + (f_t \cdot A O).$$

From the first of these equations we get :—

$$\frac{f'}{f_s} = \frac{\frac{A C}{A B}}{\cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta, \quad (1)$$

and from the second :—

$$\begin{aligned} f' &= \frac{B C}{\frac{A B}{\sin \theta}} \cdot f_s + \frac{A O}{\sin \theta} \cdot f_t \\ &= \frac{\cos \theta}{\sin \theta} f_s + \frac{\sin \theta}{\sin \theta} f_t \\ &= f_s \cot \theta + f_t \end{aligned}$$

$$\text{Or,} \quad \frac{f' - f_t}{f_s} = \cot \theta. \quad (2)$$

Multiplying together (1) and (2), we get :—

$$\frac{f'}{f_s} \cdot \frac{f' - f_t}{f_s} = 1.$$

$$\therefore f' (f' - f_t) = f_s^2.$$

Which on being solved for f' gives :—

$$f' = \frac{f_t}{2} + \sqrt{\frac{f_t^2}{4} + f_s^2} \quad (V)$$

We take the positive sign in the solution of this quadratic equation, for obviously f' is greater than $\frac{1}{2} f_t$.

Being now in possession of the relation subsisting among the stresses, we next have to express these in terms of the T.M. and B.M. :—

Since
$$\text{T.M.} = \frac{\pi}{16} D^3 f_s$$

And
$$\text{B.M.} = \text{R.M.} = \frac{\pi}{32} D^3 f_t$$

Hence,
$$f_s = \frac{\text{T.M.}}{\frac{\pi}{16} D^3} \dots \dots \dots (3)$$

And
$$f_t = \frac{\text{B.M.}}{\frac{\pi}{32} D^3} \dots \dots \dots (4)$$

In like manner, we must have :—

$$f' = \frac{\text{B.M.}}{\frac{\pi}{32} D^3}$$

where B.M.' stands for the *equivalent* bending moment.

Making these substitutions, and reducing, (V) becomes :—

$$\text{B.M.'} = \frac{1}{2} \{ \text{B.M.} + \sqrt{\text{B.M.}^2 + \text{T.M.}^2} \} \dots \dots (VI)$$

Now, if T.M.' denotes the *equivalent* twisting moment, it easily follows from equations (3) and (4) that for equal intensities of stress we have :—

$$\text{T.M.'} = 2 \text{B.M.'}$$

Hence,
$$\text{T.M.'} = \text{B.M.} + \sqrt{\text{B.M.}^2 + \text{T.M.}^2} \dots \dots (VII)$$

It will be found that equation (VII), giving the so-called equivalent twisting moment, is the one most generally applied. It should be noted, however, that the stress concerned here is a *tensile* one and not a shear as in a proper twisting moment.

EXAMPLE IV.—Investigate an expression in terms of f_s , f' , and f_t , which will give the resultant tensile stress, f' , per square inch of section in a material which is subjected at the same time to a direct tensile stress of f_t lbs. per square inch, and to a shearing stress, f_s lbs. per square inch. A bar of iron is at the same time under a direct tensile stress of 5,000 lbs. per square inch, and to a shearing stress of 3,500 lbs. per square inch. What would be the resultant equivalent tensile stress in the material? (S. & A. Hons. Exams., 1896.)

ANSWER.—The complete investigation referred to in the first part of this question is given in the text, and equation (V,

is the expression required. It only remains to find the numerical value of f' , having given

$$\begin{aligned} f_t &= 5,000 \text{ and } f_s = 3,500 \\ \therefore f' &= \frac{5,000}{2} + \sqrt{\frac{5,000^2}{4} + 3,500^2} \\ „ &= 6,800 \text{ lbs. per sq. in. fully.} \end{aligned}$$

EXAMPLE V.—A wrought-iron shaft is subjected simultaneously to a bending moment of 8,000 inch-lbs., and to a twisting moment of 15,000 inch-lbs. Find the twisting moment equivalent to these two, and the least safe diameter of the shaft. The safe stress against shearing is to be taken at 8,000 lbs. per square inch. Prove clearly the formula you employ. (S. & A. Hons. Exam., 1890.)

ANSWER.—Here we have:—

$$\text{B.M.} = 8,000 \text{ inch-lbs.}$$

$$\text{And T.M.} = 15,000 \quad „$$

Hence, by formula (VII) we get:—

$$\begin{aligned} \text{T.M.'} &= 8,000 + \sqrt{8,000^2 + 15,000^2} \\ „ &= 25,000 \text{ inch-lbs.} \end{aligned}$$

To find the diameter of the shaft to withstand this T.M.' with a shearing stress of not over 8,000 lbs. per square inch, we employ formula (II) making:—

$$\text{T.M.} = \text{T.R.} = \frac{\pi}{16} D^3 f.$$

$$\therefore D = \sqrt[3]{\frac{\text{T.M.}'}{\frac{\pi}{16} \cdot f}} = \sqrt[3]{\frac{25,000}{\frac{3.1416}{16} \times 8,000}} = 2.51 \text{ inches.}$$

Stiffness of Shafts.—Angle of Twist.—We have already seen that the effect of a turning moment applied to a shaft is to twist one part relatively to another. Hitherto we have been dealing only with the resistance the shaft offers to being twisted—that is to say, we have been concerned only with the *strength* of the shaft without regard to the question of *stiffness*. In many cases

—especially in light machinery—the question of the stiffness of the shafting is of greater importance than that of strength.

The stiffness of a shaft is measured by the smallness of the angle of twist per unit length of the shaft.

Turning back to the figure illustrating strain in a shaft, let dl be the axial distance, in inches, between the two sections whose diameters are AB, ab , and let $d\theta$ be the circular measure of the angle between those diameters when the shaft is twisted; then the torsional, or shearing *strain* at the surface of the shaft, is

$$= \left(\frac{D}{2}\right) \times \frac{d\theta}{dl}.$$

D , as before, being the extreme diameter of the shaft in inches,

Let f = Surface stress in the material of the shaft in lbs. per sq. inch.

„ C = Modulus or coefficient of shearing elasticity or of rigidity in lbs. per sq. inch.

Then, since

$$C = \frac{\text{stress}}{\text{strain}} = \frac{f}{\left(\frac{D}{2}\right) \cdot \frac{d\theta}{dl}}.$$

$$\therefore d\theta = \frac{2f}{CD} \cdot dl.$$

Hence, for a shaft L inches long we have, by a simple integration, the angle of twist.

$$\theta = \frac{2f}{CD} \int_0^L dl = \frac{2fL}{CD}.$$

To express this result in terms of the twisting moment and the diameter of the shaft, we have:—

$$f = \frac{T.M.}{\frac{\pi}{16} D^3} \text{ for solid shafts.}$$

$$\text{And, } f = \frac{T.M.}{\frac{\pi}{16} \frac{D^4}{d^4} D} \text{ for hollow shafts.}$$

Making these substitutions and simplifying, we get:—

Angle of twist for solid shafts,

$$\left. \begin{aligned} \theta &= \frac{10.2 \text{ (T.M.) } L}{C D^4} \cdot \text{radians.} \\ \text{Or, } \theta^\circ &= \frac{584 \text{ (T.M.) } L}{C D^4} \cdot \text{degrees.} \end{aligned} \right\} \quad \text{(VIII)}$$

And, for hollow shafts,

$$\left. \begin{aligned} \theta &= \frac{10.2 \text{ (T.M.) } L}{C (D^4 - d^4)} \cdot \text{radians.} \\ \text{Or, } \theta^\circ &= \frac{584 \text{ (T.M.) } L}{C (D^4 - d^4)} \cdot \text{degrees.} \end{aligned} \right\} \quad \text{. . . . (IX)}$$

By the equations just established, we see that, while the strength of shafts vary as the *third* power of their diameters, their stiffness varies as the *fourth* power.

EXAMPLE VI.—Establish a formula for the moment of resistance to torsion of a solid shaft of circular section. The angle of torsion of a shaft is limited to 1° for each 10 feet of length; find the diameter of a solid round shaft to transmit 100 H.P. at 50 revolutions per minute, the modulus of resistance to torsion being 10,000,000 lbs. per sq. inch. (S. & A. Hons. Exam., 1892.)

ANSWER:—

Here, $\theta = 1^\circ$ when, $L = 10 \times 12 = 120$ inches

And, $C = 10,000,000$.

Also, $\text{T.M.} = 63,024 \times \frac{\text{H.P.}}{n} = 63,024 \times \frac{100}{50}$

„ $= 126,048$ inch-lbs.

Now, applying formula (VIII) the given conditions are that:—

$$1^\circ = \frac{584 \times 126,048 \times 120}{10,000,000 \times D^4}$$

Hence, solving for D, we get:—

$$D = \sqrt[4]{\frac{584 \times 126,048 \times 120}{10,000,000}} = 5.45 \text{ inches.}$$

POWER THAT STEEL SHAFTING WILL TRANSMIT AT VARIOUS SPEEDS.

From *The Practical Engineer*, September 2, 1892. By A. G. BROWN, M.E.

DIAMETERS OF SHAFTS IN INCHES.														
Revs. per Minute.	1½	2	2½	3	3½	4	5	6	7	8	9	10		
HORSE-POWERS THEY WILL TRANSMIT.														
50	3.3	5.3	8.0	10.9	15.6	27	43	64	125	216	343	512	729	1000
60	4.0	6.4	9.6	13.1	18.8	32	51	77	150	259	412	614	875	1200
70	4.7	7.5	11.2	15.2	21.9	38	60	89	175	302	480	717	1021	1400
80	5.4	8.5	12.8	17.4	25.0	43	69	102	200	346	549	819	1166	1600
90	6.0	9.6	14.4	19.6	28.1	49	77	115	225	389	617	922	1312	1800
100	6.7	10.7	16.0	21.8	31.2	54	86	128	250	432	686	1024	1458	2000
110	7.4	11.8	17.6	23.9	34.4	59	94	141	275	475	755	1126	1604	2200
120	8.1	12.9	19.2	26.1	37.5	65	103	154	300	518	823	1229	1750	2400
130	8.7	13.9	20.8	28.3	40.6	70	111	166	325	562	892	1331	1895	2600
140	9.4	15.0	22.4	30.5	43.8	76	120	179	350	605	960	1434	2041	2800
150	10.1	16.1	24.0	32.6	46.9	81	129	192	375	648	1029	1536	2187	3000
160	10.8	17.1	25.6	34.8	50.0	86	137	205	400	691	1097	1638	2333	3200
170	11.5	18.2	27.2	37.0	53.1	92	146	218	425	734	1166	1741	2479	3400
180	12.2	19.3	28.8	39.2	56.3	97	154	230	450	778	1235	1843	2624	3600
190	12.8	20.4	30.4	41.3	59.4	103	163	243	475	821	1303	1945	2770	3800
200	13.5	21.4	32.0	43.5	62.5	108	172	256	500	864	1372	2048	2916	4000
225	15.2	24.1	36.6	49.0	70.3	122	193	288	563	972	1543	2304	3280	4500
250	16.9	26.8	40.0	54.4	78.1	135	214	320	625	1080	1715	2560	3645	5000
275	18.6	29.5	44.0	59.8	85.9	149	236	352	688	1188	1886	2816	4009	5500
300	20.3	32.2	48.0	65.3	93.7	162	257	384	750	1296	2058	3072	4374	6000
325	21.9	34.8	52.0	70.7	101.6	176	279	416	813	1404	2229	3328	4739	6500
350	23.6	37.5	56.0	76.2	109.4	189	300	448	875	1512	2401	3584	5103	7000
375	25.3	40.2	60.0	81.6	117.2	202	322	486	938	1620	2572	3840	5468	7500
400	27.0	42.9	64.0	87.0	125.0	216	343	512	1000	1728	2744	4096	5832	8000
425	28.7	45.6	68.0	92.5	132.8	230	364	544	1063	1836	2915	4352	6197	8500
450	30.4	48.2	72.0	97.9	140.6	243	386	576	1125	1944	3087	4608	6562	9000
475	32.1	50.9	71.0	103.4	148.4	257	407	603	1188	2052	3258	4864	6926	9500
500	33.7	53.6	80.0	108.8	156.2	270	429	640	1250	2160	3430	5120	7290	10000

For power of wrought-iron shafts take 70 per cent. of steel shafts of the same size.

LECTURE IV.—QUESTIONS.

1. A 10-inch shaft has a 4-inch hole run through it; what fraction of its weight is removed? To what extent is its strength in resisting torsion affected? *Ans.* 16 per cent.; 2·5 per cent. nearly.

2. A hollow shaft is 10 inches external diameter and 4 inches internal diameter; compare its strength to resist torsion with that of a solid shaft of the same weight.

3. Cylindrical bars of metal, each of 1 inch diameter, are exposed to torsion by weights applied at the end of a 12-inch lever. What would be the probable ultimate strength in the case of good specimens of wrought iron and cast iron. State the law according to which the strength of shafting increases by increasing its diameter.

4. If a wrought-iron shaft of 1 inch diameter is broken by the torsion of a load of 800 lbs. acting at the end of a 12-inch lever, find the weight which, when applied to the end of the same lever, would break a shaft of the same material, but 3 inches in diameter. State, in general terms, the reasoning by which you arrive at the result. *Ans.* 21,600 lbs.

5. If a shaft of 3 inches diameter transmits safely 33 horse-power at 100 revolutions per minute, what size of shaft will transmit safely 20 horse-power at 150 revolutions per minute. *Ans.* 2·22 inches.

6. If 800 lbs. at the end of a 12-inch lever be a safe stress to apply to a wrought-iron bar 1 square inch in section, find the effort which a shaft 2 inches in diameter can transmit at the circumference of a pulley one foot in diameter, and making 300 revolutions per minute. Find also the horse-power transmitted. *Ans.* 8,912 lbs.; 254 H.P.

7. A shaft is of given material and given diameter, find an expression for the moment of resistance to torsion. Given the maximum stress to which the material may be subjected, find the diameter of a shaft which will transmit a given horse-power at a given number of revolutions per minute.

8. A twisting moment of 9,600 inch-pounds is sufficient to break a wrought-iron shaft of 1 inch diameter. Use 6 as a factor of safety, and hence determine what horse-power can be safely transmitted through a shaft of 3 inches diameter when running at 120 revolutions per minute. Prove the formula which you employ.

9. Investigate an expression for the moment of resistance to torsion of a given cylindrical shaft when subjected to a given twisting moment. What is the maximum horse-power which could be transmitted by a shaft 3 inches in diameter when making 150 revolutions per minute, it being given that the shearing stress in the material is not to exceed 7,500 lbs. per square inch? *Ans.* 94·5 H.P.

10. If θ be the angle of twist expressed in circular measure in a length of shafting l , M the twisting moment, C the modulus of transverse elasticity, and d the diameter of the shaft, prove that—

$$\theta = \frac{10 \cdot 2 M l}{C d^4}.$$

11. A horizontal bar of round iron, 1 inch diameter, 6 feet long, hinged at the ends, is subjected to equal and opposite pushing forces of 1,000 lbs. at its ends, and a load of 10 lbs. is hung at the middle so that it is both a beam and a strut. Find the greatest stress anywhere. $E = 29 \times 10^6$ lbs. per square inch.

12. Find the inside and outside diameters of a hollow steel shaft, the internal diameter being $\frac{5}{8}$ of the external diameter. The shaft is to transmit 6,000 H.P. at 116 revolutions. Suppose the maximum twisting moment to be 1.3 times its mean value and the maximum stress allowed in the material to be 10,000 lbs. per square inch. Prove the truth of the formula which you use.

13. A shaft transmits 35 horse-power at 130 revolutions per minute; what is the twisting moment in pound feet? What is the nature of the strain and stress in the shaft? *Ans.* 1,413 lbs. (B. of E. Adv., 1900.)

14. A wire of Siemens' steel, 0.1 inch diameter, is to be twisted till it breaks. Sketch the arrangement; show how the angle of twist and the twisting moment are measured; how the results may be plotted on squared paper, and the sort of results that may be expected. In what way may a wire of twice the diameter be expected to behave? After twisting such a wire much beyond permanent set, suppose the twisting torque to be removed, in what state of internal strain might one expect to find the material? (B. of E. Adv., 1901.)

15. A hollow tube of aluminium bronze, 1 inch diameter inside, $1\frac{1}{4}$ inches diameter outside, is to be twisted till it breaks. How would you arrange the experiment without any special testing machine? Show on squared paper what sort of results you would expect. If the modulus of rigidity of the material is 5.3×10^6 lbs. per square inch, what twisting moment will produce a twist of 0.001 radian per inch? What is now the greatest stress? (B. of E. H., Part I., 1901.)

16. Suppose that a shaft of 1 inch diameter may be safely subjected to a torque of 2,000 pound inches, what torque will a $2\frac{1}{4}$ -inch shaft safely resist? Calculate the horse-power which may be transmitted by the latter shaft if its speed is 150 revolutions per minute. How does the shear stress in a circular shaft, subjected to twisting, depend upon distance from the centre? (B. of E. Adv., 1902.)

17. A steel shaft is to be used to transmit power a distance of 75 feet; the twist on the whole length is not to exceed $22\frac{1}{4}^\circ$, nor the stress to exceed 8,500 lbs. per square inch: what must be the diameter of the shaft, and what H.P. can be passed through it at 135 revolutions a minute? (Modulus of transverse elasticity = 12×10^6 lbs. per square inch.

(C. & G., 1900, H., Sec. A.)

18. Compare the strength of two cylindrical shafts, subjected to pure torsion, if their diameters are $1\frac{1}{2}$ and $2\frac{1}{4}$ inches respectively. Assuming that the cost of such shafting is directly proportional to its weight, what would be the relative costs? (C. & G., 1901, O., Sec. B.)

19. Obtain a formula for the diameter of a shaft, given the twisting moment to which it is exposed, and the maximum stress allowed in the material:—A shaft 15 inches in diameter is exposed to a twisting moment of 227,800 foot-pounds, and to a maximum bending moment of 43 foot-tons: find (a) the maximum stress set up in the material, (b) the twisting moment required to produce the same stress if there were no bending moment acting with it. (C. & G., 1901, H., Sec. A.)

20. The two halves of a flange coupling are fastened together by four round wrought-iron bolts. The diameter of the bolt circle is 10 inches, and the shaft transmits 120 H.P. when making 120 revolutions per minute. Assuming a suitable shear stress for the bolts, find their diameter.

(C. & G. 1902, O., Sec. B.)

21. Find the diameter of the propeller shaft for a ship, each engine of which develops 10,000 H.P. when going at 120 revolutions per minute.

You may assume that the shaft is subjected to pure torsion, and that the safe shear stress of the material is 5 tons per square inch.

(C. & G., 1902, O., Sec. B.)
22. The screw shaft of a high-speed vessel is 6 inches external and 3 inches internal diameter, and rotates 400 times a minute. If the intensity of stress is limited to 5 tons per square inch, find the maximum horsepower that can be safely transmitted, the shaft being supposed subjected to twisting only. If the length of the shaft between the thrust block and the screw be 60 feet, find the angle of torsion of the shaft, the modulus of rigidity being 4,500 tons per square inch. (C. & G., 1902, H., Sec. A.)

23. In an overhanging crank, the crank-arm radius is 16 inches and the distance between the centre of the crank-pin and the centre of the near crank-shaft bearing is 12 inches. When the connecting-rod is at right angles to the crank, the thrust along the rod is 5,000 lbs. Estimate the maximum tensile and shearing stresses in the crank-shaft, the diameter of the crank-shaft being 5 inches. (C. & G., 1902, H., Sec. A.)

N.B.—See *Appendices B and C* for other questions and answers.

LECTURE IV.—A.M. INST. C.E. EXAM. QUESTIONS.

1. The bolts of a cylinder cover are screwed down until they are subject to an initial stress of, say, 1 ton per square inch. Discuss the question of the effect of steam-pressure in the cylinder on the tension in the bolts.

(I.C.E., *Feb.*, 1898.)

2. Find the relative weights of a solid shaft and of a hollow shaft which will transmit a given torque (twisting moment) with the same maximum stress, the external and internal diameters of the hollow shaft being as 3 to 2. Find the ratio of the torsional stiffnesses of the two shafts.

(I.C.E., *Feb.*, 1898.)

3. For a shaft subject to torsion and bending deduce a formula for the simple torque that is equivalent—as regards the maximum stress induced—to the given torque and bending moment combined. (I.C.E., *Feb.*, 1898.)

4. Define the terms:—Efficiency of machine, H.P.-hour, torque, moment of momentum, moment of inertia, radius of gyration. If a torque of 0.245 ton-foot is acting on a shaft which makes 100 revolutions per minute, find the H.P. transmitted. (I.C.E., *Feb.*, 1899.)

5. Show that the resistance to twisting of a shaft varies as the cube of the diameter. Compare the strengths of two shafts of the same diameter, one solid, the other hollow of half the weight. (I.C.E., *Oct.*, 1899.)

6. Find a formula for the diameter of a shaft to transmit N horse-power at n revolutions per minute, and state the circumstances on which the value of the coefficient depends. Obtain a numerical result for 2,700 H.P. at 100 revolutions, using such a value of the coefficient as you consider suitable for some specified case. (I.C.E., *Oct.*, 1899.)

7. When a force is applied to the pin of an overhung crank, show that it is equivalent to a couple tending to turn the crank shaft, a couple tending to bend the shaft at a section between the crank and its journal, and a shearing force at the same section. Find their amounts in terms of P the applied force, r the length of crank, and a the distance of section considered from the point of application of P . (I.C.E., *Feb.*, 1900.)

8. Distinguish between "sheering stress" and "normal stress," and find their values at any point of an oblique section (1) of a compressed block, (2) of a tube subjected to torsion. What is the essential difference of these two cases? (I.C.E., *Feb.*, 1900.)

9. Find the relation between the stress on a twisted shaft and the moment producing it. Find the least diameter of a solid steel shaft to transmit 269 H.P. at 150 revolutions per minute, the stress being limited to 9,000 lbs. per square inch. (I.C.E., *Feb.*, 1901.)

10. A steel shaft has to transmit 25 H.P. at 75 revolutions per minute. Of what diameter would you make this shaft? Choose your own safe working-stress. (I.C.E., *Oct.*, 1901.)

11. If the end of a rod of 1 inch diameter is twisted by the turning effort of a force of 80 lbs. acting at the end of a 12-inch lever, find the force which, when applied to the end of the same lever, would twist equally the end of a rod of the same material but of $1\frac{1}{2}$ inch diameter and of half the length. (I.C.E., *Feb.*, 1902.)

12. Deduce an expression for the amount of twist in a shaft under a given twisting moment, and apply your result to find the angle through which a $2\frac{1}{2}$ inch steel shaft will be twisted if it is 80 feet long and subjected at one end to a twisting moment of 19,000 inch-lbs. (I.C.E., *Feb.*, 1902.)

13. A shaft transmits 120 H.P. at 75 revolutions a minute. The diameter of the circle of the bolt centres of the couplings is $10\frac{1}{2}$ inches. Each coupling has 6 bolts, and the shearing stress per square inch allowed in these bolts is $2\frac{1}{2}$ tons. Find the diameter of these bolts.

(I.C.E., Feb., 1902.)

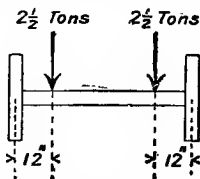
14. Obtain an expression for the work done by a couple. The pitch of a screw propeller is 14 feet and the twisting moment is 120 ton-inches. Find the thrust. (I.C.E., Oct., 1902.)

15. How is the strain caused by shearing stress measured, and what is the modulus of rigidity? If a rod $\frac{1}{2}$ inch in diameter and 20 inches long is fixed at one end and the other end is twisted through an angle of 15° relatively to it, what is the strain in the outer fibres of the rod?

(I.C.E., Oct., 1902.)

16. A bar of iron $\frac{1}{2}$ inch diameter is twisted to destruction; calculate what twisting moment is required for this purpose, assuming that the shearing stress becomes uniform all over the section and equals in the limit 19 tons per square inch. (I.C.E., Oct., 1902.)

17. An axle fixed to two wheels is driven at the centre by a chain and supports two bearings, distant 12 inches from the centre of the wheels, the load on each bearing being $2\frac{1}{2}$ tons. If the power exerted is 20 H.P. and



the rate of turning is 30 revolutions per minute, find the maximum intensity of stress in the axle if its diameter is 5 inches.

(I.C.E., Oct., 1902.)

18. How is the strain measured in the case of a bar in tension or compression; also in the outside fibres of a shaft subject to torsional stress? If the modulus of elasticity is given in the first case, and the modulus of rigidity in the second, how would you find the stresses from the measured strain? (I.C.E., Feb., 1903.)

N.B.—See Appendices B and C for other questions and answers.

LECTURE V.

STRENGTH AND ELASTICITY OF MATERIALS.
TESTING AND TESTING MACHINES.

CONTENTS.—Short History of Mechanical Testing Machines—Advantages of Laboratory Testing to Young Engineers—Mechanical *plus* Chemical and Microscopic Tests of Engineering Materials—Different Mechanical Tests—Necessity for a Careful Prior Investigation of the Kind of Stresses to which the Structure may be Subjected—Investigations of Accidents and Breakages—Prof. Barr's Wire-Testing Machine—Line Diagrams of Three Leading Types of Testing Machines—Thirty-Ton Single-Lever Vertical Testing Machine, with Autographic Recorder—Fifty-Ton Single-Lever Testing Machine—Buckton & Co.'s 300-Ton Universal Testing Machine for Full-sized Structural Members—Equipment of this Machine for Different Kinds of Tests—A Convenient Method of Obtaining and Regulating the Pressure and Flow of Water to the Ram of a Testing Machine—Proportions of Test-Bars and their Attachment in Tensile Tests—Transverse Bar-Testing Machines—Cross Bending, &c.—Admiralty Rules for Testing Materials for Machinery—French Methods of Testing Materials of Construction—Publications Dealing with Engineering Tests and Materials—Questions.

Short History of Mechanical Testing Machines.—Instead of actually loading a specimen direct with dead weights, we may save both time and labour by using a testing machine which measures by some more or less convenient "advantage" or "moment" the stress or load applied to a specimen. The greater the load required to test the specimen, the greater will be the saving between direct loading and the application of the desired stress in an up-to-date accurate testing machine, which is simply an ingeniously devised and refined weighing apparatus.

Naturally, the most primitive form of testing machine was to fix one end of the specimen to a rigid foundation, then to attach the other end of the specimen to the end of the shorter arm of a balancing lever, and put known weights into the scale pan hung from the end of the longer arm until the desired effect was produced on the specimen.

In 1813 an improvement upon this simple arrangement was made and introduced into the Royal Dockyard at Woolwich by the well-known inventor of the hydraulic press, Mr. Bramah of Pimlico, London, for the purpose of testing the chain cables for mooring ships, which were then taking the place of rope hawsers. This machine had a fixed hydraulic cylinder with its ram-head hooked on to one end of the chain to be tested. The other end of the chain was attached to levers with balancing weights, whereby the stresses produced upon the chain by the pressure of water on the ram were indicated.

In 1852 Ludwig Werder, of Nürnberg, made a machine for applying both tension and compression stresses to test specimens.

From April, 1858, to September, 1861, Mr. David Kirkaldy, head draughtsman to Robert Napier & Sons, Engineers and Shipbuilders, Glasgow, carried out a most extensive and careful set of tests and other experiments upon "homo" and "puddled" steels for marine steam boilers and machinery, as well as upon the iron plates and angle irons of H.M.

armour cased ships "Black Prince" and "Hector"; and The Institution of Engineers and Shipbuilders in Scotland awarded him their Prize Gold Medal for his communication "Experiments on Iron and Steel."*

From the beginning of 1862 to June, 1864, Mr. David Kirkaldy devoted two and a half years to the designing, drawing, and patenting of a specially large new form of horizontal testing machine, which he then entrusted to Greenwood & Batley, Leeds, to make under his supervision. This machine was finally set to work at The Grove, 99 Southwark Street, London, on the 1st January, 1866, for the purpose of public and private testing, and is still doing good work to the present day. By this machine, columns of any length up to 21 feet 6 inches by 2 feet by 2 feet 8 inches can be tested up to 270 tons, which gives some idea of its size and capabilities. It was arranged for tension, compression, and deflection tests.† The author had the pleasure of seeing this machine at work in 1873, and of having its construction, action, and uses explained to him by the designer.

There can be no doubt, that to Mr. David Kirkaldy, M.Inst.C.E., &c., belongs the credit of first designing and setting to work an accurate, large testing machine, as well as of carrying out thousands of different reliable experiments with it for various governments, firms, and persons. The more important and larger sizes of testing machines used in many works and laboratories in Europe have been based upon the leading principle of this machine—viz., that of applying the load to the test-piece by water pressure in a hydraulic cylinder and of measuring the load by a weight.

From 1860 to 1870, Herr Wöhler carried out many experiments on iron and steel, and devised several testing machines for subjecting specimens to off-repeated stresses, or fatigue.

In 1872-73, Lord Kelvin's Hydrostatic Testing Machine was first used for testing thousands of miles of the sheathing wires for submarine cables. (See the author's *Manual of Applied Mechanics* for figure and description.)

In 1879, Mr. A. H. Emery built and started a testing machine at the Watertown Arsenal, U.S.A., with a peculiar diaphragm-piston or press cylinder, &c., which could do all the different tests of Kirkaldy's machine, and subject columns 30 feet long by 2 feet 6 inches by 2 feet 6 inches to at least the same thrust, or 357 imperial tons.‡

In August, 1882, Mr. J. H. Wicksteed, of Leeds, described in his paper, read before the Institution of Mechanical Engineers, his special form of Single-Lever Testing Machine; and, in February, 1886, before the same Institution, his "Autographic Test-Recording Apparatus" for use with his vertical or horizontal testing machines.

In 1894, the great Charlottenberg testing machine was installed. It tests up to 492 imperial tons, and takes in columns 49 feet long by 2 feet 7 inches by 2 feet 7 inches.

* See *Trans. Inst. E. and S.*, 1862-63, vol. vi., p. 27, for Mr. Kirkaldy's paper with conclusions and discussion; also vol. vii., p. 134, for an account of the presentation of the medal.

† All students interested in Mechanical Testing and Testing Machines should try and see the large book by W. G. Kirkaldy on *Strength and Properties of Materials with Description of the System of Testing*, published by Sampson Low, Marston, Searle & Rivington, Limited, London, since they will find therein a large quantity of useful data, with illustrations and description of the above-mentioned 1866 Kirkaldy Testing Machine.

‡ See vol. lxxxviii., *Proc. Inst. C.E.*, p. 14, for Prof. Kennedy's paper on "Engineering Laboratories," where further historical and other data will be found.

In 1903, Mr. J. H. Wicksteed, President Inst. M.E., designed, and his firm, Joshua Buckton & Co., Limited, of Leeds, made and installed a 300-ton Universal Testing Machine at the Testing Laboratory of the Conservatoire des Arts et Métiers. This machine is 130 feet in length and 120 tons in weight. It will take in columns 88 feet long by 3 feet 3 inches (each way), and tension members of the same length and diameter.*

Advantages of Laboratory Testing to Young Engineers.—During an ordinary apprenticeship in a works or pupilage in a civil engineer's office, the average young engineer does neither have much opportunity of studying scientifically the physical, chemical, and microscopic (or minute structural) properties of the metals with which he may be brought into contact, nor their strengths and elasticities under different kinds of stresses. Moreover, he has seldom full access to specifications wherein the various tests to be applied to the work in hand are detailed. True, he can attend evening science and technological lectures, or study at home under a system of "tuition by correspondence." But, neither his apprenticeship nor his evening studies, however well performed, will afford him the same comprehensive grasp, or give him the same intimate and useful insight into the properties, behaviour, and efficiencies of engineering materials and machines as a thoroughly good set of day engineering laboratory courses. There, he can be taught not only how to arrange for, but to make by himself, or in conjunction with other students, accurate measurements, tests, and a variety of experiments.

Professor A. B. W. Kennedy was the first person to start, in 1878, at University College, London, such an Engineering Laboratory; and now, most of our Universities and larger Technical Colleges possess imitations of, and even improvements upon, what he initiated.† Consequently, young engineers who can spare the time and the money may attend and be greatly benefited by such practical Laboratory courses.

This slight diversion here on the author's part from his otherwise strict adhesion to explanatory diagrams, calculations, and technical explanations, is partly due to the fact, that he is indebted to Profs. Kennedy and Ewing, as well as to the Institution of Civil Engineers, for permission to reproduce in this lecture a number of figures with explanations of the actual testing machines used by them and their students in their laboratories. A careful study of these descriptions, as well as of the other testing machines for works, with the references to papers and books, will no doubt prepare those who have served an apprenticeship to mechanical engineering to make tests with similar machines. They will also help pupils who have not yet done so, but who intend to enter upon a laboratory course or who have begun the same, to understand the actual machines and the different tests far better than if they had previously never read about them.

* See Mr. J. H. Wicksteed's 1904 B.A. paper on "A Universal Testing Machine of 300 tons for Full-sized Structural Members," and description of this machine in this lecture.

† The author started a combined workshop and laboratory in 1881 at the College of Science and Arts, Glasgow, without having had the privilege of seeing or of having read any account of Professor Kennedy's laboratory. But, due to seven years' continuous mechanical and electrical testing of everything pertaining to submarine cables, these early efforts produced far better results than mere lectures with the aid of a blackboard and chalk.

Mechanical plus Chemical and Microscopic Tests of Engineering Materials.—It is now generally recognised, that prior to the designing and during the manufacture of all important new engineering structures, specimens of the proposed and of the actual materials should be subjected to certain tests and investigations which shall, as far as possible, imitate and prove their fitness to withstand the very stresses and strains, as well as the other affecting influences, to which the different parts of the structure may have to be subjected, during their useful existence.

In many cases, mere mechanical tests will not reveal the complete all-round knowledge of the fitness of a specimen—or even of a complete member—to withstand in the best manner possible, and for the longest time, the various influences which may be brought to bear upon engineering materials during their lifetime. Hence, it is frequently necessary, that these mechanical tests—however skilfully applied and their results accurately measured, tabulated, and considered—should be accompanied by exact chemical analysis, as well as by careful microscopic observations with photo-micrographs.*

Micrographs.—For example, the researches of Professor J. O. Arnold,† at the University College, Sheffield, have revealed, that almost invariably *small* steel castings exhibited in the first stage of their manufacture the Widmannstätten figures, when the carbon in the steel was near the semi-saturation point of steel—viz., 0.45 per cent. His researches involved a close investigation of the influence of mass, and hence the experimental castings varied from 28 lbs. to 2 tons. In the heavier castings the particular Widmannstätten figures already mentioned are seldom to be found, since the slow cooling of the mass exerted an influence similar to that of annealing, which operation causes a change in the structure so profound as almost always to destroy these figures.

The Steel Casting Micrograph Figure.—The structure of the metal from a 30-lb. casting is shown by the upper half-section of the micrograph figure. It exhibited only two of the constituents since the original magnification of 66 diameters was too low to reveal the third and fourth constituents—viz., the sulphides of manganese and of iron—which were present in minute quantities.

The dark etching constituent is pearlite ($2\text{Fe} + \text{Fe}_3\text{C}$). Its colour is due to the liberation during etching of an automatic stain composed of that dark, carbonaceous colouring matter upon which the well-known carbon colour test depends. The pale constituent is ferrite, or nearly pure iron, and has assumed that crystalline structure characteristic of the Widmannstätten figures.

* In order that students may the more thoroughly grasp the importance of carrying out their investigations of metals in a complete manner, they should study "Naval Accidents," by Thomas Andrews, F.R.S., in *Engineering*, p. 737, December 2, 1904, *et seq.*

† I am indebted to Prof. Arnold for kindly presenting me with the original photo-micrographs of 66 diameters, from which my publishers reproduced the accompanying plate (Fig. 1) to half size, or 33 diameters, as well as drawings of the two test-pieces, and to *Nature* of November 10, 1904, p. 32, for the data by Prof. Arnold and Mr. A. M'William.



Fig. 1.—Test Specimens of Cast Steel.
Made by Professor J. O. Arnold and A. M'William at the
University College, Sheffield, 1904.

Chemical Analysis.—The mean analyses of drillings taken from a portion of the casting were as follows:—

Carbon, ...	0.39 per cent.
Silicon, ...	0.08 „
Manganese, ...	0.03 „
Sulphur, ...	0.03 „
Phosphorus, ...	0.02 „
Aluminium, ...	0.03 „
Iron by difference, ...	99.42 „
Total, ...	100.00 „

The Annealed Steel Casting Micrograph.—The lower half-section delineates the very different structure of the cast steel after careful annealing. The result shows a total re-arrangement of the pattern presented by the ferrite and pearlite, and a consequent elimination of the above-mentioned figures.

Mechanical Tests of the Cast and Annealed Steels.—The very complete change in structure due to annealing was also accompanied by a great change in the mechanical properties of the steel. This change will be seen by comparing the results of bending tests made before and after annealing, upon two bars each 10 inches long by $\frac{3}{4}$ inch diameter from the casting. The metal as cast snapped sharply after bending through an angle of 43° over a radius of $\frac{3}{8}$ inch; whereas, the annealed bar bent through an angle of 180° over the same radius without any signs of fracture. The following results were obtained from this steel casting:—

Upper and Lower Halves of Plate, Fig. 1.	Specific Gravity.	Elastic Limit. Tons per Sq. In.	Maximum Stress. Tons per Sq. In.	Elongation on 2 Ins.	Reduced Area.	Compression at 100 Tons per Sq. In.
Casting, .	7.956	13.33	24.62	8.4 %.	12.3 %.	57.3 %.
Annealed, .	7.978	12.21	23.50	14.0 %.	16.0 %.	62.3 %.

This particular steel casting was not intended for commercial purposes; but the results as stated are sufficient to show, that a thorough investigation into the properties of metals for engineering structures should be accompanied by exact chemical analysis and by precise metallographic reproductions of the structures, before and after any special treatment, such as annealing or tempering in water or oil, &c., and even before and after the mechanical tests.*

* Students who are specially interested in the various phases of this subject should study the several reports of "The Alloys Research Committee," as started and maintained by The Institution of Mechanical Engineers, London, from 1891 to 1905. A synopsis of five reports will be found in the General Index to the *Proceedings*, 1885 to 1900, of that Institution, as issued in November, 1904, under the heading "Alloys Research."

Different Mechanical Tests.*—Messrs. Seaton and Jude in their paper on "Impact Tests" remark, that—"At present everybody makes tension tests and trusts more or less to them alone"!

The tension test is no doubt a good and sufficient one when the predominant stress is that of pure static tension or steady load as in boilers, buildings and tanks, &c.; or, where the structural work may be subject to recurrent loads of *one kind (with intervals of rest)*, in addition to the steady load due to its own weight as in the several members of bridges, &c. But, in the case of structures subjected to rapidly-repeated loads of one kind, all more or less suddenly applied, as with bolts, studs, rails, &c.; or, structures subjected to alternating loads, such as in the fixed and moving parts of machinery in general, and many parts of a ship, the simple tension test is not sufficient.

But, the following tests can be made on most pieces of metal and on steel in particular :—

1. Tension and elongation.
2. Compression.
3. Transverse bar tests.
4. Cross bending, or folding, or doubling cold or hot.
5. Shearing, boring, drifting, punching; hammering to a point or edge.
6. Fatigue of metals by gradual reversal of stress either by bending in one plane or by oscillations, as adopted by Wöhler and others.
7. Fatigue of metals by reversals of the mean stress, as carried out by Prof. Osborne Reynolds and J. H. Smith, M.Sc.
8. Impact on unnotched and on notched bars. Drop or falling tests.
9. Torsion.
10. Hardness, temper, and brittleness.

The above-mentioned authors naturally ask the question, "Which of the tests in such a long list is really a true universal gauge of the suitability of (say) a piece of steel for any purpose it may be put to?" And, as a help to answer this question, they give the following interesting table of the different kinds of stresses in the steel parts of an up-to-date (reciprocating) steam engine of moderate size :—

* See the paper read before The Institution of Mechanical Engineers, on November 18, 1904, by A. E. Seaton of London and A. Jude of Birmingham—viz., "Impact Tests on the Wrought Steels of Commerce," and the discussion. Also, the reports and articles on the same in *The Engineer and Engineering* of November 25, 1904, *et. seq.* Students may also refer to Appendices D to M at the end of Mr. A. E. Seaton's *Manual of Marine Engineering*, 15th or later editions, as published by Charles Griffin & Co., for the Admiralty, Board of Trade, Lloyds, British Corporation, and Bureau Veritas Rules for Boiler Shells, Plates, Stays, Shafting, and their respective tests. The British "*Admiralty Rules for Testing Materials for Machinery*," and French "*Methods of Testing Materials of Construction*," will be found at the end of this lecture. Books containing all the rules and regulations of each of these authorities may be obtained by direct application to their secretaries. These books are to be found in the libraries of most engineering institutions.

DIFFERENT KINDS OF STRESSES TO WHICH THE STEEL PARTS OF A RECIPROCATING STEAM ENGINE ARE SUBJECTED AND THEIR PERCENTAGE VALUES.

Constant tension, ...	3.91 per cent.
Constant tension and compression (range from 0 to a max.), ...	1.30 "
Constant tension and shock, ...	48.80 "
Alternating tension and compression with shock, ...	2.81 "
Repeated tension (from a constant to a max.) with shock, ...	36.00 "
Miscellaneous and doubtful, ...	7.17 "
Total, ...	100.00 "

"It will therefore be seen, that 87.6 per cent. of the whole of the engine's stresses are more or less due to shock, whilst pure tension stresses form an insignificant percentage of the total stress." They argue that, if various other machines be examined, it will be found that 9 out of 10 are working under similar conditions.

Necessity for a Careful Prior Investigation of the Kind of Stresses to which Structures may be Subjected.—The above short quotations and abstracts are chiefly made here, to draw the student's attention to this important paper, and to the fact, that it is the duty of the engineer to consider most carefully whilst he is designing any fixed engineering structure or moving machine, what are the various and the chief kinds of stresses to which the structure or machine will be subjected during their existence. Further, to determine and specify clearly how each specimen test-piece for each important member should be tested, so that he may ascertain to the best advantage its suitability for its future duties.

Investigations of Accidents or Breakages of Materials.—When making investigations of accidents or causes which have led to the breakage of materials, the engineer should get or produce—

1. A complete set of scale and sized drawings of the parts in question.
2. Photographs of these parts taken from different points of view, and, if possible, before any alterations have been made.
3. As logical a statement as can be gathered of the conditions under which the breakage or accident occurred.
4. Accurate chemical analysis of the parts near the fracture.
5. Large photo-micrographs of these parts to show the structure of the materials.
6. Carefully machined test-pieces (at least in two right-angled directions) taken from different places near the fracture and elsewhere.
7. Such tests of these machined pieces as will most likely prove their qualities of resisting maximum stresses under fair conditions.
8. He should then draw up a concise, clear, illustrated description of the whole of the circumstances and investigation.

The Order of this Subject.—In this and the following lectures, the order of all the previously mentioned tests will be adhered to as far as possible, beginning with a testing machine for the elongation and breaking stress of wires, and then proceeding to larger machines for making tension, compression and transverse bending tests, &c. Admiralty rules for and French methods of testing, and stress-strain diagram recording apparatus will be illustrated and described with results, as well as how Young's modulus of elasticity is obtained for different cases, and, finally, the strength of struts and columns will be considered.

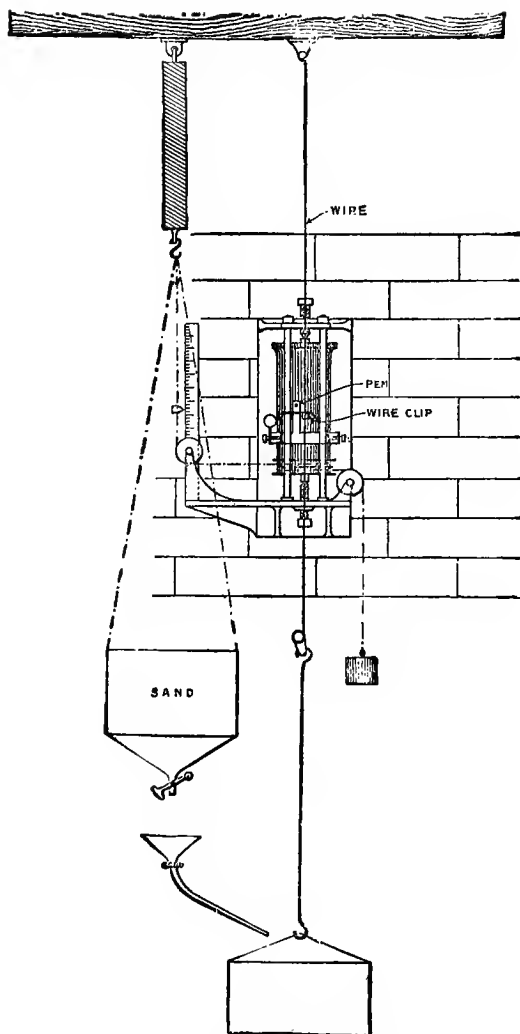


FIG. 2.—PROF. BARR'S WIRE-TESTING MACHINE.

Prof. Barr's Wire-Testing Machine.*—Fig. 2 illustrates clearly the general arrangement and method of performing the test, viz. :—By admitting sand from the sand-box through a filler to the box attached to the end of the test-piece. The weight of sand allowed to pass is registered by the pointer on the graduated vertical scale.

If test-pieces of annealed wires differed from an ordinary specimen, it would only be in their having greater homogeneity. Therefore wire testing has this great advantage, that a large number of practically identical specimens can be obtained and tested. Hence, in testing two wires cut from one coil with the small machine shown by Fig. 2, if the two diagrams are made upon one paper with a fine pen under like conditions, it will be found, that the lines agree so closely for a considerable part of their length as to be undistinguishable from each other.

Line Diagrams of Three Leading Types of Testing Machines.†—In the previous article on the history of testing machines it should have been mentioned, that for small machines up to 5, 10, or even 15 tons stress it is usual to employ screw and gearing instead of the hydraulic ram. This can be done without any loss of accuracy, and for ordinary laboratory work such sizes and method of applying the load are fully as convenient.

The following three diagrams are simply intended to illustrate the principle of each machine to the student and not details or proportions :—

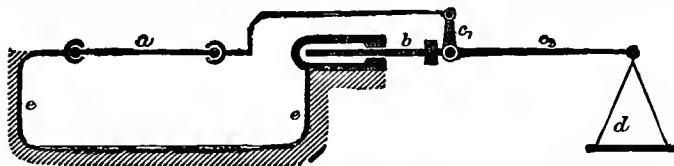


FIG. 3.—LINE DIAGRAM OF A WERDER TESTING MACHINE.

As used by Bauschinger at Munich in 1871.

Werder Machine.—Here the test-piece *a* is held at one end by a grip on the frame of the machine *e*, and the other end is pulled by a connecting-rod from the short arm *c*₁ of a knee-lever, whilst a scale pan *d* hangs from the outer end of the long arm *c*₂ of the lever. The central fulcrum of the lever rests upon the end of the ram *b*, so that the whole of the measuring apparatus moves along coincident with the motion of the ram head and the extension of the test-piece. The arm *c*₂ is kept horizontal by aid of a spirit level.

University College, London, Testing Machine.—This type of machine was designed and made by Greenwood & Batley in 1878 to meet Prof. Kennedy's requirements in his laboratory at University College, London. In principle it is the same as the much larger horizontal machine previously referred to as designed by Mr. Kirkaldy. It consists of a

* This wire-testing machine is used in the Watt Engineering Laboratory at Glasgow University. For a figure and description of Lord Kelvin's hydrostatic wire-testing machine, see the author's *Manual of Applied Mechanics*.

† Figs. 3, 4, 5, 8, 9, and 10 are from Prof. Kennedy's paper on "Engineering Laboratories," by his kind permission, and by the favour of the Institution of Civil Engineers. See *Proc. Inst. C.E.*, vol. lxxxviii.

hydraulic ram b , connected directly to one end of the test-piece a , whose other end is attached to the knee-lever $f_1 f_2$ (5 to 1) and then by a link to a steelyard lever $c_2 c_1$ (20 to 1). The total leverage is therefore 100 to 1.

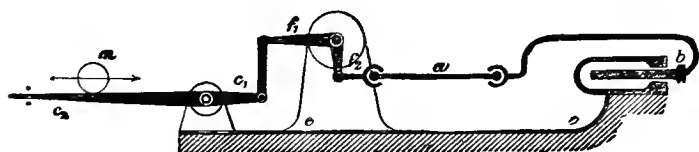


FIG. 4.—LINE DIAGRAM OF THE TESTING MACHINE AT THE UNIVERSITY COLLEGE, LONDON.

As made by Greenwood & Batley, Leeds, in 1878.

The load is applied by the ram b and measured by the position of the poise-weight m on the steelyard. The carriage and its hanger weigh 50 lbs. by themselves, but the poise-weight m is variable by the addition or subtraction of 50 lbs. at a time up to 1,000 lbs.

Wicksteed's Machine.—This vertical machine has a single straight lever $c_2 c_1$ (50 to 1) placed horizontally on the top of the column e of the machine. A movable poise-weight m measures the load applied by the

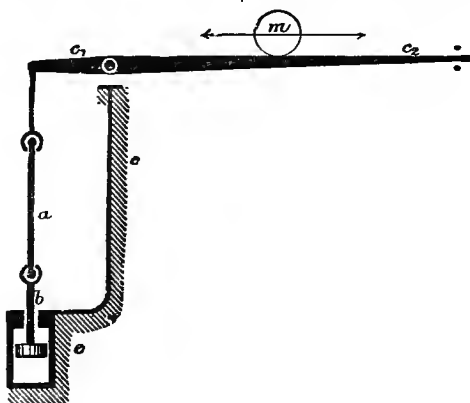


FIG. 5.—LINE DIAGRAM OF A WICKSTEED VERTICAL SINGLE-LEVER TESTING MACHINE. As made by Buckton & Co., Leeds, 1882.

ram b to the test-piece a . Thus, when a certain poise-weight is run out to its stop at the end of the lever c_2 , it balances 50 times the stress put upon the test-piece a by the ram b . The poise-weight m runs on a four-wheel carriage, which is moved along the steelyard lever by a central screw, that is generally turned by belt power in the larger and by hand and a lever in the smaller machines. The pull on the test-piece is read by aid of a vernier attached to the poise-weight, whilst the scale proper is fixed to the steelyard. This type of machine is generally speaking simpler and better appreciated than the two former types.

Thirty-Ton Single-Lever Vertical Testing Machine, with Autographic Recorder.—*Objects to be attained with the Machine.*—The machine illustrated by Fig. 6 is for testing the strength of different materials in tension, bending, and compression. It is made by Greenwood & Batley, Ltd., Leeds, to test up to 30 tons, and to admit test-pieces 3 feet in length for tension, 24 inches long for compression, also beams 14 inches deep by 7 inches wide for bending. The supports are made adjustable from 6 inches to 5 feet apart (see Fig. 5 for the line diagram).

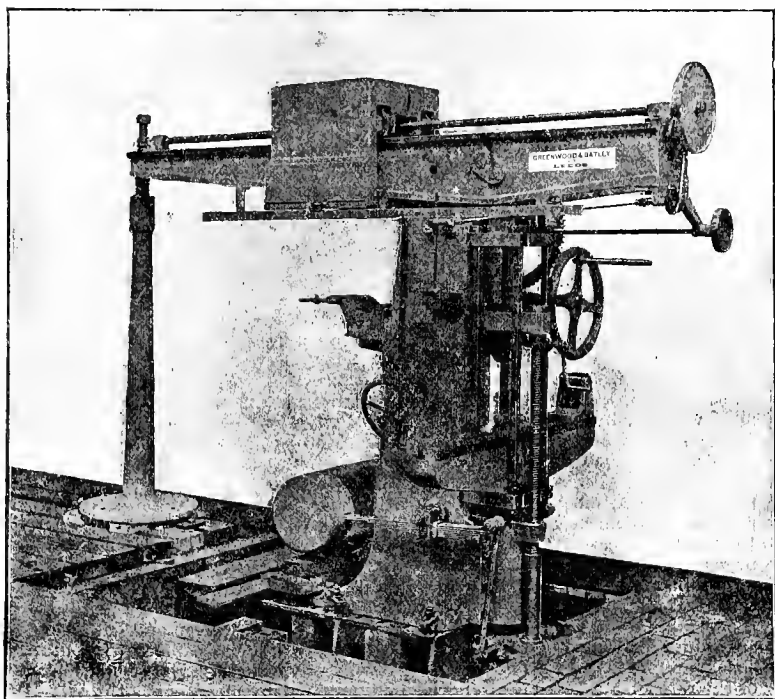


FIG. 6.—THIRTY-TON SINGLE-LEVER VERTICAL TESTING MACHINE.

By Greenwood & Batley, Ltd., Leeds.

Description of the Machine.—It consists of a heavy, vertical cast-iron support, upon which is mounted the weigh beam, with its travelling weight of 1 ton. This travelling weight is moved along the beam by means of a quick threaded screw, which may be rotated either by hand or by hydraulic power. The beam carries a graduated scale to read direct in tons and fractions of a ton. The weigh beam and die holders rest on hardened steel knife edges. The lower die holder is attached to

an adjustable crosshead connected to the hydraulic ram crosshead by two steel screws, and is balanced by a weight and spring beam, shown at the bottom of the left-hand side of the vertical support. This cylindrical balance weight serves to minimise the shock when the test-piece breaks.

The testing machine is provided with different forms of clips for holding the various lengths of test-pieces, as well as an autographic recorder, shown on the side of the vertical column, for plotting mechanically a stress-strain diagram of the test-piece.

Fifty-Ton Single-Lever Vertical Testing Machine* (*See Frontis-plate*).—A convenient form of testing machine for use in laboratories is illustrated by the frontisplate of this edition. Here, a single self-balanced lever, with a single heavy travelling poise-weight, is the means for measuring the load applied to the specimen by a direct screw or by the hydraulic ram. This testing machine consists of a strong upright column to which the cylinder of the hydraulic ram is attached near the foot. A long lever or weigh-beam rests by a knife-edge on the top of the column and carries a travelling poise-weight. This travelling weight weighs 1 ton and carries a vernier, by means of which its position is read against a divided scale on the beam. The test-piece is placed vertically, with its lower end secured in a crosshead, which is pulled downwards by the hydraulic ram underneath it. The upper end of the test-piece is secured by a shackle, which hangs from an inverted knife-edge on the beam. The beam oscillates about its knife-edge, which is placed at a short distance to the left of the first one, and is supported by a fulcrum plate on the top of the upright column. The travelling poise-weight is moved by means of a screw concealed within the beam. This screw receives its motion through spur-wheels from a parallel shaft provided with a Hooke's joint in the axis of oscillation of the beam. This shaft is turned either by a hand-wheel or by a power-driven counter-shaft.

The cylindrical counterpoise, which is seen projecting behind the central column and near the floor level, serves to force up the ram when the hydraulic pressure is relieved. The pressure in the hydraulic cylinder may be applied either by means of a belt-driven, hydraulic, screw compressor, or by an accumulator, or, better still, by a hydraulic intensifier. In the latter case, the work is done by admitting water from a low-pressure supply main behind a large piston, which forces forward a small hydraulic plunger and produces an intense pressure in the fluid upon which the small plunger acts, from which it is in turn transmitted by a pipe to the straining ram of the testing machine. This arrangement has the advantage of allowing the load to be applied to the test-piece without shock, and at as quick or as slow a rate as may be desired. The regulation of the rate of application of the load is obtained by means of a throttle valve, through which water from the low-pressure supply main has to pass on its way to the large cylinder of the intensifier.

Arrangements for Different Tests.—This testing machine is arranged for compressive tests by admitting a test-piece 24 inches long; for deflection tests, with a maximum length of 60 inches; shearing, $1\frac{1}{2}$ inch diameter; torsion, for a load of 50,000 lbs. at 1 inch-moment, with a maximum length for observation of 10 inches, as well as for tension tests.

For compression tests, the upper shackle is connected to a platform by means of four columns in the shape of a cross-beam.

* See *Proc. Inst. Mech. Engs.*, 1882, for paper on "A Single-Lever Testing Machine," by J. Hartley Wicksteed, of Leeds.

This cross-beam hangs below the cross-head, which is pulled down by the hydraulic ram. It will be seen, that the arrangement is two stirrups linked with one another, but having one of them inverted, so that, when the two are acted upon, they pull against each other and cause a block of material, or test-piece placed between them, to be compressed.

For tests in bending, the stirrup or beam which hangs by columns from the upper shackle is made between 4 and 5 feet long, and carries supports at its ends for the ends of the test-piece, while the cross-head presses down on the middle of the piece (see figure). In both instances, the force which is exerted is measured by means of the weigh-beam and travelling poise-weight, in the same way as shown for the tension tests.

For torsion tests, the worm and worm-wheel shown in the figure at the top of the upright serve to twist the test-piece, one end of which is secured in the axle of the worm-wheel, while the other end is secured in a socket which projects from the side of the weigh-beam. The axis of the test-piece under torsion is in the same line as the knife-edge about which the beam is free to oscillate.

Knife-edges in Testing Machines.—Each of the knife-edges in the weigh-beam of Buckton & Coy.'s testing machines is the edge of a square cut-bar of steel, and they are made long enough to prevent the load on them from exceeding 5 tons to the linear inch.

Buckton & Co.'s 300-Ton Universal Testing Machine for Full-sized Structural Members.*—This machine is composed of three principal parts:—(1) The apparatus which applies the load to the test-piece, (2) the apparatus which measures the load, and (3) the frame or bed.

There are two great advantages in the arrangement of this testing machine:—*First*, the bed is *movable*, instead of being fixed as in ordinary machines; and, *second*, the weighing levers are placed at the same end as the stressing cylinder, and can be worked by one man. These levers are mounted upon this stationary cylinder, instead of being carried on the moving ram, as in the Werder machine. It is this combination, that gives the machine the facility which it possesses for testing such a great variety of sizes, and for subjecting full-sized members to compression or tension without the necessity of re-arranging the machine.

Stressing Apparatus.—This consists of a cast-steel hydraulic cylinder, with a ram 26 inches in diameter, and a 7-foot stroke. It is actuated by a water supply of 1,700 lbs. per square inch from an accumulator system. The ram passes through a U leather gland, coated with a sheath of electrically-deposited copper, to avoid the rusting to which steel rams are liable. The main ram pushes the movable bed of the machine, which slides on rollers. These rollers travel on planed and side-flanged cast-iron bed-plates which are bolted to the concrete foundations. The side flanges of the fixed bed-plates, guide and maintain the truly axial motion of the sliding bed.

General Description of the Working of the Testing Machine (see Fig. 7).—When the hydraulic ram H R advances from its hydraulic cylinder H C, the sliding bed S B carries with it the moving crosshead C₁, which is locked to it by means of four square locking bolts contained and guided

* This Universal Testing Machine was illustrated and described by Mr. J. H. Wicksteed, M. Inst. C.E., President of The Institution of Mechanical Engineers, before Section G of the 1904 Cambridge Meeting of the British Association, and printed in *The Engineer* of September 2nd, 1904, p. 236.

in the body of its crosshead. Parallel to the moving bed is a system of tie-rods TR, coupled near the cylinder HC by a cast-steel crosshead C_2 , and at the other end of the machine by the crosshead C_3 . These tie-rods TR are connected, as shown, to two weighing levers L_1 , L_2 by the crosshead C_4 .

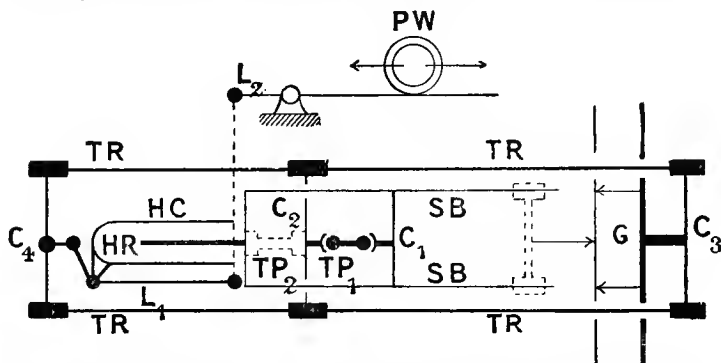


FIG. 7.—DIAGRAMMATIC LINE DRAWING OF BUCKTON & Co.'s 300-TON UNIVERSAL TESTING MACHINE FOR FULL-SIZED STRUCTURAL MEMBERS.

(This sketch shows a test-piece TP_1 in tension, and in dotted lines a test-piece TP_2 in compression, also a bending test at G.)

Equipment of the Machine for Different Kinds of Tests.*—

Tension Test.—The test-piece TP_1 is held between the moving crosshead C_1 and the fixed or weighing crosshead C_2 . Each crosshead is made with a spherical seat, in which a spherical block is free to adjust itself, and contains the clips actually holding the specimens to be tested. For round or flat bars, serrated wedge clips, fitted in two semi-cylindrical backings, are inserted in the spherical block. By this means, even bars of unequal thickness, such as an angle-iron-flange, can be held. For testing specimens with solid heads, holders are inserted into the spherical blocks, which carry suitable dies made in halves. Tests on chains are made by means of four links. Keys pass through slot-holes in these links and hold the chains. Flat ropes are tested by means of two large drums, and are gripped at their extremity by clips. These same drums also lend themselves, by a slight modification, to testing round ropes.

Compression Test.—A compression test is effected by using the opposite sides of the crossheads C_1 and C_2 to those which serve for tension tests. In Fig. 7, a compression test upon a short test-piece TP_2 is shown between the crosshead C_2 and the end of the sliding bed SB, by means of compression plates 2 feet or 1 foot square respectively. To avoid any deviation of the crosshead C_2 from an axial line (which the

* This machine will test a strut 88 feet long by 3 feet 3 inches by 3 feet 3 inches; or a chain 78 feet long, $4\frac{1}{2}$ inches diameter, of iron; or a beam 3 feet 3 inches broad, 6 feet 8 inches deep, by 20 feet between the supports. It will also test in single shear a bar 8 inches wide by $2\frac{1}{2}$ inches thick.

crushing of a heterogeneous specimen might induce), this crosshead is guided in either direction by rollers mounted upon wedges, and the whole contained in strong vertical brackets well tied together. Compression tests on long specimens are made between the crosshead C_1 and the deflection girder G , or the crosshead C_2 , for the deflection beam G rests upon rollers and can be removed without difficulty.

Deflection Test.—For small specimens, this test can be carried out between the fixed crosshead C_2 and the end crosshead C_3 of the sliding bed; to which are attached two small adjustable supports, as indicated by arrowheads.

Shearing and Punching Tests.—The punch and dies are inserted between the crosshead C_2 and the end of the sliding bed at C_1 . For shearing, there is attached to the crosshead C_2 a box in which the test-piece to be sheared is inserted. The shear blade is guided by a roller inside the box, and by this means a simple shear is obtained exactly as in an ordinary shearing machine.

Autographic Diagrams.—By attaching recorder wires to the crossheads, autographic records can be taken of all the above tests.

A Convenient Method of Obtaining and Regulating the Pressure and Flow of Water to the Ram of a Testing Machine.*
 —The method adopted by Prof. Kennedy in working the testing machine at University College, London, is shown by Fig. 8.

Here *a* is the delivery pipe to the testing machine; *b* an ordinary force pump, with its lever *c*; *d* an accumulator loaded to about $1\frac{1}{2}$ tons per square inch. The pump is worked by a Davey motor *e*, on the shaft of which an eccentric *f* drives the free end of a double bar link *g*. The inner end of this link swings on a lever *h*, which can be thrown in or out by a hand lever *k*. The stroke of the block driving the pump lever may be made to vary from zero to the full stroke of the eccentric, according to the position of the link. In this way, the stroke of the pump can be modified, or its action entirely stopped by the hand lever, without stopping the

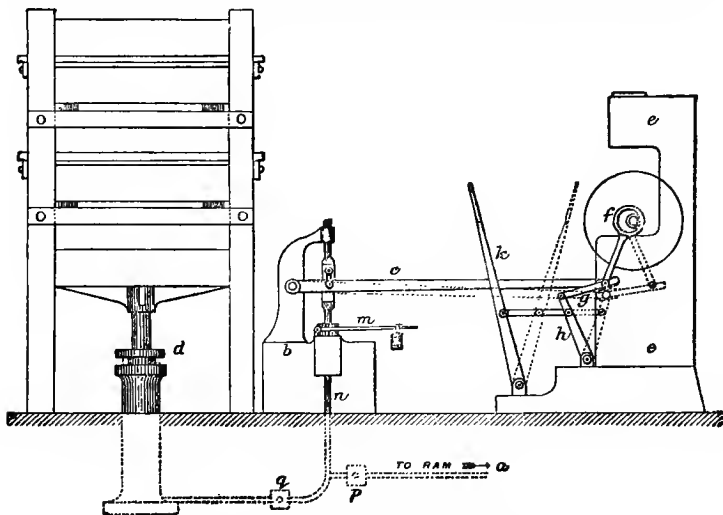


FIG. 8.—PROF. KENNEDY'S METHOD OF WORKING THE TESTING MACHINE.

engine. A relief valve *m* is fitted in the pump itself. And, *n* is the delivery pipe from the pump, with two regulating valves *p* and *q*. The valves allow the pump to pump either (1), to the accumulator, or (2), to the ram direct, or (3), the pump and accumulator to work simultaneously on the ram, or (4), the accumulator alone to work on the ram. For the taking of stress-strain diagrams and other delicate test-work, it is found, that it is better to use the accumulator alone, its speed being regulated by throttling the water passing through both the valves *p* and *q*. For ordinary work,

* In the new *James Watt* Laboratory at Glasgow University, an accumulator with removable weights is used in connection with the large Buckton & Co.'s horizontal testing machine. The force pump is driven by belting from shafting, and is automatically set to work or stopped by the accumulator in a similar way to that in the figure in Lecture II., Vol. IV., "Hydraulics."

where a large number of specimens are to be tested and speed is an object, then the pump and the accumulator are employed simultaneously. The accumulator is pumped up the distance it has fallen during the replacement of the broken test-piece. The accumulator load is divided into four parts, any or all of which can be used simultaneously, or shelved, according to the required load.

The maximum of steadiness, as well as of convenience in working, will be found in some such system as that shown by Fig. 8, although it may not be necessary to use an accumulator for ordinary works' testing.

Proportions of Test-Bars and their Attachment in Tensile Tests.*—With test-pieces of any proportions it is possible to determine the elastic properties of a material, but the plastic and associated qualities can only be determined numerically by the use of test-pieces which are so formed that proper regard is given to the laws of the transmission of stress and of internal friction. The following proportions may prove useful for tensile tests:—(1) When possible, the test-pieces should be either of circular or square section. (2) The ends of the bar should be equal and symmetrically formed, and the scale on the material must be either removed entirely or maintained uninjured. (3) The centre-punch marks for measuring the extension should be from $1\frac{1}{2}$ to twice the diameter or width of the test-piece from its shoulders. The length between the centre-punch marks should be from ten to twelve times the diameter or width of test-piece, or, as previously stated, it is taken in some cases from 8 to 10 inches. (4) In the case of thin plates or sheets of a material of small extensibility, their width and length should be equal and about one or two hundred times the thickness of the plates. (5) For plates and sheets of material having great extensibility the width should be reduced to one-tenth of the length.

For compression tests it is better to make the test-pieces of a right rectangular or circular cylindrical shape with no shoulders, and of a length equal to from two to two and a half times the diameter.

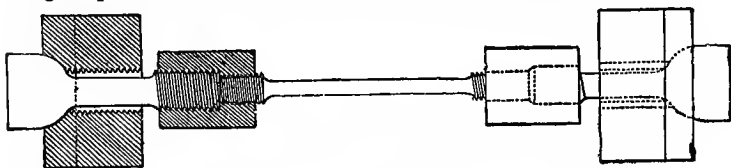


FIG. 9.—HOLDER FOR SECURING AND ADJUSTING THE TEST-PIECE IN THE MACHINE.

The method of holding a test-piece so fair, that the pull will be symmetrically distributed about the axis of the test-piece, and also that fracture may not occur at or near the grips due to any inequality in the stress, always presents some difficulty, more especially when the material is of the rigid, non-plastic type. The test-piece is sometimes made with enlarged ends on which screw-threads are lathe-cut. Each end of the

* See *Proc. Inst. C.E.*, vol. lxxvi., Part ii., 1884, for paper on "The Adoption of Standard Forms of Test-Pieces for Bars and Plates," by William Hackney, B.Sc., Assoc.M.Inst.C.E. *Ibid.*, vol. cxxvii., p. 400, for paper on "The Shape of Compression Test-Pieces," by A. Martens. *Ibid.*, vol. cxviii., p. 395, for paper on "Forms of Tensile Test-Pieces," by S. L. H. Appleby.

test-piece is then screwed into a nut. The seat of the nut is so shaped as to form part of a sphere, thus forming a ball-and-socket joint at each end of the test-piece. Or, the enlarged ends terminate in shoulders inside of which two half rings are placed to form a collar, and the half rings are spherical-curved where they form the bearing with the shackle.

When testing plastic materials, such as the various kinds of wrought iron and mild steel, very little difficulty is experienced in obtaining a fair test of their ultimate strength, even with the very simplest appliances for holding the test-piece, because the plastic yielding which precedes the rupture of the material wipes out any inequality there may be in the distribution of the stress to begin with. Therefore, the trouble of screwing the ends of the test-piece or of forming shoulders may be dispensed with, and a much simpler method of attachment by wedge grips may be adopted. In this arrangement each end of the test-piece lies between two wedges of hard steel. The faces of these wedges where they press on the test-piece are rough, while the backs are made smooth and greased in order to make them slip easily along the tapered recess in the shackle. When the pull comes on the bar, the wedges are drawn along with it and press against the test-piece with sufficient force to cause the rough faces of the wedge to bite into the plastic surface of the test-piece and thus hold it securely. When testing flat plate strips the ends of the test-piece are cut a little wider than the main body of the piece, thereby giving an enlarged surface for the wedge to act on. No enlargement of the ends is required with round or square test-pieces, because the wedges, instead of being made plain, have a groove with roughened sides, so that each end of the test-piece is gripped at four places around its circumference. The tapered hole in which the wedges are placed is made from two half rings which are separately free to turn round in the shackle, thus permitting of its adoption to cases where the opposite sides of the strip are not perfectly parallel.

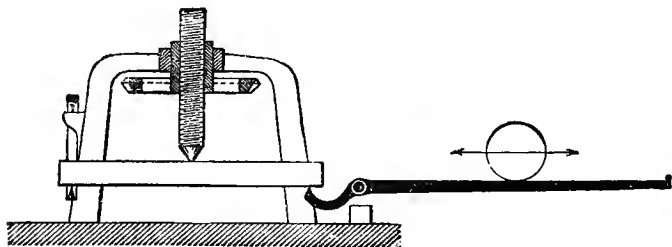


FIG. 10.—PROF. KENNEDY'S SMALL TESTING MACHINE FOR TRANSVERSE TESTS.

3. Transverse Tests.—Transverse and torsional tests usually require so much smaller loads than those used for tension and compression, that it is probably better to carry out the two former kinds of tests upon separate machines. Some arrangement for transverse tests is frequently fitted to large tensile testing machines, but Fig. 10 shows a simple, small, separate machine designed by Prof. Kennedy for laboratory transverse tests up to central loads of 4 tons and spans of 5 feet. It will be seen from the figure, that the load is applied by screw gearing to the centre of the span, and that its value is measured by a steelyard and poise-weight at one end.

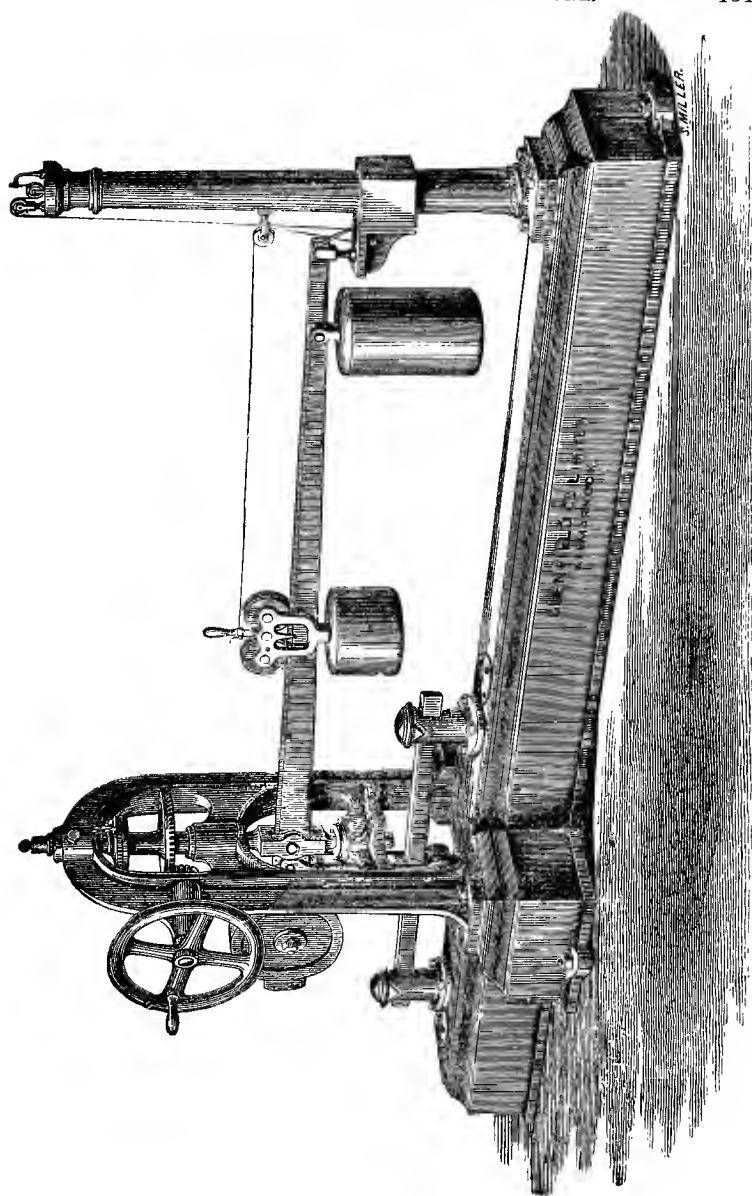


FIG. 11.—TRANSVERSE BAR TESTING MACHINE.

Made by Glenfield & Kennedy, Limited, Kilmarnock.

Transverse Bar Testing Machine.—Engineers and architects usually specify that cast-iron beams, pipes, columns, &c., shall be made of a certain brand of iron, and that standard specimens cast from the same ladle shall withstand certain deflections and bending stresses. It is, therefore, of great importance that cast-iron founders should possess a simple and reliable machine whereby such tests may be quickly and accurately made.

The accompanying illustration shows one form of such a machine for testing the ordinary standard sizes of specimens of cast-iron bars—viz., 2 inches $\times$ 1 inch, or 1 inch square, with centres 3 or 4 feet apart. After introducing the test-bar to the position shown by its supports on the sole plate of the machine, the hand-wheel is turned until the stress just makes the test-bar taut on the upper knife-edges of the supports. Then, the “deflection index” is set at zero. Turning the hand-wheel still further causes the right-hand end of the weigh-beam lever to rise. This action releases the small catch, fixed to the end of the lever, from the ratchet wheel, which allows a weight in the cataract cylinder placed in the right-hand end pillar to pull the travelling weight towards the right hand along the top of the graduated lever. Should the travelling weight and its index pointer arrive at the end of the lever before the specimen is broken, the travelling weight is brought back to zero and an additional weight is then added to the hanging rod, and the above process is repeated. When the bar breaks, an index, not shown on the figure, is connected to the machine when in use, which shows the maximum deflection of the test-piece to the third decimal place of an inch, whilst the stress in lbs. required to break the specimen is shown by the position of the index on the movable weight opposite the scale figure on the weigh-beam bar together with the weights on the vertical rod.

4. Cross Bending, Folding, or Doubling Tests—Cold, Hot, or after Annealing.—The mechanical tests of the cast and of the annealed steels, illustrated above and below the photo-micrograph plate of Fig. 1 in this lecture, consisted of simple *cross bending* over a radius $\frac{3}{8}$ inch in the case of the test-piece from the fresh casting, and of doubling after careful annealing over a rod $\frac{3}{8}$ inch in diameter.

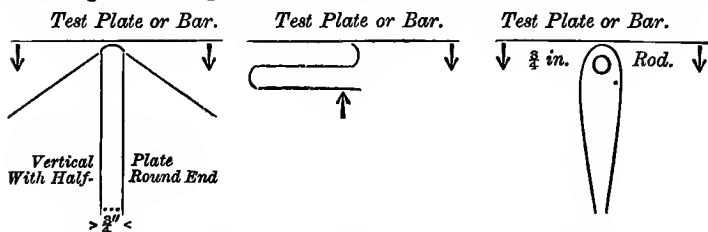


FIG. 12.—CROSS BENDING.

FOLDING.

DOUBLING.

Fig. 12 shows these three different tests. They may be carried out cold as the test-piece comes from the foundry or the forge; or, after any amount of specified annealing, according to circumstances.

In the following set of rules by the British Admiralty, and in the suggestions by the French Commission on Testing, we see such tests are applied. Sometimes, they produce a very severe stress in the material, and, if properly carried out and fulfilled, they are most certainly a surety that the metal possesses ductility, pliability, and tenacity.

Admiralty Rules for Testing Materials for Machinery.—

1. *Steel Castings for Machinery.*—Steel castings for the machinery are to satisfy the following conditions:—Tensile strength, not less than 28 tons per square inch, with an extension of 2 inches of length of at least 23 per cent. Bars of the same metal, 1 inch square, should be capable of bending cold, without fracture, over a radius not greater than $1\frac{1}{2}$ inches, through an angle depending on the ultimate tensile strength, this angle to be not less than 90° at 28 tons ultimate strength, and not less than 60° at 35 tons ultimate strength, and in proportion for strengths between these limits. For intricate thin castings the extension in 2 inches of length is to be at least 10 per cent., and the bending angle is to be not less than 20° at 28 tons ultimate strength, and 15° at 35 tons ultimate strength, and in proportion for strengths between these limits. Test-pieces are to be taken from each casting. All steel castings are also to satisfactorily stand a falling test, the articles being dropped from a height of 12 feet (or as may be approved) on a hard macadamised road or a floor of equivalent hardness.

Steel castings Tests.

It is to be distinctly understood that contractions or defects in steel castings are not to be made good by patching, burning, or by electric welding, without the sanction of the Admiralty overseers.

2. *Steel Forgings for Machinery.*—All steel forgings are to satisfy the following conditions:—Ultimate tensile strength, not less than 28 tons per square inch, with an extension in 2 inches of length of at least 30 per cent. Bars of the same metal, 1 inch square, should be capable of being bent cold, without fracture, through an angle of 180° over a radius not greater than $\frac{1}{4}$ inch. Test-pieces are to be taken from each forging. Crank and propeller shafts are to have test-pieces taken from each end, and the ultimate tensile strength of the material of these shafts must not exceed 32 tons per square inch.

Steel forgings.

For all important forgings, such as crank and propeller shafts, connecting- and piston-rods, the forgings are to be gradually and uniformly forged from solid ingots, from which at least 30 per cent. of the top end of the ingot has been removed before forging, and at least 3 per cent. of the total weight of the ingot from the bottom end after forging. The sectional area of the body of the finished forging is to be not more than $\frac{1}{4}$ the original sectional area of the ingot.

3. *Cast Iron.*—Test-pieces to be taken from such castings as may be considered necessary by the inspecting officer. The minimum tensile strength to be 9 tons per square inch, taken on a length of not less than 2 inches. The transverse breaking load for a bar 1 inch square, loaded at the middle between supports 1 foot apart, is not to be less than 2,000 lbs.

Cast iron.

4. *Gun-metal, Naval Brass, and White Metal.*—The gun-metal used for all castings throughout the whole of the work supplied by the contractors is, unless otherwise specified, to contain not less than 8 per cent. of tin, and not more than 5 per cent. of zinc, the remainder to be of approved quality copper. The exact proportion of tin above 8 per cent. being arranged as may be required, depending on the use for which the gun-metal is intended. The ultimate tensile strength of gun-metal is to be not less than 14 tons per square inch, with an extension in 2 inches of length of at least $\frac{7}{8}$ per cent. The composition of any naval brass used is to be:—

Composition, analysis, and tests of gun-metal and naval brass, &c.

Copper, 62 per cent.; zinc, 37 per cent.; and tin, 1 per cent. All naval brass bars are to be cleaned and straightened. They are to be capable of (1) being hammered hot to a fine point, (2) being bent cold through an angle of 75° over a radius equal to the diameter or thickness of the bars. The ultimate tensile strength of naval brass bars $\frac{3}{4}$ inch diameter and under is not to be less than 26 tons per square inch, and for round bars above $\frac{3}{4}$ inch diameter, and square bars not less than 22 tons per square inch, whether turned down in the middle or not. The extension in 2 or 4 inches of length is to be at least 10 per cent. Breaks within less than $\frac{1}{2}$ inch of the grip are not to count. Cuttings from the propellers and other important gun-metal castings and naval brass work will be sent to Portsmouth dockyard for analysis. The white metal used for bearing surfaces is to contain at least 85 per cent. of tin, not less than 8 per cent. of antimony, and about 5 per cent. of copper; zinc or lead should not be used. The brasses are to be carefully tinned before filling with white metal.

Castings
and
forgings.

The gun-metal castings are to be perfectly sound, clean, and free from blowholes. The steel castings are required to be clean, sound, and to be out of twist, and as free as possible from blowholes; the steel forgings are required to be quite sound, clean, and free from all flaws. All castings and forgings must admit of being machined, planed, and bored to the required dimensions; and no piecing, patching, bushing, stopping, or lining, will be permitted, nor will any manufactures in which these conditions have been infringed be accepted. In cases of doubt as to the suitability of castings or other materials, for the purpose intended, early reference should be made to the inspecting officer to avoid subsequent delay by the rejection of such parts after delivery. The whole of the steel plates, angles, and rivets used in the construction of any part of the work supplied is to be manufactured by the Siemens-Martin process. All castings are to be of steel or gun-metal, except where otherwise specified; and all forgings, plates, bolts, &c., are to be of steel. No steel is to be toughened without sanction from the Admiralty.

Copper.

5. *Copper for Pipes.*—Strips cut from the steam and other pipes, either longitudinally or transversely, are to have an ultimate tensile strength of not less than 13 tons per square inch when annealed in water, with an elongation in length of 2 inches or 4 inches of not less than 35 and 30 per cent. respectively. Such strips are also to stand bending through 180° cold until the two sides meet, and of hammering to a fine edge without cracking.

Hammer
test.

Scale to
be re-
moved.

6. *Treatment of Mild Steel.*—All plates and furnaces for boilers and steam pipes are to be treated as follows, with a view of removing the black oxide or scale formed during manufacture:—The plates and furnaces, previous to their being taken in hand for working, are to stand for not less than 8 hours in a liquid consisting of 19 parts of water and 1 of hydrochloric acid. The plates should be placed in the bath on edge, and not laid flat. When the plates and furnaces are removed from the dilute acid both the surfaces are to be well brushed and washed to remove any scale which may still adhere to them. They should then be placed in another similar bath filled and kept well supplied with fresh water, or be thoroughly washed with a hose, as may be found necessary. The plates on removal from the fresh water should be placed on edge to dry. This treat-

ment is to be carried out on the premises where the boilers and pipes are made.

All plates or bars which can be bent cold are to be so treated; Cold and if the whole length cannot be bent cold, heating is to be had bending. recourse to over as little length as possible.

The front and end plates of the boilers, and also all other plates, Hydraulic including those for the combustion chambers are to be flanged by flanging. hydraulic pressure, and in as few heats as possible.

In cases where plates or bars have to be heated, the greatest care should be taken to prevent any work being done upon the material after it has fallen to the dangerous limit of temperature known as a "blue heat"—say from 600° to 400° Fahrenheit. Should this limit be reached during working, the plates or bars should be reheated. Dangerous temperature to be avoided.

Plates or bars which have been worked while hot are to be subsequently annealed simultaneously over the whole of each plate or bar. Annealing.

In cases where any bar or plate shows signs of failure or fracture in working it is to be rejected. Any doubtful cases are to be referred to the Admiralty. Failure of metal.

French Methods of Testing Materials of Construction.*—Vol. I., in addition to the constitution of the commission, contains the reports of the two sections. That of Section A is divided into four parts as follows:—

Part I. Physical tests:—(a) Microscopic and other examination of the exterior appearance and fractures, and tests by resonance; (b) determination of density and thermal and electrical conductivity; (c) determination of the critical temperature and variations of conductivity with temperature; (d) study of temper.

Part II. Chemical tests:—(a) Analysis; (b) corrosion and means of protection.

Part III. Mechanical tests:—(a) General remarks; (b) on samples; (c) the influence of heat; (d) the influence of duration of test; (e) testing machines; (f) dimensions; (g) mechanical terminology; and (h) technical terminology.

Part IV. Methods of mechanical testing:—(a) Gradually applied tests, tensile, compressive, bending, folding and doubling, torsional, shearing, and punching; (b) tests by blows, bending, indentation, and perforation; (c) study of hardness and brittleness; (d) hot and cold working, hammering, &c.; (e) special tests of wires, ropes, chains, rivets, pipes and tubes, and hydraulic pressure tests.

The report of Section B, which deals only with cements, limes, and kindred materials, is divided into the following parts:—

Part I. General considerations, choice of tests, normal tests, &c.

Part II. Tests of cement, fineness, specific weight, apparent density, analysis, homogeneity, making of sample briquettes and mortars, setting, tensile, compressive, and bending tests; stability, efficiency, porosity, permeability, solubility in sea water, and adhesiveness.

Part III. Tests of hydraulic and fat limes.

Part IV. Tests of puzzolanas, &c. Part V. Sand for use in mortar.

Part VI. Plasters. Part VII. General conclusions.

*The original report of the proceedings of the French Commission on "Methods of Testing Materials of Construction" will be found in the library of the *Inst. C.E.*, dated Paris, 1894.

Publications dealing with Engineering Tests and Materials.

—The following is a list of publications which students should endeavour to study in connection with this subject:—

See *Engineering*, vol. xi., January-June, 1871, for Wöhler's apparatus for applying different kinds of stress to bars, with an abstract of his results.

See *Chain Cables and Chains*, by Thomas W. Traill, M.Inst.C.E., &c., published by Crosby Lockwood & Co.

See *Proc. Inst. Mech. Engs.*, 1886, for paper giving "Description of Tensile Tests of Iron and Steel Bars," by the late Mr. Peter D. Bennett, of Tipton.

See *Proc. Inst. C.E.*, vol. cxvii., 1894, for paper on "Testing Some Specimens of Malleable Cast Iron," by Andrew G. Ashcroft, Assoc.M.Inst. C.E.; vol. cxviii., 1895, for abstract of paper on "The Resistance of Materials under Impact," by Mansfield Merriman; vol. cxxix., 1897, for paper on "A New Indentation Test for Determining the Hardness of Metals," by William Cawthorne Unwin, B.Sc., F.R.S., M.Inst.C.E.; vol. cxxxiv., 1898, for paper on "Testing the Strength of Materials," by A. H. Jameson, M.Sc., Stud.Inst.C.E.; vol. cxxxv., 1899, for paper, "Note on the Endurance of Steel Bars Subjected to Repetitions of Tensional Tests," by E. G. Coker, B.A., B.Sc.; vol. clviii., 1904, for paper on "Axle-Loads on Railway Bridges," by John Graham, M.Inst.C.E.; *ibid.*, paper on "Maximum Bending Moment in a Girder under a Moving Group of Loads," by Evelyn H. Young, Assoc.M.Inst.C.E.; *ibid.*, paper on "Moving Loads on Railway Under-Bridges," by W. E. Lilly, M.A.; *ibid.*, paper on "The Relation of the Constants of the Elongation Equation to Contraction of Area," by Prof. Elliott.

See *The Engineering Review*, September, 1904, for an article on "The Fatigue of Metals," by Arthur N. Kemp.

See *Phil. Trans. of the Roy. Soc. of London*, vol. 199, November, 1902, Series A, p. 265, for paper on "A Throw-Testing Machine for Reversals of Mean Stress," by Profs. Osborne Reynolds, F.R.S., and J. H. Smith, M.Sc., Wh.Sc., &c.

See *Proc. Inst. Mech. Engs.*, December 16, or *Engineering*, December 23, 1904, for paper on "Heat Treatment Experiments with Chrome-Vanadium Steel," by Captain H. R. Sankey and Mr. J. K. Smith.

PUBLICATIONS OF THE ENGINEERING STANDARDS COMMITTEE.

Properties of British Standard Sections, containing the Areas, Weights, Moments of Inertia, and other Properties of British Standard Rolled Sections, including Diagrams, Definitions, and Formulæ. Cloth, 5s. net, post free 5s. 3d.

British Standard Specification for Tubular Tramway Poles. 5s. net, post free 5s. 2d.

British Standard Specification and Sections for Bull-Headed Railway Rails. 10s. 6d. net, post free 10s. 8d.

Properties of Standard Beams. Fcap. folio, 1s. net, post free 1s. 1d.

British Standard Tramway Rails and Fishplates: Sections and Specification. Fcap. folio, 21s. net, post free 21s. 3d.

Influence of Gauge Length and Section of Test-Bar on the Percentage of Elongation, by Professor W. C. Unwin, F.R.S. Fcap. folio, 2s. 6d. net, post free 2s. 7d.

Published by Crosby Lockwood & Son, 7 Stationers' Hall Court, Ludgate Hill, London, E.C., and can also be obtained at the Offices of the Committee, 23 Victoria Street, London, S.W.

LECTURE V.—QUESTIONS.

1. State a few of the chief cases in the history of testing machines.
2. What are the advantages to be derived by young engineers from a laboratory course on the testing the strength and elasticity of materials, and why?
3. Why is it advisable to obtain not only accurate mechanical tests, but also chemical analysis and photo-micrographs of specimens of the materials to be used before designing an engineering structure and during their manufacture?
4. State the different mechanical tests which you could apply to, say, steel bars or plates. Explain by aid of an example why impact tests are important.
5. Suppose that you were called upon to investigate the case of a broken connecting-rod from a high-speed reciprocating engine, state the different things and information you would require before drawing up your report.
6. Sketch and describe Prof. Barr's wire-testing machine, state how you would use it and what information you would get from your results.
7. Give free-hand line diagram sketches of three leading types of testing machines.
8. Sketch and describe a good 30- or 50-ton single-lever vertical testing machine. State how it works and how you would obtain reliable results from it.
9. Give a line diagram of a 300-ton universal testing machine, and state how it works for tension, compression, and bending stresses.
10. Sketch and explain any convenient method of obtaining and regulating the pressure and flow of water to the ram of a testing machine.
11. In making tensile tests of bars and plates what proportions would you adopt, and why? Sketch and describe a suitable holder for round test-pieces. When testing round or square bars of wrought iron or mild steel how would you shape and grip them?
12. What is the simplest form of machine that you know for carrying out simple cross-bending or transverse tests?
13. Sketch and describe the construction and action of an accurate transverse bar testing machine for measuring the deflections and bending stresses obtained with standard sized specimens of cast iron.
14. Explain, by aid of sketches, how you would carry out cross bending, folding, and doubling tests upon a steel bar or copper plate. Give instances where one or more of these tests are required by the Admiralty or the French Government, and mention the circumstances attending a proper fulfilment of the test.

N.B.—See *Appendices B and C* for other questions and answers.

LECTURE V.—A.M. INST. C.E. EXAM. QUESTIONS.

1. Describe the method of conducting a tension test of a bar of mild steel. State what precautions should be taken in preparing the test bar and what measurements should be made. (I.C.E., *Oct.*, 1897.)
2. Describe and sketch one form of testing machine. State any special conditions you would require in such a machine for testing to 50 tons load. State how or to what extent its accuracy can be ascertained by trial. (I.C.E., *Oct.*, 1897.)
3. Describe the methods of holding tension specimens, and mention any special advantages or defects of any of these. (I.C.E., *Oct.*, 1897.)
4. Give a specification of tests of mild steel bars and plates for bridge-work. (I.C.E., *Oct.*, 1897.)
5. Having ascertained that a certain mixture of cast iron possesses an ultimate tensile strength of 9 tons per square inch, calculate the breaking weight of a standard bar 2 inches by 1 inch, placed upon supports 3 feet apart and loaded in the middle. In using for this purpose the ordinary theory of transverse flexure, add any comments that you may think necessary in regard to the result so obtained. (I.C.E., *Feb.*, 1899.)
6. Mention some practical tests by which we can estimate or measure the ductility of metals, and some of the reasons which make it important to obtain a test of this property. (I.C.E., *Feb.*, 1901.)
7. Describe with the help of sketches the general arrangement of any testing machine you are familiar with, suitable for tensile and compressive tests. Explain how the machine may be tested for accuracy. (I.C.E., *Oct.*, 1901.)
8. Explain how a cube of cast iron can be tested to destruction by compression in a testing machine; how is it likely to fracture? Give your reasons. (I.C.E., *Oct.*, 1902.)

V.B.—See *Appendices B and C* for other questions and answers.

LECTURE VI.

STRENGTH AND ELASTICITY OF MATERIALS.
TESTING AND TESTING MACHINES.

CONTENTS.—Shearing, Planing, Boring, Punching, Drifting, and Riveting, Hammering to a Point or Edge—Fatigue of Metals—Historical Notes—Wöhler's Testing Machines, with an Example—General and Particular Deductions from Wöhler's Experiments—Gerber's Formula—Elastic Limits—Heating due to Stress—Rest and Heating Relieve Stress—Microscopic Examination of the Inter-Molecular Crystalline Action in Metals due to Fatigue—Prof. Ewing's Conclusions—Prof. Reynolds and Dr. J. H. Smith's Method of Testing Metals by Tensile and Compressive Reversals of the Mean Stress—Determination of the Tensile, Compressive, and Range of the Reversal Stresses, with Formulæ and Results on Mild Steels—Impact Tests—Seaton and Jude's Impact Testing Machine—Advantages of the Impact Test—Cumulative Effects of Small and Medium Shocks—Effect of Allowing a Factor of Safety for both Fatigue and for Impact—Drop or Falling Tests—Torsional Tests—Torsion of Rods and Wires—Hardness, Temper and Brittleness Tests—Methods of Determining the Hardness of a Metal—Workshop Test—Calvert and Johnson's Test for Alloys—Prof. Unwin's Test—Comparison between Unwin's Indentation Test and the Scratch Method—Questions.

5. Shearing, Planing, Boring, Punching, Drifting, and Riveting, Hammering to a Point or Edge.—If this had been a lecture upon these headings in the sense of "Workshop Appliances and Methods," then many well-illustrated and instructive pages might be written about them. But here, in this instance, the attention of the student has simply to be drawn to the fact, that under certain circumstances, and in certain specifications, one or more of these methods are used as tests for iron, steel, and certain other metals, in addition to one or more of the special tests detailed in the previous, this, and the following lectures.

When Mr. H. E. Yarrow, of the well-known torpedo-boat building firm at Poplar, London, delivered his address to the Junior Institution of Engineers on December 2nd, 1904,* he pointed out, that one of the most important considerations in the design of torpedo vessels was, how to realise (as far as possible) the maximum of strength with the minimum of weight, so that their hulls might have uniform elasticity throughout their length. The importance of realising this fact will be understood when it is remembered, that in a rough sea such vessels are subjected to severe bending

* See *Proc. Inst. Jr. Engs.* for 1905. Students should endeavour to get a copy of this paper, or of the abstracts thereof contained in the engineering papers—e.g., *The Engineering Times* for December 15, 1904.

His laborious and most valuable researches into the "Fatigue of Wrought Iron and Steel" are published in *Zeitschrift für Bauwesen*, Berlin. A very full abstract of his report, with figures of the different machines designed and used by him, appeared in *Engineering*, vol. ix., 1871. A shorter article by Arthur N. Kemp on the "Fatigue of Metals" appeared in the September number of the *Engineering Review* for 1904, wherein some of Wöhler's machines and experiments are given, as well as results by other engineers.

In 1873, Launhardt's experiments are given in *Zeitschrift des Architekten und Ingenieur-Vereins*, Hanover.

In 1874, Spangenberg repeated Wöhler's experiments with the very machines devised by the latter, and obtained similar results, which are also published in *Zeitschrift für Bauwesen*.

In 1874, Gerber published in *Zeitschrift für Bankunde*, München, a formula representing the best results of Wöhler's experiments. It is certainly simpler and easier of application than Launhardt's or Weyrauch's formulæ, and is the same or similar to that used by Prof. Unwin (see a following page).

In 1879, Lippold published his views in *Organ für die Fortschritte des Eisenbahnwesens*, Wiesbaden.

In 1881, Prof. Mohr wrote on this subject to *Der Civil Ingenieur*, Leipzig.

In 1886, Sir Benjamin Baker, M.Inst.C.E., read a paper before the *American Society of Mechanical Engineers* on the results of his experiments upon iron and steel, which were obtained by using a similar rotating machine to one of those designed and used by Wöhler. Here, a horizontal spindle rotated 50 to 60 times a minute, having chucks beyond its two midbearings gripping test bars, whose further or outer extremities were hooked on to vertical tension springs. Hence, during each revolution, each test-piece was subjected by its spring to alternate tension and compression. In one case these were repeated over 14×10^6 times before fracture of the test-piece occurred.

In 1886, Bauschinger published in *Mittheilungen aus dem Mech. Techn. Laboratorium in München* his important communication on the "Variation of the Elastic Limits of Materials."

In 1899, Prof. J. A. Ewing, F.R.S., and Walter Rosenhain, in their Bakerian lectures on the "Crystalline Structure of Metals," and again in the *Philosophical Transactions of the Royal Society* (see Series A, vols. 193 and 195), give some interesting and useful information bearing upon this subject.

In 1902, Prof. Osborne Reynolds, F.R.S., and Dr. J. H. Smith, M.Sc., read their paper on a "Throw-testing Machine for Reversals of Mean Stress" before the Royal Society (see Series A, vol. 199 of *Phil. Trans. Roy. Soc.*). I am indebted to the kind permission of Dr. J. H. Smith for leave to make abstracts of this paper.

In 1903, Prof. Ewing and J. C. W. Humphrey read a paper before the Royal Society, and gave the results of their recent investigations into the "Fracture of Metals under repeated Alternations of Stress."

The more important conclusions arrived at by some of the above-mentioned experimenters will now be referred to, but as each of these communications are long and important, students are advised to try and also study the original papers or abstracts of them which may be found in leading engineering journals about the above dates.

Wöhler's Testing Machines, with an Example.—By means of ingeniously contrived machines, he submitted test-pieces to—(1) Direct pull, alternated with complete or partial relaxation from the pull. (2) Repeated bending in one direction. (3) Bending in opposite directions. (4) Twisting the test-piece towards one side  only; and, (5) towards opposite sides 

Unfortunately, Wöhler's experiments, although extensive, were not carried out on a sufficient variety of materials to be of universal use since his experiments were restricted to iron and steel. Hence, the results of his tests cannot be generalised, as it is unsafe to assume, that the behaviour of a metal will be the same under different kinds of stress. Therefore, to make experiments of one kind of test and deduce values for others is not feasible, and his results must be used *only* for the same materials as those tested by him. He found, that wrought iron, having an ultimate tensile strength of 19·5 tons per square inch, would stand an indefinite number of alternations of stress, provided the limits did not exceed the following:—

Kind of Stress.	Stress in Tons per Square Inch.
Alternating load, or from a pull to a push, ...	+ 7 to - 7
Load varying between zero and maximum, or from a certain pull to no stress, ...	+ 13 ,, 0
Load varying between limits, or from a certain pull to a less pull, ...	+ 19 ,, + 10½

From these results it appears, that the relative values of the maximum loads for iron under these three conditions are approximately 1 : 2 : 3. Moreover, he found similar results in the case of certain specimens of steel.

General and Particular Deductions from Wöhler's Experiments.—(1) That wrought iron and steel are ruptured by stresses *much* below their statical breaking stress, if such stress be repeated a sufficient number of times.

(2) That within certain limits the range of stress, and not the maximum stress, determines the number of reversals for rupture.

(3) That as the range of stress is diminished, the number of repetitions for rupture increases.

(4) That there is a limiting range of stress for which the number of repetitions of stress for rupture becomes infinite.

(5) That this limiting range of stress diminishes as the maximum stress increases, which was clearly indicated by the results under the previous heading.

Gerber's Formula.—As previously mentioned in the historical notes, Gerber founded a formula upon Wöhler's experiments. He did so by simply plotting the ranges of stress as ordinates and the corresponding minimum stresses as abscissæ, when he found, that their points of intersection, if joined, formed a parabola. Consequently, the formula to fit in with this curve is known as—

GERBER'S PARABOLIC EQUATION.

$$f(\text{max.}) = \frac{1}{2} \Delta + \sqrt{(f^2 - c \Delta f)}.$$

Where $f(\text{max.})$ = the maximum stress,
 „ $f(\text{min.})$ = the minimum stress,
 „ f = the statical breaking stress,
 „ Δ = the range of stress = $f(\text{max.}) \pm f(\text{min.})$,
 And c = a constant.

For a statical load, $\Delta = 0$, and $f(\text{max.}) = f$.

For a load varying between 0 and a maximum—

$$\Delta = f(\text{max.}).$$

And approx., when $c = 1.5$, then $f(\text{max.}) = \frac{2}{3}f$.

For an $\sim$ load varying between $f(\text{max.})$ and $f(\text{min.})$ —

$$\Delta = 2f(\text{max.}).$$

And, when $c = 1.5$, then $f(\text{max.}) = \frac{1}{3}f$.

Hence, the relative values of the above maximum loads, which are—

	Alternating :	Variable :	Statical,
Are as,	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$,
Or as,	1	2	3,

just as Wöhler found them to be, by his testing machines, for the wrought iron referred to in a previous paragraph. Of course, the constant c has various values for different materials, and, whilst it may be about 1.5 for a certain kind of wrought iron, it will rise as high as 2.5 for the finest spring steel.

EXAMPLE I.—Two steel bars, having a static breaking load of 28 tons per square inch, are stressed in tension, the one from 4 to 5 tons per square inch and the other from 1 to 5 tons per square inch. Find the breaking strengths for the respective methods of loading.

The stress variation must first be put in terms of maximum static stress, and this again in terms of the new breaking stress $f(\text{max.})$, thus :—

$$\Delta = \frac{\text{Highest stress} - \text{lowest stress}}{\text{Highest stress}} \times f(\text{max.}).$$

In the first case, where the ultimate static stress = 30 tons per sq. inch,

$$\Delta = \frac{5-4}{5} f(\text{max.}) = \frac{1}{5} f(\text{max.}).$$

Now, taking Gerber's formula and substituting the values for the symbols.

Where f = Original breaking stress in tons per square inch = 28.
 „ Δ = Stress variation in terms of $f(\text{max.})$ in tons per sq. inch.
 „ c = A constant deduced from experiments = 2 for hard steel.
 „ $f(\text{max.})$ = The new max. breaking stress in tons per sq. inch.

We get, $f(\text{max.}) = \frac{\Delta}{2} + \sqrt{(f^2 - c \Delta f)}$

$$= \frac{1}{10} f(\text{max.}) + \sqrt{(28)^2 - \left(2 \times \frac{1}{5} f(\text{max.}) \times 28\right)}.$$

$$\frac{9}{10} f(\text{max.}) = \sqrt{784 - 11.2 f(\text{max.})}.$$

Simplifying, squaring, and solving this quadratic equation, we find—

For *First Case*—

$$f(\text{max.}) = 22.5 \text{ tons per square inch.}$$

In the second case, where the ultimate static stress = 28 tons per sq. inch,

$$\Delta = \frac{5-1}{5} f(\text{max.}) = \frac{4}{5} f(\text{max.}).$$

Therefore, substituting the values in the formula, we obtain—

$$\begin{aligned} f(\text{max.}) &= \frac{\Delta}{2} + \sqrt{(f^2 - c \Delta f)} \\ " &= \frac{2}{5} f(\text{max.}) + \sqrt{(28)^2 - \left(2 \times \frac{4}{5} f(\text{max.}) \times 28\right)}. \\ \frac{3}{5} f(\text{max.}) &= \sqrt{784 - 44.8 f(\text{max.})}. \end{aligned}$$

Simplifying, squaring, and solving the quadratic as in the first case, we find—

For *Second Case*—

$$f(\text{max.}) = 14.6 \text{ tons per square inch.}$$

Elastic Limits.—Bauschinger showed, in his 1886 paper, that when the elastic limit in tension is raised, the elastic limit in compression is lowered. Also, that by subjecting a material to a few alternations of equal stresses, the elastic limits tend towards fixed positions, which positions he called the *natural* elastic limits. Further, he explained the results which he obtained, when subjecting a metal to repeated stresses, by showing that the limiting range of stress coincided with the difference of the two elastic limits.

Heating due to Stress.—All have noticed, that when bars of metal are subjected to varying stresses they become heated due to the work done upon them. Hence, perfect elasticity does not exist in such materials.

Rest and Heating Relieve the Molecular Stress.—It has been noticed, that when a metal has been fatigued, due to varying stresses, a period of rest or of annealing will apparently restore its elasticity. Profs. Ewing and Muir of Cambridge are reported to have found, that a few minutes' immersion of a fatigued bar in hot water effects such a recovery; thus showing, that high temperatures are not necessary under certain conditions for annealing or for recuperation. But any old *hard-bristled shaver* could have told them these things, since from time immemorial it has been customary for them to put their razors into hot water before action; and, further, to keep two or three of these useful tools within their cases, in order to give them alternate rests of a week or so at a time, which they said made them cut better!

Microscopic Examination of the Inter-Molecular Crystalline Action in Metals due to Fatigue.—Prof. Ewing and Mr. J. C. W. Humphrey,* in some recent experiments, applied the microscope to watch the process by which iron breaks down and becomes fatigued under repeated reversals of stress. The tests which they carried out were on

* See *Proc. Roy. Soc.*, 1903, paper on "The Fracture of Metals under repeated Alternations of Stress," by Prof. Ewing and J. C. W. Humphrey.

Swedish iron of high quality, having a statical breaking strength of 23·6 tons per square inch on the original area, an extension of 26 per cent. in a length of 8 inches, and a contraction of 61 per cent. in the area.

The test-bars were in the form of rectangular rods measuring ·3 inch by ·1 inch, and were first prepared by annealing in a muffle furnace, being enclosed in a tube of lime, and the whole kept at a dull-red heat for two hours. One face of each rod was polished and etched prior to its being subjected to alternating stress actions, so that under test the polished surface was alternately stretched and compressed. The machine used was similar in form to that of Wöhler's, and ran at 400 revs. per minute.

Prof. Ewing's Conclusions.—"Whatever the selective action of the stress is due to, the experiments demonstrate, that in repeated reversals of stress certain crystals are attacked and yield by slipping, as in other cases of non-elastic strain. Then, as the reversals proceed, the surfaces upon which slipping has occurred continue to be surfaces of weakness. The parts of the crystal lying on the two sides of each surface continue to slide back and forward over one another. The effect of this repeated sliding or grinding is seen at the polished surface of the specimen by the production of a burr or rough and jagged irregular edge, broadening the slip band and suggesting the accumulation of *débris*. Within the crystal this repeated grinding tends to destroy the cohesion of the metal across the surface of slip, and in certain cases this develops into a crack. Once a crack is formed it quickly grows in a well-known manner, by tearing at the edges, in consequence of the concentration of stress which results from lack of continuity."

"The experiments throw light on the known fact, that fracture by repeated reversals or alternations of stress resembles fracture resulting from a 'creeping' flaw in its abruptness, and in the absence of local drawing out or other deformation of shape."

The formation of cracks does not appear to be so serious in the case of unidirectional stress, as in the case of alternating stress, there being some especial weakness induced by reason of the alternations. It is a curious fact, that a piece of material after being fatigued by varying stress actions and which has nearly arrived at its limit of endurance, does not appear to show any marked deterioration either in tensile strength or in its plastic properties when tested by the static method. Prof. Ewing says this is not to be expected, so long as the reversals have not been sufficient to produce a crack, but it will have a *yield-point* corresponding to the amount of permanent extension it has received.

7. Fatigue of Metals (Continued).—Prof. Osborne Reynolds' and Dr. J. H. Smith's Method of Testing Metals by Tensile and Compressive Reversals of the Mean Stress.*—In the year 1899, Prof. Reynolds suggested a novel method of testing the "Fatigue of Metals," and gave a general idea of the design of the apparatus and means by which the experiments should be carried out to Mr. J. H. Smith, the 1851 Exhibition Scholar, at his Whitworth Laboratory, Owens College, Manchester. The credit for the details of the design, calculations, and execution of the work has been honourably given by the former to the latter engineer in their joint paper on a "Throw Testing Machine for

* See Vol. 199, *Philosophical Transactions of the Royal Society of London*, Series A, November, 1902; also, T. E. Stanton and Leonard Bairstow's paper on "The Resistance of Iron and Steel to Reversals of Direct Stress," in the *Proc. Inst. C.E.* (Paper 3,630), Session 1905-1906.

Reversals of Mean Stress," as read before the Royal Society on March 20, 1902. This arduous research occupied about two years, and the author is indebted to Dr. Smith for his kind permission to reproduce in this book a concise outline of the objects, methods, and results whereby the undertaking was achieved.

Objects of the Research.—“(1) The stress should be direct tension and compression, and (2) of approximately equal amounts, such tension and compression being obtained by means of the inertia force of an oscillatory weight; (3) the rapidity of repetitions should be much higher than in the experiments of Wöhler, Spangenberg, Bauschinger, and Baker; in fact, ranging as high as 2,000 reversals per minute.”

Hitherto most of the experiments on “repeated stress” had been carried out on bars subjected to bending, where the ordinary formula for stress in a bent bar was used to calculate the stress at breaking. In such experiments, it had been assumed, that the distribution of the stress at the breaking-down point was the same as for an elastic bar. But, no such assumptions are necessary with this method. Further, the tensile and compressive stresses being nearly of the same value, the elastic limits would soon be changed to their *natural* positions, and the “range of stress” for unlimited reversals would be this *natural elastic range*. If the *limiting range* coincided with the *natural range*, it would be constant whatever the rate of the reversals. Most of the experiments were conducted to find out the *variation of the limiting range of stress* as the rapidity of the reversals increased; but the importance of extending the experiments to very high speeds was fully recognised, in view of the tendency of engineers to adopt fast-speed reciprocating engines, tools, and machinery.

The Apparatus and the Method of Applying the Stresses.—The line diagram (Fig. 13) merely indicates the more important parts as far as this concise description is concerned, but it will be understood from it, that the test specimens consisted of round short rods of small diameter. These rods were about $3\frac{1}{2}$ " over all, with screwed ends of $\frac{3}{4}$ " diameter by $1\frac{1}{2}$ " long, turned down in their middle part to $\frac{1}{4}$ " diameter by $\frac{1}{2}$ " long, and then filleted out to short collars the size of the bottom of the threads of their $\frac{3}{4}$ " screwed ends, in a similar way to the test-pieces shown by Fig. 9, but much shorter in the body. The ends of each specimen were screwed home into the top and bottom chucks, or upper and lower spindles, and made

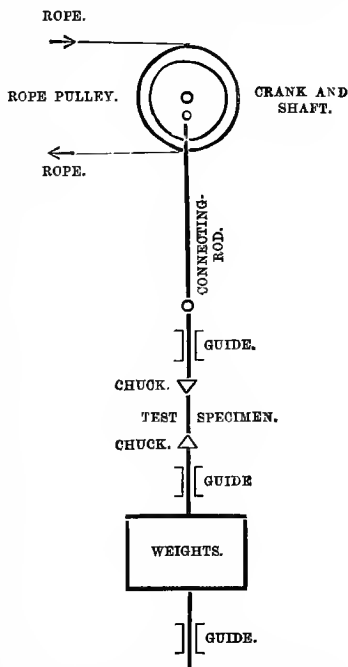


FIG. 13.—THROW TESTING MACHINE FOR REVERSALS OF MEAN STRESS.

fast by lock-nuts. Upon the lower spindle there were suspended any desired number of flat round disc weights; whilst the top end of the upper spindle was attached to a crank of $\frac{1}{2}$ " radius or "throw" by a connecting-rod 12" in length, and the whole truly arranged for vertical reciprocation by the three guide bushes, as shown by Fig. 13. The overhanging crank-pin was turned out of the solid from one end of its crank-shaft. This shaft was very strong and rigid, being supported by two bearings. It was uniformly rotated by a rope and coned pulley drive, the pulley being fixed to the back end of the crank-shaft, which extended beyond the after bearing.

The periodic, vertical, up-and-down motions of the two spindles (between which the test-piece is fixed) are approximately simple harmonic motions, due to the length of the connecting-rod being that of 24 cranks ($\frac{12''}{.5''}$), and

with good lubrication of the guide bearings, the friction was reduced to a minimum. The weight attached to the lower spindle *plus* the weight of the latter produced, by their inertia, a *tension stress at the bottom end* and a *compression stress at the top end of the stroke*. For a fixed load and speed, the stress per square inch of cross-section on test-pieces was varied by inserting specimens of different diameters; or, the load or the revolutions per minute of the crank could be varied at pleasure to produce different stresses.

Uniform Torque, Total Energy, and Balancing of the Moving Parts.—In order to enable the calculations of the stresses in the specimens to be correctly determined, the axis of the crank-shaft must be at rest, whilst the crank revolves with uniform angular velocity. This was effected by driving the crank-shaft with a constant turning effort or "torque," making the total kinetic energy of the moving parts constant for each experiment, and by good all-round balancing of these parts.

The total energy of the moving parts was rendered invariable by *first* attaching a second horizontal connecting-rod to a lug-pin on the large crank-head of the vertical connecting-rod, at the level line of the crank-shaft centre. This second connecting-rod was equal in length to 18 cranks, and gave a reciprocating horizontal motion to adjustable sliding weights (not shown in Fig. 13). Since the vertical and the horizontal weights each received their motions from the same crank-pin, the *velocity* of the one varied as the *sine* and the other as the *cosine* of the angular displacement of the crank.

Hence, the sum of the squares of their velocities were constants, since $\sin^2 \alpha + \cos^2 \alpha = 1$. Consequently, the *total kinetic energy* of the parts were rendered constant when the total weight moving in a horizontal direction was made equal to that moving in a vertical direction. *Second*, by a rigid system of balancing the rotating parts, the whole of the moving parts of the machine were dynamically adjusted.

Determination of the Tensile, Compressive, and Range of the Reversal Stresses, with Formulæ and Results on Mild Steels.—Referring back to Lecture XII., Vol. I., we see, on pp. 314 to 316, the formulæ and figure connecting *centrifugal force* with the *weight* and *acceleration* of a particle moving uniformly in a circle. In the figure (p. 315) alluded to, PA represents the connecting-rod at the top of the stroke. Here r/l vanishes, and we are left solely with the upward momentum of the vertical weight *minus* the dead weight, thus causing *compression*.

Now, when the crank moves round clockwise to its lowest position, r/l also vanishes, and we are again left with the downward momentum of the weight *plus* the dead weight, causing a *tension stress*.

Again, during the time that the crank arm moves clockwise at a uniform angular velocity from its highest to a horizontal position, the vertical

weight moves downwards, from zero velocity to a maximum velocity, with a certain acceleration. Hence, the rate of change of its momentum is equal to the product of its mass ($W \div g$) and its acceleration during that time; or, the force of the moving weight on the specimen is equal to its mass $\times$ its acceleration $\pm$ its dead weight.

Let r = Radius of crank in feet.

„ l = Length of connecting-rod in feet.

„ W = Weight of the moving mass in lbs.

„ v = Velocity of mass in feet per second.

„ ω = Angular velocity in radians per second.

„ N = Revolutions per minute.

„ n = Revolutions per second = $N \div 60$.

„ g = Acceleration due to gravity.

„ A = Area of the cross-section of specimen in square inches.

„ F_c = Force or stress due to compression in lbs.

„ F_t = Force due to tension in lbs.

In the "Throw Testing Machine," let the ratio of the crank radius r to the length of the connecting-rod l be taken into account, and let us note that the acceleration of the moving weight = $\frac{v^2}{r}$, whilst its mass is = $\frac{W}{g}$.

Then, the Force of Compression } $F_c = \left(\frac{W}{g} \times \frac{v^2}{r} - \frac{W}{g} \times \frac{v^2}{r} \times \frac{r}{l} \right) \frac{1}{A} - \frac{W}{A}$ lbs.
per sq. in. on the specimen

Or, " " $F_c = \left(\frac{W \omega^2 r^2}{g r} - \frac{W \omega^2 r^3}{g r l} \right) \frac{1}{A} - \frac{W}{A}$ lbs.,

i.e., " " $F_c = \frac{W \omega^2 r}{g A} \left(1 - \frac{r}{l} \right) - \frac{W}{A}$ lbs.

In the same way and from the previous reasoning, we obtain the formula for the tensile stress per square inch on the test-piece when the weight arrives at its lowest position.

Thus, the Force of Tension } $F_t = \frac{W \omega^2 r}{g A} \left(1 + \frac{r}{l} \right) + \frac{W}{A}$ lbs.
per sq. in. on the specimen

And the Range of Stress is equal to the sum of these two stresses.

Or, The Range of Stress = $F_c + F_t = \frac{2 W \omega^2 r}{g A}$ lbs. per square inch.

Results on Mild Steel.—In order to apply this formula to one or more of Dr. Smith's results, we here quote two of them:—

Oscillatory weight, 12.42 lbs. Diameter of specimen, .2469 inch.

SET A. MILD STEEL.	Revolutions per Minute.	Maximum or Tensile Stress per Sq. Inch.	Minimum or Compression Stress per Sq. Inch.	Range of Stress per Sq. Inch.	Number of Reversals before Rupture.
Annealed, . .	2,126	7.99 tons	7.11 tons	15.1 tons	248,700
Unannealed, .	2,122	8.03 tons	7.17 tons	15.2 tons	226,500

Now, applying the previous formula for *range of stress* to the first specimen, we get—

$$F_C + F_T = \frac{2 W \omega^2 r}{g A \times 2,240} \text{ tons per square inch.}$$

$$\text{Here, } W = 12.42 \text{ lbs.; } \omega^2 = (2\pi n)^2 = \left(\frac{2\pi N}{60}\right)^2; N = 2,126;$$

$$r = \left(\frac{.5067}{12}\right) \text{ feet; } g = 32.18;$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.2469)^2 = .04788 \text{ square inch;}$$

$$2,240 = \text{lbs. per ton.}$$

$$\text{Hence, } F_C + F_T = \frac{2 \times 12.42}{32.18 \times 2,240} \left(\frac{2\pi}{60}\right)^2 \times \frac{.5067}{12} \times \frac{N^2}{A} \left\{ \begin{array}{l} \text{tons per} \\ \text{sq. inch.} \end{array} \right.$$

$$\text{Or, } \text{Range of stress} = 1.595 \times 10^{-7} \times \frac{N^2}{A} \text{ tons per square inch.}$$

$$\text{That is, Range of stress} = (1.595 \times 10^{-7}) \frac{(2,126)^2}{.04788}$$

$$,, \quad ,, \quad ,, = 15.08 \text{ tons per square inch.}$$

This answer corresponds with the value given in the previous table.

Prof. Reynolds, in his preface to the previously-mentioned joint Royal Society paper, says, that Dr. Smith's experiments had brought out evidence of two general laws which had hitherto been unsuspected, viz. :—

(1) That under a given range of stress the number of reversals before rupture *diminishes* as the frequency *increases*.

(2) That the *hard steels* will not sustain more reversals with the *same range of stress* than the *mild steels* when frequency of the reversals is great.

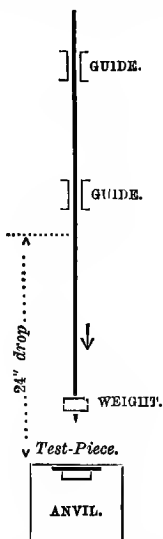
This later statement is borne out by the results given in the previous table, and some of Dr. Smith's other tables show signs of the first law, and he suggests that the explanation of the results will probably be traced to the *hysteresis effect*—i.e., to an accumulation of the lag which occurs in the elastic range.*

8. Impact Tests.—In addition to what was said in the previous lecture under the heading "Different Mechanical Tests," and to what has just been written under tests 6 and 7 on the "Fatigue of Metals," it may be remarked here, that impact tests carried out in the manner proposed by Messrs. Seaton and Jude *are necessary* for steels as well as for other metals which are to be afterwards subjected to *shocking treatment*—i.e., to rapidly-repeated loads of one kind, or to alternating loads. They found, that commercial brands of steel, which contained only a small percentage of carbon, gave the best shock-resistance results, and, that brittleness was

* *Mechanical Hysteresis.*—This term has been applied to the area enclosed between the *ascending* and *descending* stress-strain curves. These curves are obtained by gradually added loads from 0 to a maximum, and decreasing again to zero, in a similar way to the means taken to obtain *magnetic hysteresis* of iron or steel. (See Munro and Jamieson's *Pocket-Book for Electricians and Engineers*, 19th edition.)

increased very rapidly by an increase in the percentage of carbon. Further, that the line of fracture of the metal followed the direction of the ferrite by avoiding the pearlite. They also observed (as pointed out in the previous lecture under Kirkaldy's inventions), that oil quenching had the extraordinary effect of increasing the shock strength of certain commercial steels some 500 to 600 per cent.

Seaton and Jude's Impact Testing Machine.—This testing machine consists of a weight (of, say, 6 lbs.) being allowed to drop down



freely through a distance (of, say, 24 inches) upon the test-bar specimen of the metal in question, as clearly indicated by the accompanying line diagram (Fig. 14). The small specimen of a Λ notched bar $4" \times \frac{1}{2}" \times \frac{1}{2}"$, where the notch is only $\frac{1}{8}$ inch deep, is given by Fig. 15, and its position in the previous figure shows how it may be fixed upon supports having a horizontal distance between them of, say, 3 inches. After each blow the test-piece is reversed, and the apparatus is designed more for the purpose of testing the endurance of a material under a few repeated shocks rather than for testing under what shock it can be fractured by one or two blows. It is preferred to have the test specimens slightly notched, as shown by Fig. 15, because

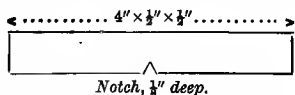


FIG. 14.—LINE DIAGRAM OF "IMPACT" TESTING MACHINE. FIG. 15.—"IMPACT" TEST-PIECE.

As used by A. E. Seaton and A. Jude.

most members in engineering structures have scratches, nicks, indentations, or even notches in some cases.

Advantages of the Impact Test.—It is affirmed by the above-mentioned advocates of this test:—

(1) That it is a gauge of the preponderant stress or "dislocating agent."
 (2) That materials which are sufficiently ductile will resist this impact or shocking test; but, that certain materials which give excellent tensile and elongation results will not stand this impact test.

(3) That 99 per cent. of all forgings are notched in some way or other, and that internal or external sharp edges cannot always be avoided in the design of machines, or even deep scratches in their construction.

Cumulative Effects of Small and Medium Shocks.—One effect which is sometimes confused with the phenomena investigated by Wöhler, but which should be treated as distinct from them, is the failure which is sometimes brought about through the cumulative effect of shocks. When a blow or shock expends kinetic energy in stressing a material or structure, the strain (which may be more or less general or local according to circumstances) is such, that the work done in producing it is equal to the energy of the blow. If the shock resulting from the kinetic energy of the blow be more than can be absorbed by the elastic property of that portion of the material or structure where it occurs, the elastic limit will be exceeded and local hardening will occur. If such shocks be oft repeated they may finally cause the collapse of the material or structure. This may be shown graphically by means of an ordinary stress-strain diagram.

Effect of allowing a Factor of Safety for both Fatigue and for Impact.—It is a matter of controversy among certain civil engineers whether it is necessary to allow for repeated impact shock effects in addition to those of mere fatigue stresses. In bridge work, for example, it is found, that the members become exceedingly large and heavy when both are allowed for, and therefore it is often assumed that, as the stresses are more or less intermittent, the allowance for fatigue alone, sufficiently covers those due to impact shocks. The stresses allowed for in the case of, say, wrought-iron bridge-work are only about 5 tons per square inch, and for steel about $6\frac{1}{2}$ tons per square inch. A period of rest for a fatigued structure most certainly tends to restore its elasticity, but whether it also actually restores the metal to its original strength (without any annealing) is not known. It is, however, certain that even annealing will not bring a material back to its best original or best normal condition when cracks or cleavage have once been started in its molecular structure.

Drop or Falling Tests.—Drop testing has always been favoured by railway companies for proving rails and car axles. The apparatus consists of a large anvil on which are supported a pair of upright guides and the beam to be tested, with a falling weight which can be raised to any definite height between the guides. Rigidity and inertia of the anvil are of considerable importance. Messrs. Cammell & Co. of Sheffield use a weight of 1 ton, a fall of 20 or 30 feet, and a rail span of 3 feet. The fracture under successive blows results from the accumulated effect which is brought about in consequence of the energy of each blow being greater than can be absorbed by the elastic strain.

9. Torsional Tests.—The usual arrangements for torsional tests in large tensile testing machines are found to be not only inconvenient, but are sometimes essentially incorrect in principle. Moreover, such machines admit but short specimens. Here, even more than for transverse tests, the use of a separate machine is very advisable. The machine shown by a side elevation and plan in Fig. 16, was used by Prof. Kennedy at University College, London. The twist is applied through a worm wheel and works with a maximum torque of 4,000 lb.-inches. The test-piece *a* is turned at its ends, centred at one end in the mandril of the worm wheel, and at the other end in the boss of the steelyard. Before the stress is applied, the test-piece lies quite freely in the bored holes which centre it.

Also, the test-piece is at all times free to move endwise, and is driven from an arm on the mandril through a friction-clutch. A similar friction-clutch at the other end of the test-piece transmits the pressure to the steelyard, and causes it to rise from its fulcrum. An arrangement is made by which the actual lift of the steelyard end of the test-piece is kept so small as to be negligible. During a test, the steelyard is kept floating by moving the poise-weight outwards along the beam as the twisting moment is increased. A machine capable of twisting asunder an inch bar of steel with a twisting moment of 14,000 to 16,000 lb.-inches is found to be sufficient for most laboratory purposes.

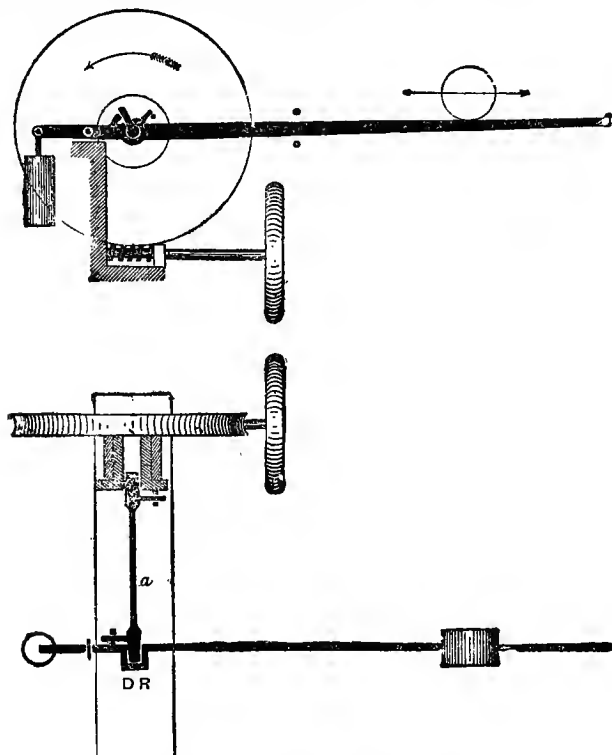


FIG. 16.—MACHINE FOR TORSIONAL STRESSES ON TEST-PIECES.

Should it be found necessary to record the torsions throughout a test, then an automatic disc recorder, faced with paper and supplied with a spring-pressed pencil, may be fixed at D R, or a plain drum recorder may be used. Specimen torsion diagrams, taken in both these ways, with an example, will be found in the next lecture.

Torsion of Rods and Wires.—The following Fig. 17 illustrates an arrangement of apparatus for attaching to a wall. The modulus of torsion in rods up to 1 inch in diameter and 2 feet long can be obtained by the aid of the two reading telescopes and mirrors as indicated in the next lecture.

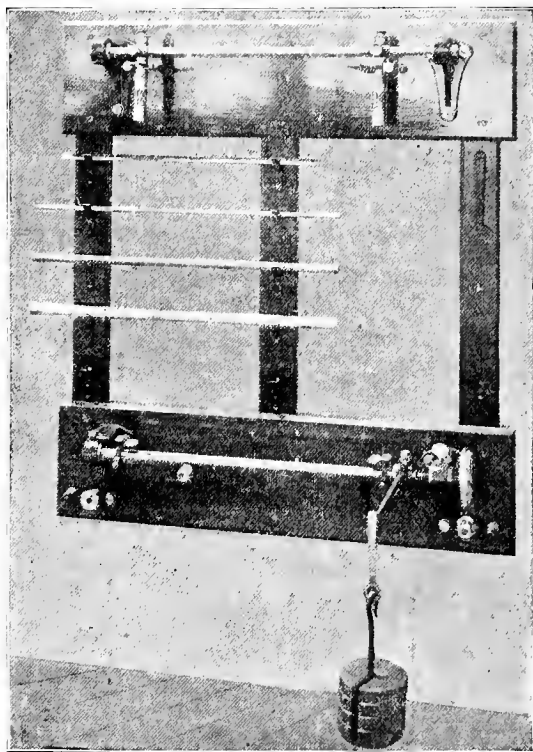


FIG. 17.—APPARATUS FOR MEASURING THE MODULUS OF TORSION IN RODS.

As made by The Cambridge Scientific Instrument Co., Ltd.

Wire Twisting and Bending.—Wires for sheathing submarine cables or ropes are tested, for *twistability*, by taking a specimen from each end of each bundle, and fixing one end of the specimen in the rigid clamp of a hand twisting machine, whilst the other end is gripped by the chuck of the turning handle. The distance between the fixed clamp and the inner end of the chuck may be about 10 inches. A straight black ink line is drawn along the upper surface of the test-piece, and the handle is turned round

slowly until the specimen breaks. The broken parts are brought together and the number of complete spirals or pitch-screw ink thread lines are counted along a length of, say, 6 inches, in order to see whether the specimen has complied with the *torsion test*, as specified in the agreement. The following table shows two of the mechanical tests applied to what is known as "Best Best Annealed Galvanised Iron Wire" for the sheathing of certain submarine cables:—

TABLE OF SIZES, TWISTING AND BENDING TESTS, FOR THE IRON SHEATHING WIRES OF SUBMARINE CABLES.

Weight of Each Wire Bundle.	Diameter of Wire.	No. of Twists in 6 Inches.	Bending and Unbending round a Spindle.
162 lbs.	0.143 inch.	13	0.572 inch.
200 lbs.	0.180 inch.	10	0.720 inch.
280 lbs.	0.230 inch.	6	0.920 inch.
290 lbs.	0.238 inch.	5	0.952 inch.

10. Hardness, Temper and Brittleness.*—It is well known, that the physical properties of ductile metals are considerably altered by different kinds of treatment. For example, cold rolling, cold hammering, wire drawing, or stretching will convert a comparatively soft and tough material into a harder and more brittle one. In the case of steel, the physical properties may be altered to a still greater extent by differences in thermal and chemical treatment. The differences in the mechanical properties of steel and other metals are of the greatest practical importance to engineers and metal workers. Under certain circumstances the hardening of a material by special treatment is very useful, such as in the tempering of tool steel. But, in other cases, a material may be quite trustworthy whilst in its normal state and be rendered useless by hardening. Consequently, to get rid of what may be termed "artificially or accidentally induced hardness," processes of annealing are applied after certain mechanical operations.

The ordinary processes of testing by which the tenacity and elongation of a material are measured, indicate indirectly the condition of the specimen, but are open to misinterpretation. For example, a simple tension test of a rail or axle does not clearly discriminate whether the material is properly annealed or not, and, in such cases, a more direct and sensitive test for hardness is required.

* Students are referred to the "Treatise on Hardening, Tempering, Annealing, and Forging of Steel," by Joseph V. Woodworth, and published by Arch. Constable & Co., Limited, 1903; and to the Final Report by Sir F. A. Abel, *Proc. Inst. M.E.*, 1885, on "Steel Hardening," &c.

Definition of Hardness.—Hardness is a property of materials about the definition of which physicists are not agreed. Moreover, they have not specially approved of any one method for accurately measuring or of comparing hardness. Excluding very brittle materials, which crush to powder under pressure, and confining ourselves specially to the metals which are used in engineering constructions, *hardness may be defined as resistance to permanent or plastic deformation.* Consequently, the resistance to indentation by a very hard tool has been taken as a measure of hardness by some experimenters.

Steel containing more than the semi-saturation proportion of carbon ('45 per cent.), when heated to a bright redness and then suddenly cooled, is found to acquire a different kind of hardness from that which is produced by overstraining; for it becomes more brittle and more elastic than before. For instance, if a piece of high-carbon tool steel be heated to a white heat and then plunged into a bath of ice-cold water or mercury, it becomes hard enough to scratch glass like a diamond, and will be very elastic. It is, however, so brittle, that the steel can only be used for such a special purpose as, say, the drilling of tempered steel or chilled iron. Steel treated in this way is called "glass-hard," and entirely loses its plastic character. When this kind of steel is tested for tension it snaps suddenly under mere elastic extension, without any appreciable diminution of its cross-section, and it has but a comparatively small tensile resistance. However, glass-hard steel may be deprived of its brittleness, its strength increased, and the range of its elastic strain greatly increased by reheating it up to a lower temperature and then allowing it to cool slowly. This process is called "withdrawing the temper," and its success or failure depends chiefly on the degree to which the temperature of the steel is thereby raised before slow cooling begins. The different grades of temper which are produced in this way are usually estimated by the colour which appears on the sandstone polished surface of the steel during its reheating. The changes of colour are due to the formation of a varying film of oxide. For example, if the glass-hard metal be reheated to, say, a temperature between 400° F. and 420° F. a light straw-colour appears on its surface. This colour shows that the steel has reached a temper suitable for the points of keen edges in hard cutting tools. Again, raise the heat and hue to, say, a violet-yellow at 500° F., when a temper fit for table knives, where flexibility as well as incisive cutting is required rather than a hard, stiff edge. A temperature of about 550° F. will produce a purple-blue surface and a temper suitable for springs and all articles where the elasticity should be very perfect throughout a wide range of loads. At a temperature of 725° F. the steel passes to a red heat, at which, some kinds of new fast-speed tool steels do the most extraordinary amount of work at a hitherto unsurpassed speed.*

The term *under-tempering* is applied to any process which tends to reduce the hardness of steel to a degree recognisable by the "colour test," and also to all processes by which the degree of hardness is lowered, modified, distempered, or lessened in brittleness. Any process which does not reduce the hardness of steel to a degree denoted by some colour within the range of the recognised "colour test" may be termed *hardening*. The general adoption of a thermometer test as a guide for tempering steels would remove the valid objection to the very old colour test—viz., that

* See 6th and later editions of the author's *Manual of Applied Mechanics* for the treatment and capabilities of this new tool steel.

the colour obtained on the piece of steel through heating it is no true indication that it possesses a temper above its natural degree of hardness; because steel, wrought iron, and cast iron will assume all the colours of the "colour test" when polished and heated to the necessary temperature. Thus, the colour which appears on the surface of a piece of steel is simply an indication that it has been heated to a certain temperature and not that it has been actually tempered, or, in fact, that the heating has in any way changed its hardness or its softness!

Workshop Test.—A mechanic tests hardness by using a file, and the earliest scientific scale of hardness was that proposed by Moh. He selected ten substances and arranged them in order, so that each would be scratched by the substance next above it, and would scratch that substance next below it. This simple "scratch test" has been found very useful in the case of brittle bodies; but it is less useful for ductile bodies.

*Turner's Scratch Test.**—Mr. Thomas Turner has introduced a more definite and exact scratch test of hardness. A diamond point is balanced at the end of a lever and loaded till it just definitely scratches the surface. The load necessary to produce a scratch is taken as the measure of hardness or hardness number. This method requires considerable skill, and it does not seem sensitive enough to discriminate the quality of ductile metals, such as iron and mild steel.

Calvert and Johnson's Test.—Some experimenters have used an abrasion test in determining the hardness of a metal, but this method is again difficult and laborious, and appears to be best adapted for brittle substances, such as stones. Consequently, the only method which has been much used in practice is an indentation test. Messrs. Calvert and Johnson took, as their measure of the hardness of alloys, the weight which caused a small truncated cone to indent the alloy to a depth of $3\frac{1}{2}$ millimetres in half an hour. Whilst, in some tests performed in the United States, the volume of indentation produced by a pyramidal point loaded with a weight of 10,000 lbs. was taken as a measure of hardness. An indentation of $\frac{1}{8}$ cubic inch was taken as unit hardness, and half that indentation was called hardness number 2.0.† This is a much more definite and scientific method of determining hardness in materials which are ductile or plastic.

Unwin's Test.‡—The apparatus devised and used by Prof. Unwin, F.R.S., is indicated by the line diagram (Fig. 18). It consists of a loosely-fitted plunger sliding down vertically in a cast-iron guide-block, and pressing an excessively hard $\diamond$ or "indenting tool" placed upon the test-

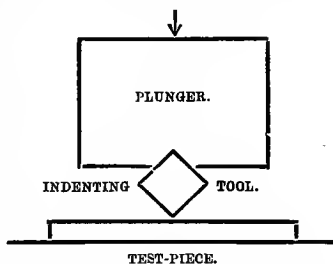


FIG. 18.—PROF. UNWIN'S TEST FOR HARDNESS.

* See *Philosophical Magazine*, 4th Series, vol. xvii., p. 114.

† See *Reports on Metals for Cannon*, 1856, Ordnance Department, U.S.A.

‡ See *Proc. Inst. C.E.*, vol. cxxix., p. 336, for paper on "A New Indentation Test for Determining the Hardness of Metals," by Prof. Unwin, F.R.S., &c.

piece.* The whole apparatus is placed under an ordinary testing machine whereby loads are applied, and the indentation corresponding to each increase of load is noted. Each load is allowed to rest on the specimen for a few minutes, until the indentation ceases to increase. In no case must the specimen be stretched during the experiment.

Comparison between Prof. Unwin's Indentation Method and Turner's Scratch Method of Determining Hardness.—The following table contains the hardness numbers for the same specimens, as determined by the *indentation method* and by the *scratch method*. In order that they may be easily compared, the relative hardnesses have been calculated by taking copper as unity. It will be seen, that the relative hardness follows the same order by whichever method it is determined. But, the scale of hardness with the indentation method is an opener scale than that with the scratch method. Further, the indentation method is found to be easier, more definite, and requires less skill. Of course, for brittle substances the scratch method is more suitable.

HARDNESS OF METALS BY INDENTATION AND SCRATCH METHODS.

Materials.	Hardness Number.		Relative Hardness. Copper = 1.	
	Indentation Method.	Scratch Method.	Indentation Method.	Scratch Method.
Cast steel (normal), .	554.0	25.0	8.94	4.17
Brass (No. 1), .	221.0	12.0	3.57	2.00
Mild steel (normal),	143.5	9.0	2.32	1.50
Copper (annealed), .	62.0	6.0	1.00	1.00
Aluminium (squirted),	41.8	4.0	.67	.66

* "The indenting tool is simply a short bar of square tool steel, ground accurately, so that the angles are right angles and as hard as possible. Any one of the edges may be used as an indenting edge, and the tool is easily re-ground. The test-piece is a square bar of the metal to be tested, usually $\frac{3}{8}$ inch square and about $2\frac{1}{2}$ inches long. A scale and vernier are attached to the guide-block for measuring the indentations."

LECTURE VI.—QUESTIONS.

1. Explain why designers of torpedo boats aim at giving the hulls of these very fast short vessels as uniform elasticity as possible throughout their length, in addition to a maximum of strength with a minimum of weight. State how they accomplished this object.

2. State your views concisely *re* shearing *versus* planing and punching *versus* boring of low-grade and of high-class plates for engineering structures. Show, by sketches, &c., how the ultimate cost of boring such plates may be minimised. Also, how both shearing and punching affect the condition of mild and the harder kinds of steel plates.

3. What objections can you offer to the punching and subsequent drifting of holes in boiler and other steel plates which have to be riveted together?

4. Why is it of importance to have reliable results on the "fatigue of metals" which are used in certain machines and structures?

5. Mention a few of the chief experimenters upon the fatigue of metals with dates, and give, in concise general terms, the results of their work.

6. Explain as concisely as you can, the methods adopted by Wöhler and the deductions derived from his experiments.

7. Give Gerber's formula as based on Wöhler's experiments, and explain the same in detail.

8. Explain in your own words Bauschinger's deductions on the "elastic limits" of materials.

9. How is heating produced in a metal when it is subjected to severe stresses? How do rest, re-heating, and annealing relieve the strains of a stressed metal bar.

10. Explain concisely Prof. Ewing's conclusions *re* the inter-molecular crystalline action of metals subjected to severe stresses and fatigue.

11. Explain, by aid of a sketch, the action of the machine, as suggested by Prof. Reynolds, which was designed and used by Dr. J. H. Smith in carrying out a series of experiments on the tensile and compressive reciprocating reversal stresses on round, short test-specimens of iron and steel.

12. Explain how the formula is derived and applied to the machine and test-pieces of the former question, by using the data for the second example in the text on unannealed mild steel.

13. Explain, by aid of line diagram figures, how Messrs. Seaton and Jude's impact tests for notched and unnotched bars are carried out. State the necessity for and the advantages of this test for certain materials, as used in certain machines and structures. State why ordinary tensile, compressive, and other tests do not give the same direct useful information which impact tests afford for such cases.

14. Explain, by aid of simple sketches, a form of torsion testing machine for round bars.

15. How are samples of wire for ropes and for the sheathing of submarine cables tested? Give a sketch of the machine you would propose to carry out such tests, and what would happen if the contractor's man drew a straight ink-line along the upper and the lower surfaces of the wire when it was tested? If you were not present during the test, how would you detect the mistake?

16. How would you test plates or bars of steel for hardness, and how would you register your results?

17. How would you test a metal for brittleness? What results would you expect in the case of a very brittle specimen?

N.B.—See Appendices B and C for other questions and answers.

LECTURE VII.

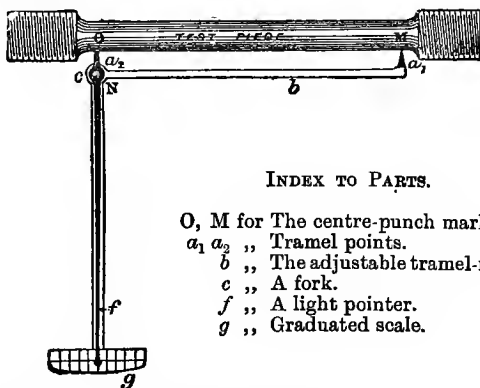
STRESS-STRAIN DIAGRAMS AND ELASTICITY OF MATERIALS.

CONTENTS.—Autographic Apparatus for Recording Stress-Strain Diagrams—Prof. Hele Shaw's Apparatus for Automatically Drawing Stress-Strain Diagrams—Prof. Kennedy's and H. G. Ashcroft's Instrument for obtaining Stress-Strain Diagrams—Automatically Drawn Stress-Strain Diagrams, giving the Elastic, Working, and Ultimate Strengths of a Material—Elastic-Strain Diagrams—Plasticity—How to obtain a Measure of the Work done per Cubic Inch by aid of the Autographic Diagram, when subjecting a Test-Piece to a Tensile Stress—Influence of Time on Stress and Strain as shown by the Diagram—Torsion Tests—To find the Work Done per Cubic Inch by aid of an Autographic Diagram when Subjecting a Test-Piece to a Torsion Test—Searle's Apparatus for Measuring the Extension of a Wire and of Determining Young's Modulus of Elasticity—Prof. Ewing's Extensometer for Measuring the Elastic Extension, and Young's Modulus of Elasticity of Specimens of Metal under Tensile Tests—Testing Machine with Extensometer attached for Measuring the Elastic Extension of Rods—Measurement of Young's Modulus in Wires—Measuring Young's Modulus by Bauschinger's Extensometer—Extensometer applied to Measure the Elastic Compression of Short Blocks—Apparatus for Determining the Deflection of Beams and Cantilevers—Measurement of Young's Modulus of Elasticity by Deflection of Beams and Cantilevers—Measurement of the Modulus of Rigidity by the Torsion of Wires—Measurement of the Modulus of Rigidity by Torsional Oscillations—Questions.

Autographic Apparatus for Recording Stress-Strain Diagrams.*—In laboratory testing, the relation of extension beyond the elastic limit to the load throughout the test, may easily be observed by applying a pair of beam compasses to two punch-marked points on the test-piece from time to time as the test proceeds, and transferring the distance to a scale upon squared paper. The original distance between the two points is usually from 8 to 10 inches. Or, Prof. Kennedy's mechanical extensometer may be used, as seen from Fig. 19. When testing is to be done quickly, and, since a knowledge of this relation is wanted afterwards, some form of autographic recording apparatus is found to be very convenient.

* Students should refer back to the first lecture on Testing Machines before studying this one. See *Proc. Inst. Mech. Engs.*, 1886, for paper giving the "Description of an Autographic Test Recording Apparatus," by J. Hartley Wicksteed, of Leeds.

In most of the arrangements which are used in practice, the diagram is drawn by the relative movement of a pencil and a sheet of paper wound on a drum, in the same way that indicator cards are taken from the cylinders of steam engines. The one component of the motion is proportional to the extension, and the other component to the travel of the weight by which the load is measured.



INDEX TO PARTS.

- O, M for The centre-punch marks.
- $a_1 a_2$ „ Trammel points.
- b „ The adjustable trammel-rod.
- c „ A fork.
- f „ A light pointer.
- g „ Graduated scale.

FIG. 19.—PROF. KENNEDY'S MECHANICAL EXTENSOMETER.*

In the single-lever vertical testing machine, as shown by Fig. 6, a mechanical autographic recorder is attached.

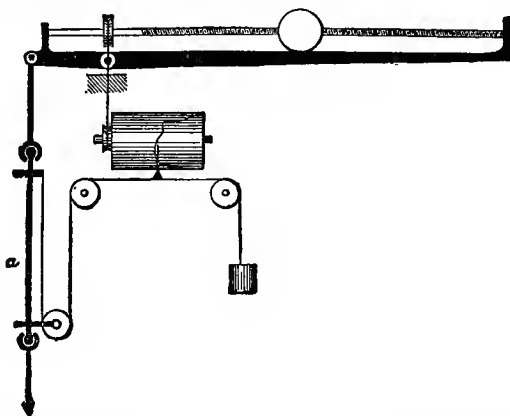


FIG. 20.—PROF. UNWIN'S AUTOMATIC TEST-RECORDING APPARATUS.

* Figs. 19 to 26 and 33 are from Prof. Kennedy's paper on "Engineering Laboratories," by his kind permission and by the favour of The Institution of Civil Engineers. See *Proc. Inst. C.E.*, vol. lxxxviii.

In the above Fig. 20 Prof. Unwin's recorder is shown supporting the paper drum horizontally on the main column. The paper drum is driven round synchronously with the rotation of the screw which moves the travelling poise-weight along the weight-beam. The extension of the test-piece is taken by having two clips firmly secured to the test-piece at points, say, 8 inches apart, by a fine wire or inextensible cord attached to the upper clip and passing under a pulley on the lower clip, then over a pair of pulleys parallel to the paper drum. The pencil is attached to the wire between the last pair of pulleys and traces on the paper of the drum the stress-strain diagram, as clearly indicated by the figure.

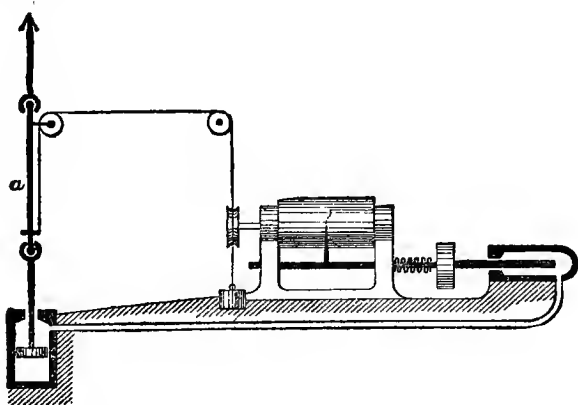


FIG. 21.—WICKSTEED'S AUTOMATIC TEST-RECORDING APPARATUS.

In the hydrographic recorder, as shown by the accompanying Fig. 21, and which can only be used with machines having a hydraulic pressure ram, the drum is pulled round by a wire from the test-piece, through distances proportional to the extension, and the pencil takes its motion, not from the travelling poise-weight, but from the piston of an auxiliary hydraulic cylinder in free communication with the hydraulic cylinder of the machine. This piston compresses a spring in its advance, consequently its displacement measures the force with which it is pressed out. The friction of the piston is eliminated by keeping it in continuous rotation, and this is found to make it indicate correctly, the pressure in the main hydraulic cylinder acting on the specimen. The net load on the test-piece does not bear a definite relation to the pressure in the main cylinder owing to the friction of the ram. A uniform scale can, however, be found, because the friction of the ram is proportional to the pressure, and the values can be interpreted by occasional reference to the weigh-beam.

Hence, we get a record of the point of first permanent set and extension at maximum load, while a stress-strain diagram is automatically drawn, showing the behaviour of the test-piece in yielding to the stresses imposed upon it from the start to the finish of the test.

Prof. Hele Shaw's Apparatus for Automatically Drawing Stress-Strain Diagrams.—The essential principles of this apparatus are identical with those of Mr. Aspinall and Professor Unwin. A front and a side elevation are given in Fig. 22, whereby its action will be easily understood. The front elevation shows a portion of a Wicksteed single-lever testing machine, to which Prof. Shaw's apparatus is attached.

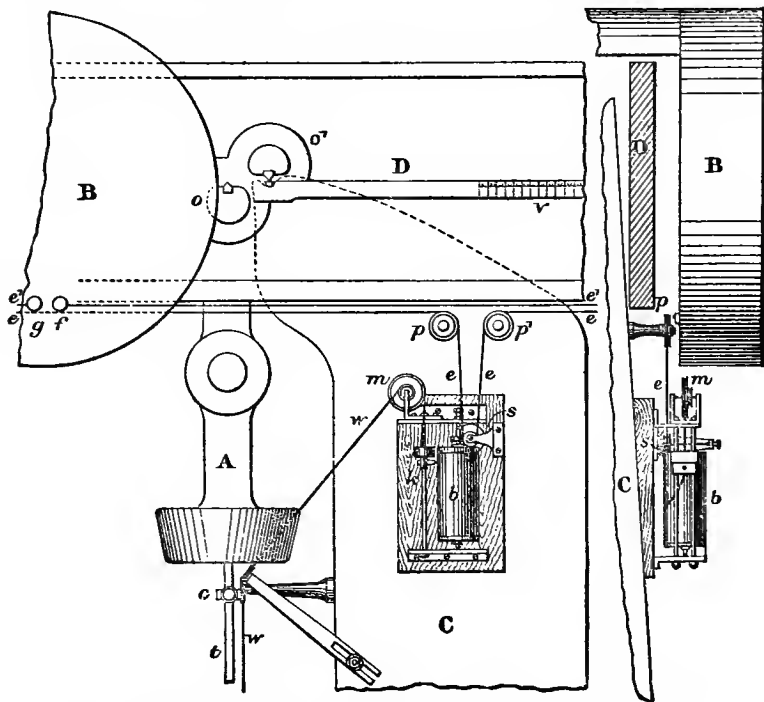


FIG. 22.—PROF. HELE SHAW'S APPARATUS FOR AUTOMATICALLY DRAWING THE STRESS-STRAIN DIAGRAM OF A TEST-PIECE.

It consists of a cord fixed at the two points *g* and *f* to the jockey weight *B*. This otherwise endless cord passes round a pulley at one end of the lever *D*, then round a pulley *p*, and finally (by means of another pulley *s*, on the axis of which is a worm) it turns the barrel *b* with the paper on which the diagram is to be drawn. The cord then passes away to the other end of the lever and is led back to the other point of attachment. Thus, inasmuch as the position of the weight on the lever gives the exact measure of the force on the specimen by means of a scale *v*, the distance turned through by the barrel *b* is directly proportional to the stress on the test-piece *t*. The strain is recorded in the usual way, for one end of a wire *w* is attached to one of the points on the test-piece, passes over the

pulley *c*, and finally to a small pulley on the same spindle as the larger pulley *m*. Another wire passes over the larger pulley *m* and carries the recording pencil *k*, whilst the tension on the wire is maintained by a suspended weight shown above *k*. Consequently, we have two motions at right angles to each other:—(1) The motion of the barrel about its axis, which gives a measure of the stress, and (2) the motion of the pencil up and down, which gives the measure of the strain. It might at first be thought that the possible stretching of the cord would lead to an incorrect result, but no one will think so when reminded, that the cord travels 15 feet for a 6-inch travel of the barrel, and therefore any possible error from this cause is reduced 30 times by this means. The side view of Fig. 22 shows the barrel *b* in position, and how the jockey-weight *B* clears the apparatus.

Prof. Kennedy's and H. G. Ashcroft's Instrument for obtaining Stress-Strain Diagrams.—*Principle of the Instrument.*—It will be seen from Fig. 23, that the test-piece *a* is placed in the machine in series with a stronger bar *b*, called a "spring-piece." The two bars are connected directly by a simple coupling, and are pulled simultaneously, the one through the other. The spring-piece is of a material such that its limit of elasticity occurs only at a load greater than that which will break the test-piece. It must also be of a perfectly elastic material, as ascertained by previous experiment, so that its extension is strictly proportional to the pull on it, and therefore to the pull on the test-piece. A very light pointer *c* is made to swing about an axis through an angle proportional to the extension of the spring-piece, and to the pull on the test-piece. The end of this pointer always touches a sheet of smoked glass *d*, to which a travel in its own plane is given proportional to the extension of the test-piece.

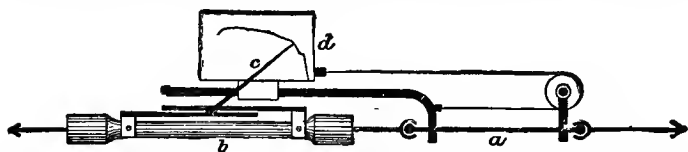


FIG. 23.—PROF. KENNEDY AND ASHCROFT'S INSTRUMENT FOR OBTAINING A STRESS-STRAIN DIAGRAM.

In this way the diagram is drawn. By an arrangement of differential levers it is assured that the motion of the glass depends solely on the extension between the marked points on the test-bar, so that no amount of extension in the coupling, or ends of the test-piece, or in any other part of the apparatus, can move the glass. Also, the instrument is so arranged that the absolute elongation of the spring-piece does not cause any motion of the pointer relatively to the glass.

Advantages and Disadvantages of the Instrument.—It cannot be said to be suitable for general use in works testing; but for laboratory testing, where the instrument is in skilled hands, and not subject to rough usage, it is believed to be, on the whole, more sensitive and to give better diagrams than any of the other forms. It has also the advantage, that it is entirely independent of either the poise-weight or the ram, or even any part of the framing of the testing machine. Its hollow, circular, tapered aluminium pointer may be made so light, that the diagram can be assumed to be free

from any errors due to inertia. The diagrams drawn by this instrument have the drawback, that their load-ordinates are arcs of circles instead of straight lines, and Prof. Kennedy said, in his Inst. C.E. paper on "The Equipment of Engineering Laboratories," that he had not yet succeeded in devising a plan for getting rid of this difficulty without unduly interfering with the sensitiveness of the instrument.

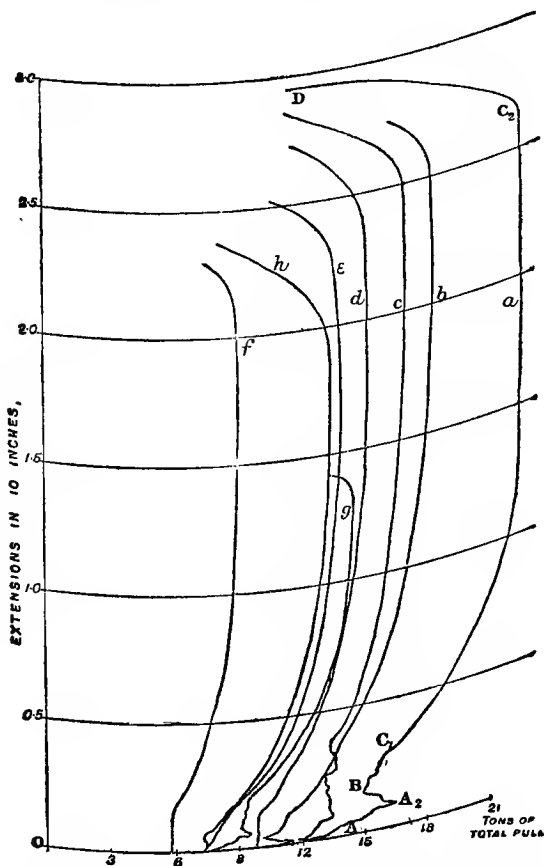


FIG. 24.—AUTOMATICALLY DRAWN STRESS-STRAIN DIAGRAMS.

By the Kennedy and Ashcroft Instrument.

Swedish bar iron, *a*; Shelton bar iron, *b*; Swedish iron, *c*; Landore rivet steel, *d*;
Landore plate, *e* and *f*; cast steel, *g*; mild steel bar. *h*.

Automatically Drawn Stress-Strain Diagrams, giving the Elastic, Working, and Ultimate Strengths of a Material.—It is well known, that the life of most ductile materials, such as wrought iron and mild steel, when of uniform section, and placed in a testing machine with the load gradually increased, will be represented by diagrams similar to those indicated in Fig. 24.

Here, $O A_1$ represents the very small elastic extensions which are strictly proportional to the stress, and therefore correspond to some constant value of the modulus of elasticity. Should the stress be removed from the test-piece at any time, during the elastic stage, the test-piece will be found to return to its original length.

The stage $A_1 A_2$ is not always present in test-pieces. In this stage, the strain increases faster than the stress. If the load be removed during this part of the experiment it will be found, that the test-piece does not return to its original length, but it will have received a small amount of permanent set.

A_2 is the point which is called the “break-down” point; or better still, the “yield-point,” where the character of the material changes.

$A_2 B$ shows a remarkable feature; for the apparent fall or “release” of stress of about 2 tons which takes place after the yield-point, is accompanied by a sudden rapid extension, without any increase of the load.

$C_1 C_2$ represents the main part of the stretch of the bar, while it is in a condition of “uniform flow” or of *semi-plasticity*. It is found from observations, that the actual extension of the test-piece in this stage, depends partly on the time during which the load acts. Also, that the extension increases with time even when the load is not increased.

C_2 shows the point of maximum load, usually termed the *breaking load*.

The curve $C_2 D$ shows the decrease of stress as the bar draws down or flows locally. The test-piece is now ruptured at the point D , which is called the *terminal load*.

The stage $A_1 A_2$ is sometimes absent or minute. $A_2 B C_1$ takes various forms; and $C_2 D$ also takes very various forms, or may be absent altogether. But a complete test-recorder ought to be able to show all the stages accurately, and where they exist. It has been said, that the stages $C_2 D$ and $A_2 B$ are not of any practical importance. It is quite possible, that for works testing, both these stages may be neglected, without the loss of much valuable information. But, it should not be overlooked, that the former gives very much the same information about any material as is obtained by measuring its reduction of area. It seems inadmissible to neglect both or either of these stages in scientific experiments, not only for the sake of accuracy, but because there is by no means a full knowledge of the practical bearing of either of these stages. In general practice, the *yield-point* is taken as the *elastic limit of the material*, and is the limit of stress in tons per square inch.

Elastic-Strain Diagrams.—The difficulty in the way of obtaining a proper diagram of the elastic part of a stress-strain diagram lies in the fact, that the extensions for ordinary test-pieces of 10 inches in length must be magnified at least 100 times, in order that the curve may be useful. Professor Unwin devised for this purpose a “semi-automatic” apparatus, which, while it did not actually draw a diagram, it determined a number of points for the curve. Professor Kennedy improved and modified this diagramming apparatus, to give the desired curve about 150 times its full size upon smoked glass. The spring-piece shown in conjunction with the test-piece in Fig. 23 is not now used, but the tracing pointer is placed on the test-piece and used to measure its exten-

sions. The frame is also carried by the test-piece, and the smoked glass is moved by the poise-weight. It requires very careful manipulation to ensure, that the glass starts at the instant at which the test-piece begins to stretch, for any inaccuracy in this matter spoils the starting-point in the diagram, although it leaves the rest of the curve unaffected. The ordinates which in this case represent extensions are curved, like those given in the stress-strain diagram (Fig. 24), and require rectification before the real nature of the curve can be seen. Although the apparatus is thus by no means in an ideal form, yet the diagrams (Fig. 25) which it gives are at least accurate and automatically drawn. The diagrams give a record of facts which are of great interest, but which have hitherto been known only as the result of long and laborious detail measurements and plotting on squared paper.

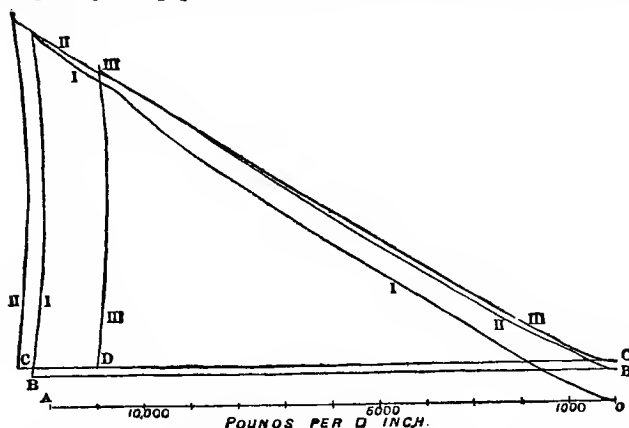


FIG. 25.—AUTOMATICALLY DRAWN ELASTIC-STRAIN DIAGRAMS.

Notes.—These diagrams were taken from a piece of Blaenavon cast iron, 75 inch in diameter and 10 inches long.

Curve I. was first drawn. It may be observed, that this test-piece shows considerable set after about 10,000 lbs. per square inch had been applied. It comes back finally to a new zero line, on removing the load, and shows a set A B of about 1+750 inch.

The load was at once re-applied and gave Curve II., which finally almost runs into Curve I., but shows a small set B C on the removal of the load.

Curve III. was then drawn, and it almost coincides with II., and finally goes back to the same zero line at the point D, thus showing no further set. The diagram exaggeration of the extension is 130 to 1. The small continuations of the extensions at the end of each experiment apparently show a "time effect."

Plasticity.—It is important not only to ascertain the breaking strength of the material to be used in a structure which is to be subjected to live loads and shocks, but also to determine its capabilities to resist deformation without rupture. In materials such as wrought iron and mild steel plasticity is combined with a high tensile strength, but with cast iron and hard steel this property is absent. Consequently, we find it usual in specifying for wrought iron and steel to require a certain percentage of elongation and contraction of area, as well as a certain breaking stress of so many tons per square inch. The plasticity of a material is measured by its final elongation and contraction of area. As previously stated, the elongation is taken on a length of 8 to 10 inches of the test-piece.

To Obtain a Measure of the Work Done per Cubic Inch by aid of the Autographic Diagram, when subjecting a Test-Piece to a Tensile Stress.—Remembering that work done is the product of the force into the linear distance through which it acts, and that the ordinates on the stress-strain diagram represent to scale the loads in tons while the extensions are measured as abscissæ; then the whole area of the stress-strain diagram represents, to some scale, the work done upon the test-piece in fracturing it. Thus, if x inches on the vertical = 1 ton, and y inches on the horizontal = 1 inch of extension of the test-piece, we get—

$$\text{Work done in inch-tons} = \frac{\text{area of stress-strain diagram in square inches}}{xy}$$

The area of the diagram can be found by means of Simpson's rule or by a planimeter, and if we divide the work done in inch-tons by the volume of the test-pieces in cubic inches, we obtain the work done per cubic inch.

$$\text{Work done per cubic inch} = \frac{\text{work done in inch-tons}}{\text{volume of test-piece in cubic inches}}$$

In order to compare results for the work done per cubic inch in breaking a bar of any material, the test-pieces should be similar, because the ultimate elongation depends on the original length, and to some extent upon the cross-sectional area for the same material.

Prof. Kennedy's Approximate Method.—Prof. Kennedy has pointed out, that the curve during the ductile and plastic stages is a very close approximation to a parabola. Consequently, if we assume it to be so, the work done in inch-tons per cubic inch upon the test-piece can be calculated without the aid of a diagram.

Let f_t = Elastic limit stress in tons per square inch.

„ $f_{max.}$ = Maximum stress in tons per square inch.

„ L = Length of the bar between the datum points in inches before extension commences.

„ l = Extension in inches.

Then, the Work done in inch-tons per square }
inch of cross-section of the test-piece

$$\left. \begin{array}{l} \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \end{array} \right\} = f_t l + \frac{2}{3} (f_{max.} - f_t) l$$

∴ Work done in inch-tons per cubic inch of }
the test-piece

$$\left. \begin{array}{l} \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \end{array} \right\} = \frac{l}{3L} (f_t + 2f_{max.})$$

But the percentage ratio of extension to original }
length

$$\left. \begin{array}{l} \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \end{array} \right\} = \frac{l}{L} \times 100 = \% e$$

∴ the Work done in inch-tons per cubic inch = $\frac{\% e}{300} (f_t + 2f_{max.})$

N.B.—The best practical method of arriving at the suitability of any given material for withstanding shocks and blows is by obtaining the work done in inch-tons per cubic inch of the test-piece.

EXAMPLE I.—In a tensile test of a piece of flat wrought-iron bar, the following results were obtained:—

- (i.) Original dimensions of cross-section 2·010 inches by 0·505 inch.
- (ii.) Final dimensions of cross-section at point of fracture 1·520 inch by 0·410 inch.
- (iii.) Gross load at limit of elasticity 34,500 lbs.
- (iv.) Gross load at fracture 53,100 lbs.
- (v.) Total extension on length of 10 inches = 1·46 inch.
- (vi.) Extension on 10" length under a gross load of 20,000 lbs. = $\frac{7}{1000}$ ".

Find from the above data:—(a) The modulus of elasticity of the material; (b) the limit of elasticity and tenacity per square inch; (c) the reduction of area per cent.; (d) the approximate work done per cubic inch of the material in fracturing the bar. Would you consider this good material?

(I.C.E., Oct., 1901.)

ANSWER.—

$$(a) \text{ Young's modulus of elasticity } \left. \vphantom{\begin{array}{l} \text{Young's modulus of} \\ \text{elasticity} \end{array}} \right\} E = \frac{PL}{A\ell}. \quad (\text{Equation IV., Lecture I., Vol. II.})$$

$$\text{,,} \quad \text{,,} \quad E = \frac{20,000 \times 10}{2\cdot01 \times \cdot505 \times \frac{7}{1000}} = \frac{200,000,000}{7\cdot105}$$

$$\text{,,} \quad \text{,,} \quad E = 28,143,000 \text{ lbs. per square inch.}$$

$$(b) \text{ Stress at elastic limit in lbs. per square inch } \left. \vphantom{\begin{array}{l} \text{Stress at elastic limit in} \\ \text{lbs. per square inch} \end{array}} \right\} = \frac{\text{load at elastic limit}}{\text{original area}} = \frac{34,500}{1\cdot015}$$

$$\text{,,} \quad \text{,,} \quad = 34,000 \text{ lbs. per square inch.}$$

$$(c) \text{ Percentage contraction of area } \left. \vphantom{\begin{array}{l} \text{Percentage contraction} \\ \text{of area} \end{array}} \right\} = \frac{\text{original area} - \text{final area}}{\text{original area}} \times 100$$

$$\text{,,} \quad \text{,,} \quad = \frac{(2\cdot01 \times \cdot505) - (1\cdot52 \times \cdot41)}{2\cdot01 \times \cdot505} \times 100$$

$$\text{,,} \quad \text{,,} \quad = \frac{(1\cdot015 - \cdot6232) 100}{1\cdot015} = \frac{39\cdot18}{1\cdot015} = 38\cdot6 \text{ per cent.}$$

$$(d) \text{ Work done per cubic inch } \left. \vphantom{\begin{array}{l} \text{Work done per cubic} \\ \text{inch} \end{array}} \right\} = \frac{\text{work done in stretching the test-piece}}{\text{volume of the test-piece}}$$

$$\text{,,} \quad \text{,,} \quad = \frac{\left\{ \begin{array}{l} \text{mean load between starting point and} \\ \text{point of fracture} \times \text{distance moved} \\ \text{through} \end{array} \right\}}{\left\{ \begin{array}{l} \text{area in square inches} \times \text{length of test-} \\ \text{piece in inches} \end{array} \right\}}$$

$$\text{,,} \quad \text{,,} \quad = \frac{30,525 \times 1\cdot46}{10\cdot15} = 4,390 \text{ inch-lbs.}$$

$$(e) \text{ Percentage elongation on 10 inches } \left. \vphantom{\begin{array}{l} \text{Percentage elongation} \\ \text{on 10 inches} \end{array}} \right\} = \frac{\text{final length} - \text{original length}}{\text{original length}} \times 100$$

$$\text{,,} \quad \text{,,} \quad = \frac{1\cdot46 \times 100}{10} = 14\cdot6 \text{ per cent.}$$

Influence of Time on Stress and Strain, as shown by the Diagram.—Prof. Barr says, another advantage in wire testing is, that any particular condition may be varied. The wire can be tested quickly or slowly, and where a considerable difference in the stress-strain diagrams is shown, it will be known to be due to the method of testing. Besides that, the material may be given rests more conveniently in experiments on testing wires than could be done with specimens in a large machine. The results obtained by varying the rate of application of the stress to the specimen as given by the stress-strain diagrams in Fig. 26, are exceedingly

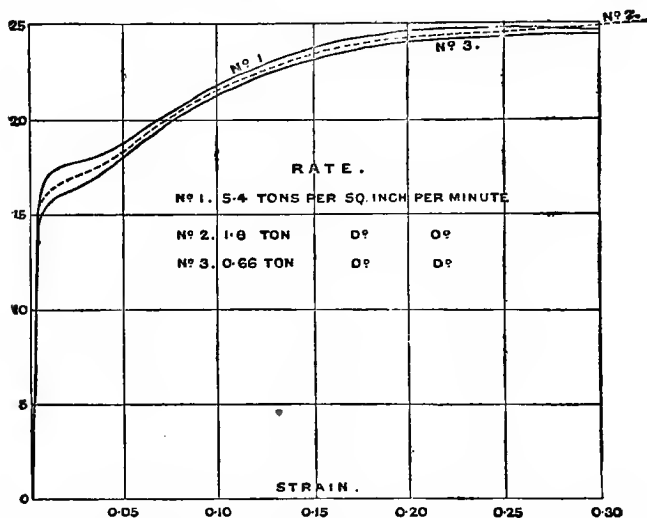


FIG. 26.—STRESS-STRAIN DIAGRAMS OF A TEST-PIECE OF CHARCOAL IRON UNDER THREE CONDITIONS OF TESTING (see Fig. 2, Lecture xxxiia.).

interesting, because it is certain, that almost the whole of the measurable difference of results on the diagram is due to the different treatment, and not to any difference in the specimens themselves. The arrangement shown in Fig. 2, is well designed for accurately obtaining the required variations in the rate of application of stress within a wide range.

Torsion Tests.—*Autographic Diagram taken with a Flat Disc Recorder.**—Fig. 27 shows in a graphic way the behaviour of an “acid steel” specimen, $\frac{3}{4}$ inch in diameter and 6 inches long, subjected to a maximum torsion or twisting moment of 4,840 inch-lbs. The specimen

* The originals of both of these torsion diagrams were kindly taken by Messrs. Buckton & Co. and presented to the Author for this book. They were then reproduced to a convenient scale by his publishers.

was twisted through an angle of $1,202^\circ$. The series of concentric circles represent the increase of stress, while the radial divisions show the amount of torsion. A complete turn of the test-piece is represented by 360° or 2π radians. It will be seen, that the characteristic behaviour of the specimen is shown to be a rapid increase of stress for minute increments of torsion, until the elastic stress is attained. It is thereafter followed by a greater and greater amount of twisting compared to the increase of stress. Hence, in the case of tough and very ductile materials, there is during the last stage of the experiment a very large proportion of twist with very little increase of stress; or, as illustrated by Fig. 27, the curve becomes almost concentric during the later stages, instead of maintaining the spiral form as in the earlier stages. The arrowhead at the very end of the curve indicates that the specimen has broken.

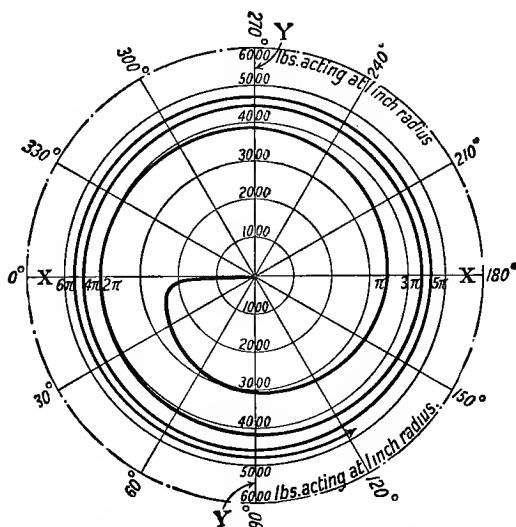


FIG. 27.—SPIRAL DIAGRAM OF TORSION TEST TAKEN WITH WICKSTEED'S FLAT DISC RECORDER. Made by Buckton & Co., Leeds.

If the curve in Fig. 27 be developed on a flat surface we get a somewhat similar diagram to that shown by Fig. 28. Here the ordinates represent twisting moments and the abscissæ angles of twist or torsion.

Autographic Diagram taken with a Drum Recorder.—Fig. 28 is a copy of the diagram taken by a drum recorder for a "basic steel" specimen 2 inches in diameter and 4 inches long between the collars. The maximum twisting moment was 103,290 inch-lbs., and the angle through which the specimen twisted was 450° , or 2.5π .

To find the Work Done per Cubic Inch by aid of an Autographic Diagram, when subjecting a Test-Piece to a Torsion Test.—Take the case of Fig. 28, which represents a stress-

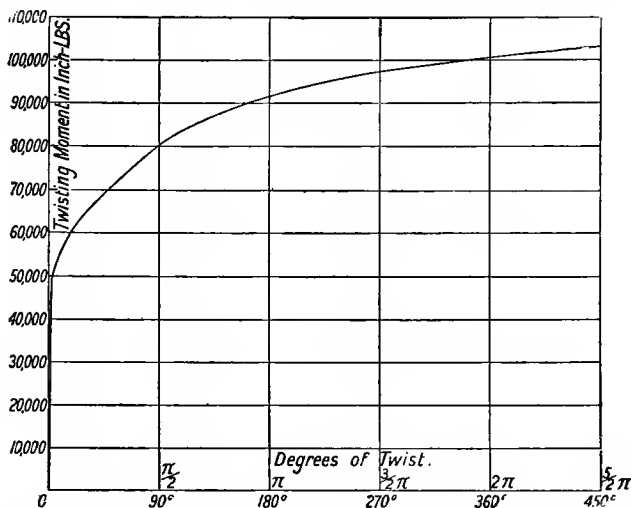


FIG. 28.—DIAGRAM OF TORSION TEST TAKEN WITH A DRUM RECORDER.

Made by Buckton & Co., Leeds.

strain diagram in which the ordinates represent twisting moments and the abscissæ angles of twist. We get—

The vertical scale } = 43,500 inch-lbs. per inch.
for twisting
moments

The horizontal scale for the angle of twist is taken as—
1 inch = $\cdot 87\pi$.

The area of dia- } = 3.75 square inches.
gram is ap-
proximately

Hence, Work done = $\frac{\text{area of diagram} \times 43,500 \times \cdot 87\pi}{2,240} = \frac{3.75 \times 43,500 \times \cdot 87 \times 22}{7 \times 2,240}$

„ „ = 199.12 inch-tons.

And, Work done } = $\frac{\text{Work done in inch-tons}}{\text{Volume of bar in cubic inches}} = \frac{199.12}{3.14 \times 4}$
per cubic inch

„ „ = 15.8 inch-tons per cubic inch.

Searle's Apparatus for Measuring the Extension of a Wire and for Determining Young's Modulus of Elasticity.*—In studying the effects of longitudinal stress upon the length of a wire some accurate means is required for measuring the elongation of the test-piece. The simplest method of magnifying these effects, consists in using a wire of considerable length, which for convenience is hung from a fixed support. The extension is produced by hanging weights to the lower end of the wire. Two chief sources of error must be avoided at the outset. These errors arise from the yielding of the support and the change in length of the wire due to a rise of temperature. Both errors are practically eliminated if, instead of finding the displacement of the lower end of the wire relative to a fixed mark, observation be made of the displacement of the end of the wire relative to the lower end of a second wire of the same material, hanging from the same support, stretched by a constant weight and thus serving as a standard of comparison. It is here assumed, that the coefficients of expansion with temperature of the two wires are identical in spite of their differences in stress.

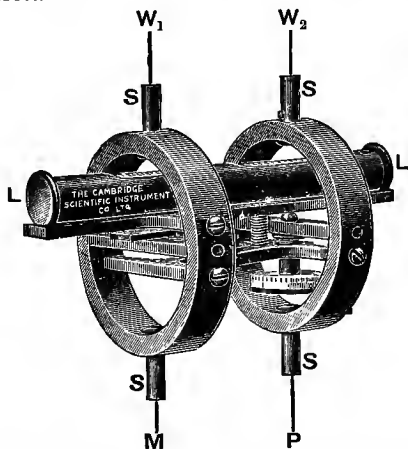


FIG. 29.—SEARLE'S APPARATUS FOR MEASURING THE EXTENSION AND DETERMINING YOUNG'S MODULUS FOR WIRES.

Description of Apparatus.—The apparatus shown by Fig. 29 furnishes a very sensitive means of measuring the relative displacement of the ends of the wires, and in addition to its sensitiveness, it possesses the advantage of giving direct readings. The two wires W_1 , W_2 have their upper ends secured to a stout piece of metal bolted to a beam. From their lower ends hang two cylindrical brass frames supporting the two ends of a sensitive level L . One end of the level is pivoted to the frame at the left-hand side

* I am indebted to Mr. G. F. C. Searle, M.A., Demonstrator at the Cavendish Laboratory, Cambridge, for kindly supplying me with a copy of his paper on "Apparatus for Measuring the Extension of a Wire," see vol. v., part v., of the *Proc. of the Cambridge Philosophical Society*, from which the above article has been compiled.

of Fig. 29 by pivots; whilst the other end of the level rests upon the end of a vertical micrometer screw, working in a nut attached to the other frame at the right-hand side of the figure. The central link connecting the two frames prevents them from twisting, relative to each other about their vertical axes, but allows a free vertical relative motion. When this link is horizontal the two wires are parallel to each other. From the lower ends of the frames hang a mass M and a pan P . The weights of M and P are sufficient to ensure that the wires are straight. The connections between the two wires W_1 , W_2 and the frames are made by the swivels S , into which the ends of the wires are soldered. These swivels enable the wires to be torsionless and thus ensure that the two wires hang in a vertical plane. Two other swivels S , connect M and P to the frames. The head of the vertical micrometer screw is divided into a number of equal divisions around its circumference, and a scale is engraved on the side of the frame at the right-hand side, which scale serves to determine the number of complete revolutions made by the screw.

Method of Using the Apparatus.—Suppose the screw has been adjusted, so that one end of the bubble of the level is at its fiducial mark. If a weight be placed in the pan P , the wire W_2 is stretched and the bubble moves towards the left-hand end of the level. The bubble is then brought back to its fiducial mark by turning the micrometer screw, so as to raise the right-hand end of the level resting upon it. The distance through which the screw is moved is clearly equal to the increase of length of the wire W_2 , and is determined at once by the difference of the reading of the screw in the two positions. The level is sensitive enough to enable the screw to be adjusted to $\frac{1}{4}$ of a division on its graduated head.

With this apparatus it is easy to investigate the deviations from Hooke's Law for copper wires.

Prof. Ewing's Extensometer for Measuring the Elastic Extension and Young's Modulus of Elasticity of Specimens of Metal under Tensile Tests.—This instrument can be quickly applied to any test-piece, and *no part of it has to be touched whilst the test is being made.*

Two clips B_1 and C are attached to the test-piece A by the points of two set-screws. The clip B_1 has a projection B_2 ending in a rounded point P , which engages with a conical hole in C . When the bar extends, this rounded point serves as a fulcrum for the clip C , and hence a point Q , equally distant on the other side, moves, relatively to B_1 , through a distance equal to twice the extension. This distance is measured by means of a microscope attached to B_1 . The microscope forms a prolongation of B_1 , and the motion of Q is brought into the field of view by means of a hanging rod R . The rod R is free to slide on a guide in B_1 , and carries a mark on which the microscope is sighted. The displacement is read by means of a micrometer scale in the eye-piece of the microscope. The pieces B_1 and B_2 are jointed to one another in such a way that the bar may twist a little, as it is sometimes liable to do during a test, without affecting the engagement of P with C . This also obviates any need of absolute parallelism in the axes of attachment of the two clips. But the joint between B_1 and B_2 forms a rigid connection so far as angular movement in the plane of the paper is concerned. This feature is essential to the action of the instrument, for it is only then that P serves as a fixed fulcrum in the tilting of A , by extension on the part of the specimen.

The figure is an illustration of the usual form of this instrument. The clips B_1 and C are set at 8 inches (200 mm.) apart.

Each division on the microscope scale corresponds to $\frac{1}{5000}$ inch of exten-

sion, so that, by estimation of tenths of a division, readings to $\frac{1}{1000}$ inch may be taken.

The screw L further serves to bring the sighted mark to a convenient point on the micrometer scale, and also to bring the mark back if the strain is so large as to carry it out of the field of view. In dealing with elastic strains there is no need for this, as the range of the scale itself is sufficient to include them, but it is useful when observations are being made on the behaviour of metals when the elastic limit is passed.

To facilitate the application of the extensometer to any rod, a clamping bar is added, by which the clips B_1 and C are held at the right distance apart with the axes of their set-screws parallel, while they are being secured to the test-piece. Such a clamping bar is especially convenient when the strain has been carried beyond the elastic limit, and it is desired to reset immediately, the clips to the standard distance apart, after the length between them has materially changed by extension of the specimen. The clamping bar must, of course, be removed before a test begins.

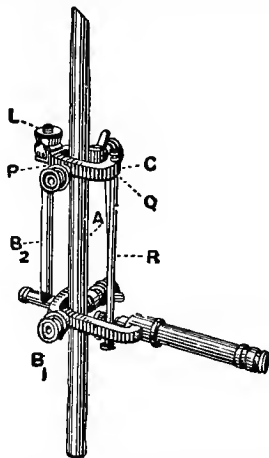


FIG. 30—PROF. EWING'S EXTENSOMETER.*

An apparatus can also be made for marking off on the specimen the 8-inch length to which the extensometer is applied when in use. To use this latter apparatus, the set-screws are turned back so that the test-piece rests in the V-grooved blocks, the upper screws are then tightened upon the test-piece, and the lower screws are also tightened sufficiently to make their points indent the bar.

* I am indebted to Prof. J. A. Ewing, F.R.S., &c., and to "The Syndics of The Cambridge University Press" for their kind permission to reproduce in this lecture the following 7 figures:—31, 32, 34, and 36 to 39. Those devised by Prof. Ewing are specially indicated with his name, and all are made by The Cambridge Scientific Instrument Co., Limited, from whom I received the other 3 blocks for figures 17, 29, and 35. Most of these figures are illustrated and described in Prof. Ewing's treatise on "The Strength of Materials," as published by The Cambridge University Press.

Testing Machine with Extensometer Attached for Measuring the Elastic Extension of Rods.—Extensometers are commonly used in conjunction with testing machines. It is also necessary to note, that for many experiments on elasticity, the testing machine is not essential, because so long as elastic strains are being dealt with there is no need of hydraulic or other gearing to take up the stretch, and an ordinary lever may be all that is required for applying the load. The figure below illustrates a laboratory apparatus for measuring Young's modulus in rods of various metals. By placing weights on the vertical rod attached to the right-hand end of the lever, loads up to 1 ton can be applied to the test-piece, which may have a diameter of $\frac{1}{4}$ to $\frac{3}{8}$ inch. An extensometer is attached to the test-piece on the left-hand side of the lever, to measure the strain produced in it.

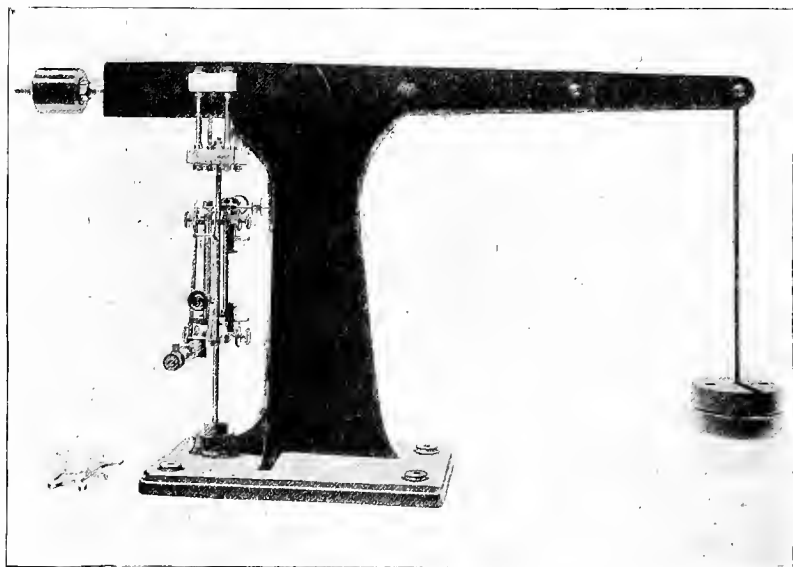


FIG. 31.—TESTING MACHINE WITH EWING'S EXTENSOMETER ATTACHED FOR MEASURING THE ELASTIC EXTENSION OF RODS.

Measurement of Young's Modulus in Wires.—When long pieces of wire are available as test-pieces, then the elastic extension can be obtained by direct measurement by means of a scale and vernier. The usual plan is to hang the two wires side by side, fastening both to the same support and then attach the scale to one wire and the vernier to the other. The advantage of using the second wire to carry the scale is, that any yielding of the support or any change of temperature, such as might take place during the test, affects both wires equally. The wire with the scale attached to

it is kept taut by a load which is not varied during the test. The other wire is first loaded with weights until it is straight, then additional weights are applied to produce the elastic extension. This extension is measured by noting the movement of the vernier along the scale. It is found, that with iron or steel wires 20 feet long, the extension is nearly $\frac{1}{8}$ inch for each ton per square inch of load. As the load may generally

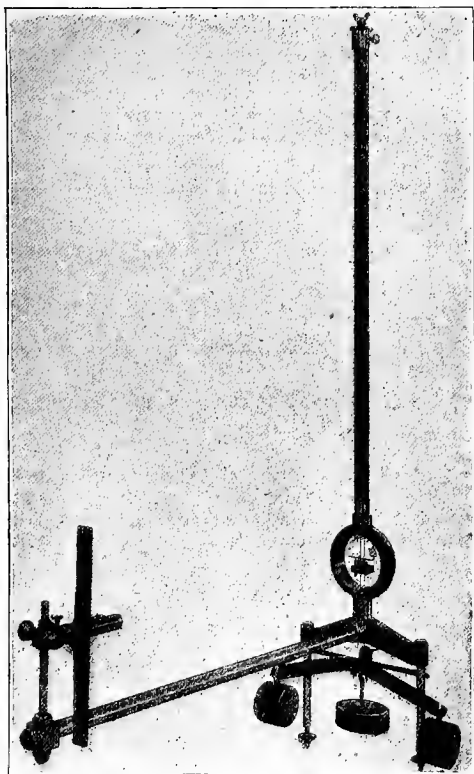


FIG. 32.—INSTRUMENT FOR MEASURING THE ELASTIC EXTENSION OF WIRES.

be raised to 10 tons per square inch, and sometimes more, without passing the elastic limit, it will be seen, that the movement of the vernier will be sufficient to give approximately accurate values of the modulus of the material.

When short pieces of wire are only available, then some means of magnifying the relative displacement will be required. One very good plan is to clamp two little blocks to the two wires. These two blocks

serve as platforms, on which is placed a small tripod carrying a mirror. Two legs of the tripod are supported in a hole and a slot respectively on one of the blocks, while the third leg rests on a plane horizontal surface on the other block.* If one wire is stretched it causes the mirror to tilt, and the amount of this tilting is measured by means of the fixed telescope and scale shown at the left-hand side of the above figure.

It will be noticed, that the two wires hang inside the vertical tubular stem from the clamp and butterfly nut at the top. A cross-bar is attached to the bottom end of one of the wires which carries a constant load, while the variable load is applied to the other wire. The reading telescope with its attached scale is supported by part of the framework of the apparatus.

It is important to note, when calculating the extension of the test-piece from the scale readings, that the angle through which the reflected ray turns is twice the angle through which the mirror is tilted.

Thus, if w = The effective width of the tripod carrying the mirror—*i.e.*, the distance from its back leg to the line joining the two front legs.

n = Number of divisions on the scale by which the telescope reading has changed when a load is applied.

d = Distance between the mirror and the scale expressed in scale divisions.

Then, the small angle turned through by the ray = $\frac{n}{d}$.

And, the angle through which the mirror is tilted = $\frac{\delta l}{w}$, where δl is the extension of the wire.

$\therefore \delta l = \frac{wn}{2d}$ and is calculated in the same units as are used in measuring the effective width w of the tripod.

Measuring Young's Modulus by Bauschinger's Extensometer.

—The small strains which take place in a tensile test of a non-plastic material, and those strains which occur during the early part of the test in any kind of material, require some form of instrument

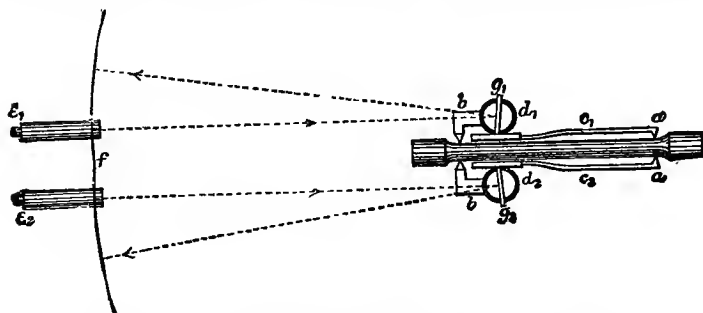


FIG. 33.—PROF. BAUSCHINGER'S OPTICAL APPARATUS FOR THE MEASUREMENT OF STRAINS.

* This automatic adjustment was first devised by Lord Kelvin in connection with the levelling of his quadrant electrometer.

capable of indicating delicate measurements. In measuring the elastic modulus and in determining the true elastic limit, it is necessary to be able to compare the fractional parts of the stretching of the test-piece, which are produced by each successive increase of load. Consequently, in order to obtain accurate measurements of the modulus of any material, there is a great advantage in being able to read the measurements to, say, $\frac{1}{1000}$ part of an inch, when we consider that the whole amount of elastic stretching in a test-piece of, say, wrought iron is only about $\frac{1}{1000}$ part of the length under observation. As the measurements taken between two marks on each side of the test-piece are so much affected by bending (due to inequality of the distribution of the stress), it is absolutely necessary, either to measure the extensions on opposite sides of the test-piece and take the mean, or to measure the displacements on each side of the test-bar by aid of two mirrors, &c., attached in such a manner as to indicate the strains on each side of the test-piece, as shown by the accompanying figure.

Extensometer Applied to Measure the Elastic Compression of Short Blocks.—The illustration (Fig. 34) shows Prof. Ewing's extensometer as adapted by him to measure the elastic compression of short blocks

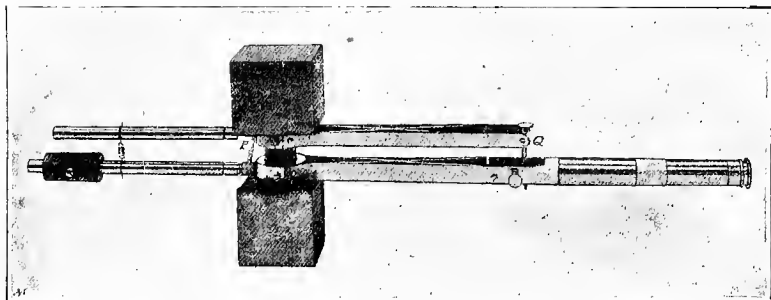


FIG. 34.—PROF. EWING'S EXTENSOMETER AS APPLIED TO MEASURE THE ELASTIC COMPRESSION OF SHORT BLOCKS.

of any material. It will be seen, that the length of test-piece which is to be tested between the clips is only $1\frac{1}{2}$ inches. The strain of the test-piece is multiplied ten times by a mechanical arrangement. This result is obtained by extending the clips to the right, and making the distance of Q from the axis nine times that of P. The prolongations of the clips and microscope, &c., on the right-hand side, so that the force with which the point P presses against its bearing or socket may be vertical. The motion of the end Q of the lever PQ is transferred to the field of the microscope by means of a vertical hanging piece jointed to the lever at Q and carries the mark on which sights are taken. With this instrument no calibrating screw is required, whilst the object to be sighted by the microscope is a small piece of glass on which two horizontal lines are engraved at a distance of $\frac{1}{100}$ inch apart. The length of the microscope is arranged by adjustment to make these lines include 500 units of the eye-piece scale. Consequently, each unit corresponds to a displacement of the glass plate through $\frac{1}{25000}$ inch, or to an extension of the test-piece of $\frac{1}{25000}$ inch.

Apparatus for Determining the Deflection of Beams and Cantilevers.—This instrument consists of two cast-iron brackets which can be fixed to a wall. An iron bed having two movable supports with knife edges in order to test the rod when placed as a beam; also a cast-iron block for fixing the end of the test-piece when being tested as a cantilever, and shown by the figure. The deflection or lowering of the

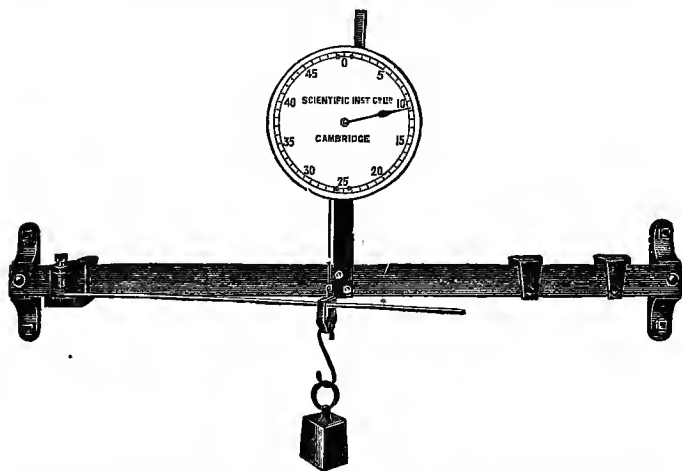


FIG. 35.—APPARATUS FOR DETERMINING THE DEFLECTION OF BEAMS AND CANTILEVERS.

middle point of the beam or the end of the cantilever may be small, so that this deflection is magnified by passing a cord from the top of the stirrup round the little axle which carries the pointer. The pointer will show on the dial the magnification of the deflection. The weights are attached by a carrier to the lower end of the stirrup.

Measurement of Young's Modulus of Elasticity by Deflection of Beams.—By noting the deflections of a loaded bar of the material, we have another ready method for finding Young's modulus for the material of the test-piece. The bar is supported as a beam on fixed knife-edged supports, or clamped at one end and free at the other. When the test-piece is long and sufficiently flexible to bend easily, then the deflection is measured by a fixed scale behind the test-piece, with a piece of mirror fixed alongside the scale. The readings may be directly taken by the experimenter bringing his eye to the level of the test-piece until the top edge of the test-piece just covers its reflection in the mirror, and then sighting this position of the edge upon the scale at its side. It is, however, found, that when dealing with less flexible test-pieces, an apparatus like that illustrated by Fig. 36, is found useful. In this arrangement, the knife-edged supports are clamped on a stiff bed similar to that of a lathe bed, and can be adjusted to suit any length of test-piece. The deflection of the

Then, as was shown in Lecture III., if a load W is applied at the centre of a beam, the deflection—

$$\Delta_1 = \frac{W L^3}{48 E I} = \frac{W x^3}{6 E I}. \quad \therefore E = \frac{W x^3}{6 \Delta_1 I}$$

Where I is the moment of inertia of the section of the rod or beam about a horizontal central axis.

Again, if a load W be applied at each extremity of the test-piece, the upward deflection at the centre is given by the equation—

$$\Delta_2 = \frac{W x^2 (L - 2x)}{4 E I}. \quad \therefore E = \frac{W x^2 (L - 2x)}{4 \Delta_2 I}$$

Young's Modulus by Deflection of Cantilevers.—The following Fig. 37 shows an arrangement for observing the deflection of the test-piece when used as a cantilever or beam fixed at one end and free at the other.

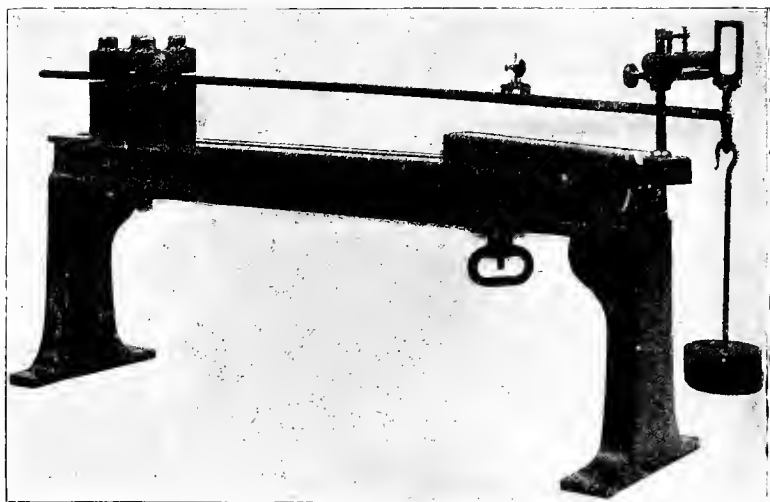


FIG. 37.—APPARATUS FOR MEASURING YOUNG'S MODULUS OF ELASTICITY BY DEFLECTION OF CANTILEVERS.

Assuming the load W to be applied at the free end, and, that L denotes the whole length of the test-piece from the clamp to the free end, we get the deflection, $\Delta = W L^3 / 3 E I$.

There is one great objection to this latter method of determining E , on account of the difficulty of maintaining the fixed end of the test-piece horizontal by means of the clamp. In this experiment, provision is made for observing the deflection at the various points along the test-piece, in order to plot the curve which a beam of uniform section will assume under a given load or system of loads. The slope of the cantilever may be determined from point to point along the test-piece by means of the jockey mirror, as previously described for slope of beams.

Measurement of the Modulus of Rigidity by the Torsion of Wires.*

The accompanying illustration shows a self-contained apparatus for performing experiments on the torsion of wires. The wire to be tested is suspended in the vertical axis of a tubular stem. It carries a cylindrical weight, round which two cords pass and are led away over pulleys on both sides of the framework to hangers. Equal weights are placed upon the hangers, and the wire is consequently twisted by a pure couple. The angle of twist is read by observing the displacement of a pointer on a fixed circular scale. This pointer is attached to the upper end of the cylindrical weight.

When a rod is twisted every part of it is in a state of shear.

Let θ = Angle of twist expressed in circular measure.

„ l = Length of wire.

„ d = Diameter of wire.

„ C = Modulus of rigidity.

„ TM = Twisting moment.

Then, within the elastic limit, we get from Lecture IV., p. 106—

$$\theta = \frac{32 l TM}{\pi d^4 C}.$$

$$\text{Or, } C = \frac{32 l TM}{\pi d^4 \theta}.$$

It is found convenient, in applying this method to measure C in rods of moderate diameter, to find θ by using two long pointers clamped on the wire near its ends. The distant ends of the pointers pass over fixed scales. Hence, the difference in the two scale readings measures the angle of twist on the length l of the wire between the positions of the two pointers. Should the diameter of the wire be so great as to make the angle of twist too small to be measured in

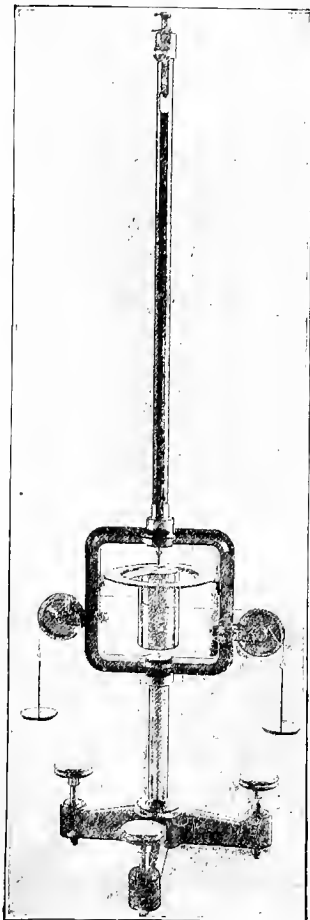


FIG. 38.—APPARATUS FOR MEASURING THE MODULUS OF RIGIDITY BY THE TORSION OF WIRES.

* See *Proc. Inst. C.E.*, vol. cli., 1903, for abstract of paper on "Iron and Steel under Torsional and Combined Stresses," by Prof. E. G. Coker, M.A., D.Sc.

this way, then a pair of mirrors are clamped on the wire and made to face sideways, and are used along with a reading telescope and scale for each. The advantage of an optical pointer over a mechanical pointer is, that of doubling the angle, and, further, it can readily be made longer.

Measurement of the Modulus of Rigidity by Torsional Oscillations.—The figure shows a complete apparatus for experiments on the torsion of wires by means of Maxwell's needle. It consists of a brass tube placed horizontally, into which there can be slipped four other equal short pieces of tube, and thus fill up its whole length. Two short pieces are filled with lead and the other two pieces of tube are empty. By placing,



FIG. 39.—APPARATUS FOR EXPERIMENTS ON THE TORSION OF WIRES BY MEANS OF MAXWELL'S NEEDLE.

first, the two empty short tubes in the interior with the two loaded pieces of tube to the outer ends of the long tube, and second, reversing this order by placing the loaded pieces in the centre and empty ones at each end, you can obtain two different values of the moment of inertia of the system.

- Let I_1 = Moment of inertia of system in the first arrangement of the short tubes.
 „ I_2 = Moment of inertia of system in the second arrangement of the short tubes.
 „ x = Half the length of the long tube.
 „ m_1 = Mass of each of the two short tubes which are filled with lead.
 „ m_2 = Mass of each of the empty tubes.
 „ t_1, t_2 = The observed periods of oscillation in the two arrangements respectively.

Then the system is changed by shifting two masses, each equal to $m_1 - m_2$, so that the distance of the centre of gravity of each mass from the axis changes from $\frac{3}{2}x$ to $\frac{1}{2}x$.

Hence to express the change in the moment of inertia—

$$I_1 - I_2 = 2(m_1 - m_2) \left(\frac{9}{16}x^2 - \frac{1}{16}x^2 \right) = (m_1 - m_2)x^2.$$

$$\text{But, } \frac{t_1^2}{t_2^2} = \frac{I_1}{I_2}, \text{ or } \frac{t_1^2}{t_1^2 - t_2^2} = \frac{I_1}{I_1 - I_2} = \frac{I_1}{(m_1 - m_2) x^2}$$

$$\text{Or, } \frac{I_1}{t_1^2} = \frac{(m_1 - m_2) x^2}{t_1^2 - t_2^2}.$$

Also, the modulus of rigidity C can be got by the following formula :—

$$C = \frac{128 \pi l I_1}{g d^4 t_1^2} \text{ (see Lecture IV., p. 105).}$$

Where t is the period of time taken to make each complete oscillation ; l is the length and d is the diameter of rod, and g is the factor for converting the twisting moment per unit of angle, or the constant ratio of the twisting moment T M to the angle θ into kinetic units.

Consequently, we obtain, without any calculations for moment of inertia I , the equation for the modulus of rigidity

$$C = \frac{128 \pi l}{g d^4} \left[\frac{(m_1 - m_2) x^2}{t_1^2 - t_2^2} \right].$$

In the next lecture we shall deal with the “Strength and Elasticity of Columns.”

LECTURE VII.—QUESTIONS.

1. Select any two of the five autographic apparatus for recording stress-strain diagrams which you consider to best fulfil the objects to be attained. Then sketch and explain their construction, use, and application.

2. Explain, by aid of a sketch of a complete stress-strain diagram, the several points of chief interest which occur in a rod of soft iron or mild steel, from the time of applying the stress until the rod breaks.

3. Explain, by aid of a sketch, how Prof. Kennedy automatically obtained elastic strain diagrams.

4. What is meant by Plasticity? How would you obtain a measure of the work done per cubic inch by aid of an autographic diagram when subjecting a test-piece to a tensile stress? Explain, also, Prof. Kennedy's approximate method and give the formula, with an example.

5. Explain the influence of time on the stress and strain of a metal as exhibited by a stress-strain diagram.

6. Explain, by aid of sketches, how you would make and take a torsional stress-strain diagram. Draw and explain torsion diagrams taken by disc and drum recorders.

7. Of the several methods given in this lecture for measuring the extension of wires and of determining Young's modulus of elasticity, select any one and describe it fully, by aid of sketches, with index to parts, &c.

8. Sketch and explain Prof. Ewing's extensometer for wires and rods.

9. Explain clearly, by aid of sketches, Bauschinger's extensometer for measuring Young's modulus of elasticity.

10. Sketch clearly and explain concisely Ewing's extensometer for measuring the elastic compression of short blocks.

11. Explain by sketches any apparatus for measuring Young's modulus of elasticity by deflection of beams and cantilevers. Work out the formulæ for the deflection and for Young's modulus of elasticity.

12. Sketch and explain any apparatus for measuring the modulus of rigidity by the torsion of wires.

13. Show how the mere torsional oscillations of a suspended weighted wire may be used to ascertain the modulus of rigidity.

14. Suppose the vertical loads and supporting forces of a horizontal beam to be known, show how we find (1) the shearing force at the section, (2) the position of the neutral line, (3) the compressive stress at any part of the section, (4) the curvature of the beam, (5) the new shape of a portion of the section originally rectangular, its sides vertical and horizontal. (B. of E., Adv. & H., Part I., 1900.)

15. Describe carefully the behaviour of a mild-steel bar tested gradually in tension up to rupture, and sketch a stress-strain curve for it. The following data were obtained in such a test:—Original diameter of bar $1\frac{1}{2}$ inches, final diameter at point of fracture $\frac{1}{2}$ of an inch, total load when limit of elasticity was reached 20·8 tons, total load at fracture 36·7 tons, total extension at fracture (on a length of 10 inches) 2·23 inches, elongation (of 10-inch length) under a total load of 10 tons ·005 inch. Deduce from these data the following:—(a) The modulus of tensile elasticity. (b) The reduction of area per cent. (c) The elongation at fracture per cent. (d) The limit of elasticity in tons per square inch. (e) The maximum load in tons per square inch (both for original and final area). (C. & G., 1901, H., Sec. A.)

16. A cylindrical steel pin is used to couple together two rods, and the joint is arranged in such a way that the pin is subjected to double shear. If the total tensile force tending to pull the joint asunder is $18\frac{1}{2}$ tons, what diameter would you make the pin? Choose for yourself the working shearing stress which can be permitted. (C. & G., 1901, O., Sec. B.)

17. Answer only *one* of the following:—(a) Describe a laboratory experiment by which you could find E , Young's modulus of elasticity for an iron wire 10 feet long and 0.05 inch diameter. How would you secure the upper end of the wire? How apply the load? And how measure the elongation? How would you plot your results, and how deduce the value of E ? About how much elongation would you expect for a load of 15 lbs.? (B. of E., H., Part I., 1902.)

18. Answer only *one* of the following:—(a) Describe a laboratory experiment by which you could find E , Young's modulus of elasticity, for an iron wire, 10 feet long and 0.05 inch diameter. How would you secure the upper end of the wire? How apply the load? And how measure the elongation? How would you plot your results, and how deduce the value of E ? About how much elongation would you expect for a load of 15 lbs.? (b) Sketch an apparatus for determining the coefficient of sliding friction between two planed surfaces of oak. If you have made this or a similar experiment, describe the behaviour of the sliding piece and any troubles you may have had. State how you would conduct the experiment, so as to establish the principal facts concerning such friction. (B. of E. Adv., 1902.)

19. In a tensile test of mild steel the original section was a rectangle of dimensions 1.96 inches by .4 inch, and the breaking load was 25 tons. The extension at fracture, in a length of 8 inches, was 2.44 inches and the contracted dimensions were 1.41 inches by .27 inch. Under a load of 14 tons (within the elastic limit) the extension in 8 inches was .0107 inch. Calculate the stress at breaking point, the percentage extension, the percentage contraction of area, and the value of Young's modulus. Express an opinion whether the specimen is a good or a bad one. (C. & G., 1902, O., Sec. B.)

N.B.—See *Appendices B and C* for other questions and answers.

LECTURE VII.—A.M. INST. C. E. EXAM. QUESTIONS.

1. Sketch a stress-strain diagram for cast iron, mild steel, and gunmetal. Indicate on each what are the characteristic points. (I.C.E., Oct., 1897.)

2. Describe carefully a form of micrometer for determining the elastic extensions of a bar. State how its accuracy can be tested. (I.C.E., Oct., 1897.)

3. Find the relation between the coefficient of direct elasticity and the coefficient of rigidity. (I.C.E., Oct., 1897.)

4. Describe the successive effects of an increasing pull stress applied to a specimen of mild steel, and illustrate your answer by reference to a stress-strain diagram sketched to a scale. Define Young's modulus, and illustrate its meaning by reference to a stress-strain diagram. (I.C.E., Feb., 1898.)

5. A specimen of mild steel 1 inch in diameter and 10 inches long between the gauge points gives the following results:—Yield-point, 15 tons per square inch; maximum load, 30 tons per square inch of original area; extension, 26 per cent. What results would you expect to obtain in testing two other specimens of the same quality of material, one $\frac{1}{2}$ inch in diameter and 5 inches long and the other $\frac{1}{2}$ inch in diameter and 10 inches long between gauge points? Criticise in this connection the specification:—"The steel is to have a strength of 28 tons per square inch and to give an elongation of 25 per cent. on a length of 8 inches." (I.C.E., Feb., 1898.)

6. Sketch and describe one form of stress-strain indicator suitable for use in tension tests of iron or steel specimens. (I.C.E., Feb., 1898.)

7. When a 1-inch bolt of mild steel has a screw-thread cut upon its end, how will its strength and toughness be affected? And how will they be affected again if the shank is turned down to the diameter of the screw inside the thread? Explain these effects by reference to the stress-strain diagram. (I.C.E., Oct., 1898.)

8. In testing materials under direct tension, describe the manner in which a stress-strain diagram is usually constructed, explaining what are the quantities represented by its co-ordinates; and sketch the typical form of such a diagram for a 10-inch test-bar of mild steel, with explanatory remarks. (It is not necessary here to describe the mechanical details of any particular apparatus.) (I.C.E., Oct., 1898.)

9. Describe some form of testing machine suitable for performing an experiment on stretching a test-piece till rupture occurs. Show how the results of such an experiment are exhibited graphically. Sketch the curve obtained for a specimen of mild steel, marking the most important points with the names commonly applied to them. (I.C.E., Oct., 1899.)

10. Explain the terms "limit of elasticity," "modulus of elasticity." Describe the changes of form which occur when a piece of material is stretched or compressed within the elastic limit. Calculate these changes in fractions of an inch for a piece of wrought iron of circular section 2 inches in diameter, 6 feet long, under a pull of 15 tons, assuming probable values of the coefficients. (I.C.E., Feb., 1900.)

11. Describe fully the several stages of an experiment on the stretching of a piece of wrought iron till rupture occurs, stating the average results of such experiments. Also explain how the results are influenced by the form of test-piece adopted. (I.C.E., Feb., 1900.)

12. Find the position of the neutral axis in a cast-iron beam whose section has the following dimensions:—Upper flange 4 inches wide and 2 inches deep; lower flange 10 inches wide and $2\frac{1}{2}$ inches deep; web 2 inches thick; total depth of section 20 inches. (I.C.E., Feb., 1901.)

13. If the elastic twist of a wire $\frac{1}{4}$ inch diameter and 4 feet long from the fixed end is 36° with a certain twisting moment, find the twist of the end of a wire of the same material $\frac{1}{2}$ inch diameter and 5 feet long from the fixed end, with the same twisting moment. (I.C.E., Oct., 1901.)

14. In a test of a cylindrical cast-iron beam on a 20-inch span it is found that with a load of 660 lbs. in the centre the deflection is 0.027 inch. If the diameter of the cross-section is 1.5 inch, what is the modulus of elasticity of the material? What hypothesis is assumed in deducing the formula you use in your calculation? Is it true for such a material as cast iron? (I.C.E., Oct., 1901.)

15. Describe fully the behaviour of a piece of good mild steel during a tensile test from the first application of the load until it reaches the rupture value. Sketch the stress-strain curve for such a specimen, and mark on it the limit of elasticity, the yield point, the maximum load, and the rupture load. (I.C.E., Feb., 1902.)

16. Make a sketch of any form of autographic apparatus you are acquainted with, for causing a stress-strain diagram to be taken during the testing of a specimen to destruction in tension. (I.C.E., Feb., 1903.)

17. What quantities must be recorded in an autographic diagram when a specimen is being tested to destruction by torsion in order that the work done upon the bar may be found. Draw such a diagram for a steel bar $\frac{1}{2}$ inch diameter and 8 inches long, and show how to find the work done on the bar. (I.C.E., Feb., 1903.)

N.B.—See *Appendices B and C* for other questions and answers.

Case (1).—When the Column is Hinged or Free at both Ends.

Let OA = The original straight axis of the column. Taking this line as axis of x , and the extremity O for origin.

„ P = Breaking load of the column.

„ A = Area of cross-section of the column.

„ l = Length of the column.

„ y = Co-ordinate of the centre of area of a normal cross-section, and the deflection at abscissa x respectively.

„ Δ = Maximum deflection of the middle of the column from the vertical, or d in the plotted results of Fig. 4.

„ $f_{c \max.}$ = Maximum intensity of compressive stress.

„ f_c = Mean compressive stress.

„ f_b = Intensity of stress due to bending = $f_{c \max.} - f_c$.

„ k = Least radius of gyration of the cross-section of the column.

„ c = Coefficient whose value depends upon the material.

$$f_{c \max.} = \frac{P}{A}. \quad \dots \quad (I.)$$

We know, that $f_b = \frac{My}{I}$ (see equation V., Lecture XXXII.).

But maximum $M = P \Delta$. $\therefore f_b = \frac{P \Delta y}{I}$. $\dots$ (II.)

Let ρ be the radius of curvature of the bent axis of the column, then to find the maximum deflection Δ , we get—

$$\left(\frac{l}{2}\right)^2 = \Delta(2\rho - \Delta) = 2\rho\Delta. \quad \therefore \Delta = \frac{l^2}{8\rho}. \quad \dots \quad (III.)$$

But, $\frac{f_b}{y} = \frac{E}{\rho}$. Consequently, from equation (III.)

$$\Delta = \frac{l^2}{8\rho} = \frac{l^2 f_b}{8 E y} = c \frac{l^2}{y}, \text{ where } c \text{ is a constant} = \frac{f_b}{8 E}.$$

Substituting this value in equation (II.), we obtain—

$$f_b = \frac{P \Delta y}{I} = \frac{P}{I} c l^2 = \frac{P}{A} c \frac{l^2}{k^2} = c f_c \frac{l^2}{k^2}$$

where k is the radius of gyration, and $I = A k^2$ (see Lecture XXII.).

Then the maximum intensity of stress in the column

$$f_{c \max.} = f_c + f_b = f_c + c f_c \frac{l^2}{k^2} = f_c \left(1 + c \frac{l^2}{k^2}\right)$$

Or, $\dots = \frac{P}{A} \left(1 + c \frac{l^2}{k^2}\right).$

If $f_{c \max.}$ is the maximum stress allowed, and which must not be greater than the elastic limit stress in compression of the material, then

$$\text{Breaking load} = P = \frac{f_{c \max.} A}{1 + c \frac{l^2}{k^2}}. \quad \dots \quad (IV.)$$

Where the radius of gyration k of the section with respect to the axis about which the resistance to bending is least—viz., the axis about which I is least. The steady working load to be placed upon the column should

not be greater than nP , where $n = \frac{1}{2}$ to $\frac{1}{3}$ for wrought iron and steel, $\frac{1}{3}$ for cast iron, and $\frac{1}{10}$ for wood, whilst for live loads these values should be halved.

Case (2).—When the Column or Strut has one End Fixed and the other End Rounded or Jointed.—Then we must substitute $\frac{2}{3}l$ for l , and we get—

$$P = \frac{f_c \max. A}{1 + \frac{c}{9} \frac{l^2}{k^2}} \quad \text{. (V.)}$$

Case (3).—When the Column or Strut has Fixed Ends.—If both ends of the column are fixed, then the load which it will carry before bending is the same as for a strut of half the length hinged at the ends, and we must substitute $\frac{l}{2}$ for l .

$$\therefore P = \frac{f_c \max. A}{1 + \frac{c}{4} \frac{l^2}{k^2}} \quad \text{. (VI.)}$$

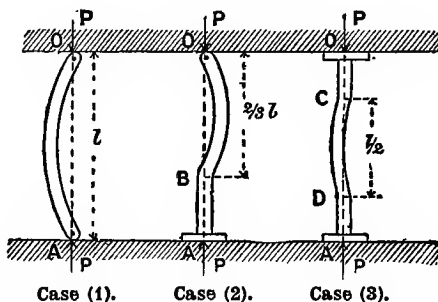


FIG. 2.—SKETCH SHOWING COLUMNS WITH HINGED, HINGED AND FIXED, AND FIXED ENDS RESPECTIVELY.

Proof of Gordon's Formula for Columns.

Taking the same notation as that given above for Rankine's formula

Let d = the greater dimension of the cross-section of column.

„ b = the least dimension of the cross-section of column.

„ $A = db$ = cross-sectional area of column.

Maximum bending moment, $M = P \Delta$, but $\Delta \propto \frac{l^2}{b}$. $\therefore M \propto \frac{Pl^2}{b}$.

Also, $f_b \propto \frac{M}{db^2} \propto \frac{Pl^2}{db^3} \propto \frac{Pl^2}{Ab^2} = cf_c \frac{l^2}{b^2}$, where c is a constant.

Hence, maximum intensity of stress,

$$f_c \max. = f_c + f_b = f_c \left(1 + c \frac{l^2}{b^2}\right) = \frac{P}{A} \left(1 + c \frac{l^2}{b^2}\right). \quad \therefore P = \frac{f_c \max. A}{1 + c \frac{l^2}{b^2}}$$

This latter equation, which is known as Gordon's formula, is similar to Rankine's one, except that the least breadth of section b is taken instead

of k the least radius of gyration, and the value of the constant c modified accordingly. This formula will be true for circular and rectangular sections, since the least dimension of cross-section is a function of the diameter of the column, but for all other sections the least dimension of cross-section is not a simple function of the diameter, hence Rankine's formula is more general than Gordon's.

VALUES OF THE CONSTANTS IN RANKINE'S FORMULÆ.

Material.	f_c max. in Tons per Square Inch.	c .
Cast iron, ...	35	$\frac{1}{1600}$
Wrought iron, ...	16	$\frac{1}{9000}$
Mild steel, ...	21	$\frac{1}{7500}$
Hard steel, ...	30	$\frac{1}{5000}$

VALUES OF THE CONSTANTS IN GORDON'S FORMULA.

Material.	Form of Section	Values of c .			f_c max. in Tons per Square Inch.
		Ends Rounded or Pivoted.	Ends Fixed.	One End Fixed and the Other End Pivoted.	
Wrought iron,	L, T, H, chan- nel, and hollow square, ...	$\frac{4}{900}$	$\frac{1}{900}$	$\frac{1}{500}$	19
	Hollow round, ...	$\frac{1}{900}$	$\frac{1}{3500}$	$\frac{1}{2000}$	17
	Solid round, ...	$\frac{1}{560}$	$\frac{1}{2250}$	$\frac{1}{1260}$	16
	Solid rectangular, ...	$\frac{1}{700}$	$\frac{1}{3000}$	$\frac{1}{1700}$	16
Cast iron, ...	Hollow round, ...	$\frac{1}{200}$	$\frac{1}{800}$	$\frac{1}{400}$	35
	Solid round, ...	$\frac{1}{100}$	$\frac{1}{400}$	$\frac{1}{230}$	35
Mild steel, ...	Hollow round, ...	$\frac{1}{825}$	$\frac{1}{2500}$	$\frac{1}{1400}$	30
	Solid round, ...	$\frac{1}{350}$	$\frac{1}{1400}$	$\frac{1}{780}$	30
	Solid rectangular, ...	$\frac{1}{620}$	$\frac{1}{2480}$	$\frac{1}{1350}$	30

EXAMPLE I.—State Gordon's formula for the strength of columns, and show how it is obtained. Apply it to find the breaking load for a cast-iron column 8 inches external diameter, $6\frac{1}{2}$ inches internal diameter, and 22 feet high. The column has flat ends. Take $f = 80,000$, and $a = \frac{1}{600}$. (I.C.E., Feb., 1898.)

ANSWER.—The first part is already answered in the text.

$$\text{By Gordon's formula, } P = \frac{f_o \text{ max. } A}{1 + c \frac{A}{b^2}}.$$

Where, $c = a = \frac{1}{600}$; and $f_o \text{ max.} = f = 80,000$ lbs.

Substituting values in the formula, we obtain—

$$P = \frac{80,000 \times (8^2 - 6.5^2) \times .7854}{1 + \frac{1}{600} \left(\frac{22 \times 12}{8} \right)^2} = \frac{80,000 \times 17}{2.77} \text{ lbs.}$$

$$\therefore P = \frac{491,000}{2,240} = 220 \text{ tons.}$$

Long Columns.—In a long column, the ratio of its length to the least radius of gyration may be sufficient to cause the column to fail by lateral flexure rather than by direct crushing. Consequently, we have a compound stress somewhat similar to that which exists in the short columns with eccentric loading. The tendency to lateral flexure in long columns may be increased by the eccentricity of the loading caused by the direction of the line of action of the load not coinciding with the true axis of the column. The strength of long columns has been investigated mathematically by Euler and Rankine, and more recently by Professors R. H. Smith, Claxton Fidler, and others.

Proof of Euler's Formula for Long Columns (see Fig. 1).

1st. *When the Column is Hinged or Rounded at the Ends.*—Let OBA be the bent axis of the column. Take the origin at O, and the vertical line OA as the axis of x . Let y be the deflection at the intermediate point c of the column, ρ = radius of curvature, and M = bending moment at xy .

$$\text{Then, } \frac{1}{\rho} = \frac{M}{EI} = \frac{Py}{EI}; \text{ and } \frac{1}{\rho} = -\frac{d^2y}{dx^2}.$$

$$\text{Or, } \frac{Py}{EI} = -\frac{d^2y}{dx^2}.$$

This negative sign is used, because, if we say that the deflection is *positive*, then the centre of curvature must lie on the negative side of OA.

$$-\frac{d^2y}{dx^2} = \frac{Py}{EI}. \text{ add } \int \frac{dy}{dx} dx \text{ to each side.}$$

$$-\int \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} \cdot dx = \frac{P}{EI} \int y \frac{dy}{dx} dx.$$

$$\text{Therefore, } \left(\frac{dy}{dx} \right)^2 = -\frac{P}{EI} (y^2 + c).$$

And, if $\frac{dy}{dx} = 0$, then $y = \Delta$, the maximum deflection; hence, $c = \frac{P}{EI\Delta^2}$.

Therefore, $\left(\frac{dy}{dx}\right)^2 = \frac{P}{EI} (\Delta^2 - y^2); \quad dx = \sqrt{\frac{EI}{P}} \frac{dy}{\sqrt{\Delta^2 - y^2}}$

Or, $\int \frac{dy}{\sqrt{\Delta^2 - y^2}} = \int \sqrt{\frac{P}{EI}} \cdot dx.$

Again integrating, $x = \sqrt{\frac{EI}{P}} \sin^{-1} \frac{y}{\Delta} + c.$

When $x = 0, \quad y = 0; \quad \therefore \quad c = 0.$

Hence, $y = \Delta \sin \left(x \sqrt{\frac{P}{EI}} \right), \quad \left\{ \begin{array}{l} \text{which is the equation of} \\ \text{the elastic curve.} \end{array} \right.$

When $x = \frac{l}{2}, \quad y = \Delta.$

Therefore, $\sin \frac{l}{2} \sqrt{\frac{P}{EI}} = 1; \quad \text{or, } \frac{l}{2} \sqrt{\frac{P}{EI}} = \frac{\pi}{2}; \quad \text{or, } \frac{3\pi}{2}; \quad \text{or, } \frac{5\pi}{2}; \quad \&c.$

The *least* value of P, and also the *minimum* thrust which will bend the column, is given by the equation—

$$\frac{l}{2} \sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \quad \text{Or, } P = \frac{\pi^2 EI}{l^2}. \quad \text{(VII.)}$$

But $P = p A$, and moment of inertia $I = A k^2$. Substituting these values in equation (VII.)—

$$p A = \frac{\pi^2 E A k^2}{l^2}. \quad \therefore \quad p = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}.$$

2nd. *When the Column is Fixed at the one end and Hinged at the other.*

Then, $P = \frac{\pi^2 EI}{\left(\frac{2l}{3}\right)^2} = \frac{9\pi^2 EI}{4l^2}. \quad \text{(VIII.)}$

3rd. *When the Column is Fixed at Both Ends.*—If the pillar or strut is fixed at both ends, the load which it will stand before yielding is the same as for a strut of half the length hinged at the ends.

Then, $P = \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2} = \frac{4\pi^2 EI}{l^2}. \quad \text{(IX.)}$

In this case, the length = $\frac{l}{4}$ in (7).

It will be noticed, that Euler's equations are for very long struts loaded under ideal conditions, viz. :—

(1) That the column is originally straight and of uniform section; (2) material of the column is homogeneous; and (3) the line of action of the load coincides with the centre line or axis of the column.

Professors R. H. Smith and Claxton Fidler have pointed out, that these conditions cannot be realised in practice, as there is nearly always some

eccentricity due to either imperfection in workmanship, want of uniformity in the modulus of elasticity throughout the column, or journal friction occurring in connecting- and eccentric-rods.

If $f_c \text{ max.}$ is the compressive or crushing strength of the material of the column and A is the cross-sectional area of the column.

Then, for short columns,

$$\text{Breaking load } P = f_c \text{ max. } A.$$

Also, according to Euler's formula for very long columns—

$$\text{Breaking load } P = \frac{\pi^2 E I}{l^2}.$$

Then—

$$\text{Breaking load } P = f_c \text{ max. } A = \frac{\pi^2 E I}{l^2} = \frac{f_c \text{ max. } A}{1 + f_c \text{ max. } A \frac{l^2}{\pi^2 E I}}. \quad (10)$$

This equation may be taken as true for columns of all lengths, because if l is small, the denominator is 1 and $P = f_c \text{ max. } A$.

When the length l of the column is great, then we neglect the 1 in the denominator, and—

$$P = \frac{\pi^2 E I}{l^2}.$$

But, $I = A k^2$, where k is the least radius of gyration of the section; then, substituting this value in equation (10)—

$$P = \frac{f_c \text{ max. } A}{1 + c \frac{l^2}{k^2}}. \quad (11)$$

$$\text{Where the constant } c = \frac{f_c \text{ max.}}{\pi^2 E}.$$

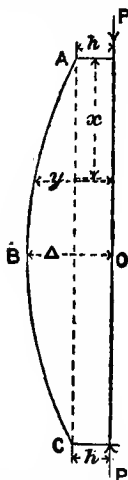


FIG. 3.—SKETCH OF STEEL ROD LOADED ECCENTRICALLY.

It is found, that if the constant is calculated from the equation, values are obtained which make the column too strong, because, as already noted, that in practice perfect straightness, loading, &c., of the column does not exist.

Therefore, the constants $f_c \text{ max.}$ and c are determined from a set of experimental tests, and the formula is treated as an empirical one.

EXAMPLE II.—A rod of steel, 4 inches diameter, 10 feet long, acts as a strut, but the resultant load does not act exactly at the centre of each end. The inexactness of loading is, say, h ; imagine it the same at both ends. If the greatest stress in the material is not to exceed 20,000 lbs. per square inch, and if Young's modulus is 3×10^7 lbs. per square inch, find the greatest loads for the strut in three cases, first when $h = 0.1$ inch, second when $h = 0.01$ inch, third when h is 0.

(B. of E. Hons., 1904—Technics.)

ANSWER.—Let l = Length of the unbent strut.

„ P = Load applied.

„ h = Distance of P from neutral axis at either end of the strut.

Consequently, if ABC represents the neutral axis of the strut, and the load line PP be chosen as the axis of x , the mid-point O of PP is the origin and OB the axis of y . The differential equation to the curve into which the strut is bent will be—

$$EI \frac{d^2 y}{dx^2} = -Py. \quad (1)$$

Where E is Young's modulus of elasticity of the material, and I is the moment of inertia of the cross-section of the strut.

Equation (1) is derived from the consideration that the curvature of the neutral axis at any point is proportional to the bending moment at that point. This latter is proportional to y , which is the distance of the point from the line of action PP of the applied forces.

It is also known that the curvature at any point on the curve

$$y = \Delta \cos \frac{\pi x}{L} \quad (2)$$

is proportional to y if Δ is small; so that we may assume (2) to be the equation to the neutral axis of the strut.

The cosine curve is shown in preference to the sine curve, since y must obviously have the same value for numerically equal positive and negative values of x . L is a constant to be determined, and we know that it cannot be equal to l unless the strut is loaded at the centre, for only in this latter case will the neutral axis cut the line PP at $x = \pm \frac{l}{2}$.

Substituting from equation (2) in (1), we obtain, after simplifying—

$$P = \frac{\pi^2 EI}{L^2} \quad (3)$$

When the strut is centrally loaded, and $c = l$, then equation (3) gives Euler's formula for the "crippling load"—i.e., the least load which will cause the strut to bend. It is easily seen that when P is less than the value given by equation (3), and c has a constant value $= l$, equation (1) can only be satisfied by (2) when $\Delta = 0$.

In this case of the eccentrically loaded strut, by substituting $x = l/2$ and $y = h$ in (2), we get—

$$y = \Delta \cos \frac{\pi x}{L}; \text{ i.e., } h = \Delta \cos \frac{\pi l}{2L} \quad (4)$$

Let f_b = The maximum stress produced in the fibres of the middle section of the strut due to bending.

„ $f_{b \max}$ = The maximum compressive stress.

„ f = Maximum compressive stress which the fibres of the strut can withstand without passing the elastic limit.

„ r = The radius of the cross-section of the strut.

„ A = The cross-sectional area of the strut $= \pi r^2$.

Then, $P \Delta = \frac{f_b I}{r}$; or, $f_b = \frac{P \Delta r}{I}$

Also, the uniform compressive stress due to the load P is $f_{c \text{ max.}} = \frac{P}{A}$.

Hence, the greatest compressive stress on the strut is—

$$f = f_{c \text{ max.}} + f_b = \frac{P}{A} + \frac{P \Delta r}{I} = P \left(\frac{1}{A} + \frac{\Delta r}{I} \right).$$

$$\therefore P = \frac{f A I}{I + A \Delta r}. \quad (5)$$

Substituting this value for P in equation (3), and simplifying, we obtain—

$$\frac{E \pi^2}{f A} = \frac{L^2}{I + A \Delta r}. \quad (6)$$

Equations (6) and (4) form two simultaneous equations in the unknown quantities Δ and L . When the deflection Δ has been calculated from these equations, the maximum permissible load can be obtained by substituting in (5).

The simultaneous equations (6) and (4) can be solved graphically. The corresponding curves are plotted, and the co-ordinates of their point of intersection give the required values of Δ and L . The curve AB in the figure is the graph of the equation—

$$1,177 = \frac{L^2}{12.56 + 25.12 \Delta}$$

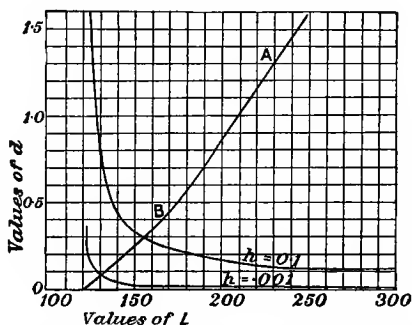


FIG. 4.—PLOTED RESULTS, SHOWING THE RELATIONS OF THE DEFLECTIONS TO THE LENGTHS OF THE STRUT.

In plotting this curve, various values of L greater than $l = 120$ inches were assumed, and the corresponding values of deflection Δ were calculated. The other two curves in the figure are graphs of the equation—

$$\Delta = \frac{h}{\cos \frac{\pi l}{2L}},$$

where $l = 120$, and h has the values .1 and .01 respectively.

It is found, that the curve for $h = .1$ cuts AB at a point corresponding to $\Delta = .295$. Substituting this value in (5), we get $P = 1.58 \times 10^5$ lbs. The curve for $h = .01$ intersects the curve AB at a point corresponding to $\Delta = .085$. Substituting in (5), we obtain $P = 2.14 \times 10^5$.

When the loading is central, $h = 0$, and we must substitute $L = l = 120$ in (3). This gives $P = 258,000$ lbs. for the "crippling load," by Euler's formula—i.e., the least load which will produce bending.

The maximum load which can be sustained by the strut without crushing is—

$$fA = 2 \times 10^4 \times 12.56 = 251,200 \text{ lbs.}$$

Therefore, when the strut is loaded centrally, it breaks from crushing, without bending, when the load is equal to 251,200 lbs.

Thus—	When $h = 0$,	$P = 251,200$ lbs.
	„ $h = .001$,	$P = 214,000$ lbs.
	„ $h = .01$,	$P = 158,000$ lbs.

In order that Euler's equation for the crippling load may not be misunderstood, a short explanation may be useful. If the central load is less than that calculated from (3), when $L = l$, the strut will not bend, though it may fail from crushing.

What happens when the load is equal to that given by (3)?

The left-hand side of equation (1) represents the resisting moment at a section xy of the strut, while the right-hand side represents the turning moment at the same section. The equation to the neutral axis is given by equation (2), and, substituting it in (1), we obtain—

$$\frac{\pi^2 EI}{L^2} \Delta \cos \frac{\pi x}{L} = P \Delta \cos \frac{\pi x}{L}. \quad (7)$$

When the loading is central, L has the constant value l ; and, since both sides of equation (7) are proportional to Δ , it follows that if bending increases, the moment producing this bending increases at the same rate as the moment to stresses in the fibres of the strut which opposes bending. Consequently, the load given by (3), which can just cause bending to commence, is sufficient to bend the rod to any extent. When the fibres are stressed beyond their elastic limit, the resisting moment increases more slowly than the bending moment, because the deflection increases and the strut breaks. Hence we see, that when the load attains the value given by (3) the strut becomes *unstable*, whatever may be its strength under crushing or tension. When the strut is eccentrically loaded, equation (4) shows, that L must diminish as Δ increases, so that the left-hand side of (7) increases at a greater rate than the right-hand side, and *stability* is secured unless the stresses in the fibres exceed the limits of elasticity.

LECTURE VIII.—QUESTIONS.

1. A cast-iron column is straight before being loaded, will it remain so when loaded? If so, why? If not, why not?

2. Illustrate and prove Rankine's formulæ for the three cases of fixing vertical columns.

3. Illustrate and prove Gordon's formulæ for the three cases of fixing vertical columns. State the difference between Rankine's and Gordon's formulæ.

4. Why do long columns fail to support loads? Prove Euler's formulæ for long columns.

5. What are the ideal conditions of loading which satisfy Euler's equations for the loading of long columns, and why?

6. A cast-steel column, 6 inches outside, 4 inches inside diameter, and 20 feet in length, is used as a strut, with the direction of the resultant load (1) centrally and (2) 5 inch from, but parallel to, its central axis. Let the greatest stress per square inch be 25,000 lbs. and Young's modulus 3×10^7 lbs. per square inch; find the greatest load which the column will sustain in each case. Plot your results to scale.

7. A structure has a hollow circular section 10 inches outside diameter and 8 inches inside. The resultant of all the loads and supporting forces acting on one side of the section has a component of 30 tons normal to the section, and it acts at 2 inches from the centre; find the maximum and minimum stresses in the section. (S. & A. H., Part I., 1899.)

8. A solid wrought-iron cylindrical strut is 8 feet 4 inches long, and has its ends solidly built in: assuming it has only a pure compressive load of 24 tons to support, what must be the diameter if the working load is only $\frac{1}{4}$ th of the collapsing load? (C. & G., 1900, H., Sec. A.)

9. It is known that a solid cast-iron column of 4 inches diameter and 7 feet in length would collapse under a load of 17.5 tons per square inch: what total load could you allow the column to support with a factor of safety of 8? (C. & G., 1900, O., Sec. B.)

10. How much would a cast-iron column 12 feet long shorten under a load of $3\frac{1}{4}$ tons per square inch, if it is a hollow section 8 inches in external diameter and 1 inch thick? The modulus of elasticity of cast iron in compression is 12,500,000 lbs. per square inch. What total compressive load will this column support under the above stress per square inch? (C. & G., 1901, O., Sec. B.)

11. Explain why the resistance of a long strut depends more on the stiffness of the material than on its strength. Quote any formula which is used in the design of long struts. (C. & G., 1902, O., Sec. B.)

N.B.—See Appendices B and C for other questions and answers.

LECTURE VIII.—A.M. INST. C. E. EXAM. QUESTIONS.

1. Explain the general relation between the formula for simple crushing and Gordon's and Euler's formulæ for long columns. Putting l for the length and d for the diameter of a column, show by a sketch the relation of the results of applying the three formulæ for different values of $\frac{l}{d}$ for columns of the same section. (I.C.E., Oct., 1897.)

2. Prove Euler's expression for the limit of elastic stability of struts, and explain in how far you consider it applicable to such columns and other struts as are commonly used in engineering structures. (I.C.E., Feb., 1898.)

3. A hollow cylindrical tower of steel plate having an external diameter of 3 feet, a thickness of $\frac{1}{2}$ inch, and a height of 60 feet, carries a central load of 50 tons, and is subjected to a horizontal wind pressure of 56 lbs. per foot of its height. Calculate the vertical stresses at the fixed base of the tower, on the windward and on the leeward side. (I.C.E., Oct., 1898.)

4. Prove that the deflection curve of a slender elastic column, in the incipient stage of buckling, is identical with the curve of a flexible chain under a load whose varying intensity at different points is proportional to the depth of the chain below the horizontal chord-line joining the points of suspension. (I.C.E., Oct., 1898.)

5. What conclusions may be directly drawn from Euler's formula, in regard to the relative strength of columns having the same dimensions, but constructed of such different materials as cast iron, wrought iron, and steel? State generally what are the conditions under which the formula would be nearly correct, and those under which it would be quite at variance with experiment. (I.C.E., Oct., 1898.)

6. Describe carefully the way in which a strut gives way when exposed to a gradually increasing crushing load, considering especially the differences due to difference in the ratio of length to diameter. Write down formulæ for the crushing load—(a) for very long, (b) for very short struts; also give any formula in common use for intermediate cases. Explain the effect of different ways of fixing the ends. (I.C.E., Oct., 1899.)

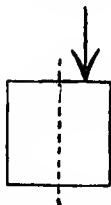
7. Find the greatest height to which a cylindrical column 4 feet diameter can safely be built so as to be stable under a wind pressure of 20 lbs. per square foot of a vertical diametrical section, the weight of the material being taken as 120 lb per cubic foot. Find the line of resistance in this case, and state the relation between the diameter of a column and its height for the same degree of stability under wind pressure. (I.C.E., Feb., 1900.)

8. A solid square pillar 10 feet in height, with a section 2 feet by 2 feet, and weighing 50 cwts., is subjected to a horizontal wind pressure of 28 lbs. per square foot on one side. Assuming its material to be uniformly elastic, find the intensity of compressive stress at the leeward edge and at the windward edge of its base. (I.C.E., Feb., 1901.)

9. A hollow cast-iron column is 9 inches in external diameter, its length is 12 feet, and its two ends are firmly built in. The compressive load it supports is 60 tons. What thickness must the metal be in order to have a factor of safety of 10? (I.C.E., Oct., 1901.)

10. Find the greatest height to which a cylindrical stone column can be built if it is to be stable under the pressure of a wind which may reach a pressure of 45 lbs. per square foot on a flat surface normal to the wind direction. The weight of the material being 145 lbs. per cubic foot, find where the line of resistance will cut the base. (I.C.E., Feb., 1902.)

11. A solid cast-iron column is 5 inches in diameter and 9 feet 7 inches long. What total load will it support if the ends are firmly built in and the working load is not to exceed one-ninth of the crushing load. (I.C.E., Feb., 1902.)



12. A cube of iron is in compression, the load is applied to one face on the centre line parallel to the sides in one direction, but is not at the centre of that line. Show that, if it deviates by one-sixth the length of the side from the centre of the face to which it is applied, there is no stress at the edge further from the load, and that, at the near edge the intensity of compression is twice the average intensity. (I.C.E., Oct., 1902.)

13. Show how Rankine's formula for the breaking load of long columns is derived, and explain why a member in constant cross-section in compression is weaker as its length increases. (I.C.E., Feb., 1903.)

N.B.—See *Appendices B and C* for other questions and answers.

APPENDICES.

APPENDIX A (p. 204 to p. 209).

- (i.) General Instructions by The Board of Education for their Examinations on Applied Mechanics.
- (ii.) General Instructions by The City and Guilds of London Institute for their Examination on Mechanical Engineering.
- (iii.) Rules and Syllabus of Examinations by The Institution of Civil Engineers for Election of Associate Members.

APPENDIX B (p. 210 to p. 245).

- (i.) Board of Education's Exam. Papers in Applied Mechanics, Stages 2 and 3. The City and Guilds of London Institute's Honours Exam. Papers in Mechanical Engineering. And, The Institution of Civil Engineers' Exam. Papers in Strength of Materials, arranged in the order of the Lectures.
- (ii.) All new Answers to Questions for the respective Lectures are tabulated under the two main headings—*either* Board of Education and City and Guilds *or* Institution of Civil Engineers.

APPENDIX C (p. 246 to p. 249).

The latest Exam. Papers pertaining to Strength and Elasticity of Materials and set by the governing bodies enumerated under Appendix A.

APPENDIX D (p. 250 to p. 255).

Tables of Constants, Logarithms, Antilogarithms, and Functions of Angles.

APPENDIX A.

MAY EXAMINATION ON SUBJECT VIIA. APPLIED MECHANICS.*

BY THE BOARD OF EDUCATION SECONDARY BRANCH,
SOUTH KENSINGTON, LONDON.

GENERAL INSTRUCTIONS.

If the rules are not attended to, your paper will be cancelled.

*Immediately before the Examination commences, the following
REGULATIONS are TO BE READ TO THE
CANDIDATES.*

Before commencing your work, you are required to fill up the numbered slip which is attached to the blank examination paper.

You may not have with you any books, notes, or scribbling paper.

You are not allowed to write or make any marks upon your paper of questions, or to take it away before the close of the examination.

You must not, under any circumstances whatever, speak to or communicate with one another, and no explanation of the subject of examination may be asked or given.

You must remain seated until your papers have been collected, and then quietly leave the examination room. None of you will be permitted to leave before the expiration of one hour from the commencement of the examination, and no one can be re-admitted after having once left the room.

Your papers, unless previously given up, will all be collected at 10 o'clock.

If any of you break any of these rules, or use any unfair means, you will be expelled, and your paper cancelled.

**Before commencing your work, you must carefully
read the following instructions:—**

Candidates who have applied for examination in the Elementary Stage must confine themselves to that stage. Candidates who have not applied to take the Elementary Stage may take Stage 2, or Stage 3, or, if eligible, Honours, but they must confine themselves to one of them.

Put the number of the question before your answer.

You are to confine your answers *strictly* to the questions proposed.

Such details of your calculations should be given as will show the methods employed in obtaining arithmetical results.

A table of logarithms and functions of angles and useful constants and formulæ is supplied to each candidate. (See end of Appendix to this Book.)

The examination in this subject lasts for three hours.

* See Appendix C for the latest Exam. Papers.

City and Guilds of London Institute.

DEPARTMENT OF TECHNOLOGY.

TECHNOLOGICAL EXAMINATIONS.

46.—MECHANICAL ENGINEERING.*

HONOURS GRADE (Written Examination).

INSTRUCTIONS.

The Candidate for Honours must have previously passed in the Ordinary Grade, and is required to pass a Written and Practical Examination. He is requested to state, on the Yellow Form, whether he has elected to be examined in A, Machine Designing, or in B, Workshop Practice (*a*) *Fitting*, (*b*) *Turning*, (*c*) *Pattern-making*. Candidates in Machine Designing must forward their work to London not later than May 6th,† and in Workshop Practice not later than May 13th.†

The number of the question must be placed before the answer in the worked paper.

The Candidate is at liberty to use divided scales, compasses, set squares, calculators, slide rules, and mathematical tables.

Five marks extra will be awarded for every answer worked out with the slide rule, provided the method of working is explained.

The maximum number of marks obtainable is affixed to each question.

Three hours allowed for this paper.

The Candidate is not expected to answer more than *eight* of the following questions, which must be selected from *two* sections *only*.

*The questions in Sections A and C may be answered from Vols. I. to V. of my *Text-Book on Applied Mechanics*. Section B is printed in my *Text-Book on Steam and Steam Engines*. The Questions for the Ordinary Grade are printed at the end of my *Elementary Manual on Applied Mechanics*.

* See Appendix C for the latest Exam. Papers.

† These dates are only approximate, and subject to a slight alteration each year.

The Institution of Civil Engineers.

EXTRACTS FROM RULES AND SYLLABUS OF EXAMINATIONS FOR ELECTION OF ASSOCIATE MEMBERS.

Note.—Engineers who desire to enter for the A.M.Inst.C.E. examinations should write *at once* to the Secretary, Great George Street, Westminster, S.W., for the complete Rules, Syllabus, and Application Forms. They will find all the questions relating to the above mentioned subjects which have been set since these examinations commenced in 1897 in my *Text-Books*.

The following extracts are simply printed here to show how far my books upon *Applied Mechanics and Mechanical Engineering; Steam and Steam Engines, including Turbines and Boilers*; as well as *Magnetism and Electricity* (including Munro & Jamieson's *Pocket-Book of Electrical Rules and Tables*), together with my "*Correspondence System of Electrical and Mechanical Engineering Science, as taught by Exercises, Drawings, and Instructions*" cover the Scientific and Practical Knowledge demanded by the Institution, under Part II., Section A (1, 2, 3a, and 3b), as well as Section B under Group i., Theory of Heat Engines; Group ii., Hydraulics and Theory of Machines; and Group iii., Applications of Electricity.

Note for Students.—In the regulations for Students one of the subjects which may be selected is that of Elementary Mechanics of Solids and Fluids, for which see my *Elementary Applied Mechanics* when studying this subject. Also, see my *Elementary Magnetism and Electricity* book when reading that part of the Elementary Physics for admission of Students, or correspond with me re those two subjects.

PART II.*—Scientific Knowledge.

SECTION A.

1. **Applied Mechanics** (one Paper, *time allowed, 3 hours*).
2. **Strength and Elasticity of Materials** (one Paper, *time allowed, 3 hours*).

* Candidates may offer themselves for examination in Sections A and B of Part II. together; or they may enter for Section A alone, and, if successful, may take Section B at a subsequent examination. In the latter case, however, such candidates will not be allowed to present themselves for examination in Section B unless or until they are actually occupied in work as pupils or assistants to practising Engineers. The Council may permit Candidates who have attempted the whole of Part II. at one examination, and have failed in Section B only, to complete their qualification by passing in that section at a subsequent examination, subject to their being then occupied as above stated.

3. *Either* (a) **Theory of Structures.**
 or (b) **Theory of Electricity and Magnetism** (one Paper,
time allowed, 3 hours).

SECTION B.

Two of the following nine subjects—not more than one from any group (one Paper in each subject taken, *time allowed, 3 hours for each Paper*):—

Group i.	Group ii.	Group iii.
Geodesy.	Hydraulics.	Geology and Mineralogy.
Theory of Heat	Theory of Machines.	Stability and Resistance
Engines.	Thermo- and Electro-	of Ships.
Metallurgy.	Chemistry.	Applications of Elec- tricity.

Mathematics.—The standard of Mathematics required for the Papers in Part II. of the examination is that of the mathematical portion of the Examination for the Admission of Students, though questions may be set involving the use of higher Mathematics.

The range of the examinations in the several subjects, in each of which a choice of questions will be allowed, is indicated generally hereunder:—

SECTION A.

1. Applied Mechanics:—

Statics.—Forces acting on a rigid body; moments of forces, composition, and resolution of forces; couples, conditions of equilibrium, with application to loaded structures. The foregoing subjects to be treated both graphically and by aid of algebra and geometry.

Hydrostatics.—Pressure at any point in a gravitating liquid; centre of pressure on immersed plane areas; specific gravity.

Kinematics of Plane Motion.—Velocity and acceleration of a point; instantaneous centre of a moving body.

Kinetics of Plane Motion.—Force, mass, momentum, moment of momentum, work, energy, their relation and their measure; equations of motion of a particle; rectilinear motion under the action of gravity; falling bodies and motion on an inclined plane; motion in a circle; centres of mass and moments of inertia; rotation of a rigid body about a fixed axis; conservation of energy.

2. Strength and Elasticity of Materials:—

Coefficients of elasticity; elastic resistance to tension, compression, shearing, and torsion; uniform and varying stress; moment of stress and of resistance in beams of various sections; distribution of shearing stress in beams; stresses suddenly applied and effects of impact; buckling of struts; effect of different end-fastenings on their resistance; combined strains; calculations connected with statically indeterminate problems as beams supported at three points, &c.; limit of elasticity, yield-point, and

ultimate resistance of various materials as tested: the plastic state; principal forms of testing machines and of appliances used in measurements for the determination of coefficients of elasticity; calculation of extension, deflection, buckling, &c., within elastic limits, and of ultimate strength and ductility for ordinary materials of construction.

3. (a) Theory of Structures:—

Graphic and analytic methods for the calculation of bending moments and of shearing forces, and of the stresses in individual members of framework structures loaded at the joints; plate and box girders; incomplete and redundant frames; theory of continuous girders and principal methods of calculation; travelling loads; riveted and pin-joint girders; rigid and hinged arches; strains due to weight of structures; theory of earth-pressure and of foundations; stability of masonry and brickwork structures.

3. (b) Theory of Electricity and Magnetism:—

Electrical and magnetic laws, units, standards, and measurements; electrical and magnetic measuring instruments; the theory of the generation, storage, transformation, and distribution of electrical energy; continuous and alternating currents; arc and incandescent lamps; secondary cells.

SECTION B.

Group i. Theory of Heat Engines:—

Thermodynamic laws; internal and external work; graphical representation of changes in the condition of a fluid; theory of heat engines working with a perfect gas; air- and gas-engine cycles; reversibility, conditions necessary for maximum possible efficiency in any cycle; properties of steam; the Carnot and Clausius cycles; entropy and entropy-temperature diagrams, and their application in the study of heat engines; actual heat engine cycles and their thermodynamic losses; effects of clearance and throttling; initial condensation; testing of heat engines, and the apparatus employed; performances of typical engines of different classes; efficiency.

Group ii. Hydraulics:—

The laws of the flow of water by orifices, notches, and weirs; laws of fluid friction; steady flow in pipes or channels of uniform section; resistance of valves and bends; general phenomena of flow in rivers; methods of determining the discharge of streams; tidal action; generation and effect of waves; impulse and reaction of jets of water; transmission of energy by fluids; principles of machines acting by the weight, pressure, and kinetic energy of water; theory and structure of turbines and pumps.

Theory of Machines:—

Kinematics of machines; inversion of kinematic chains; virtual centres; belt, rope, chain, toothed and screw gearing; velocity, acceleration

and effort diagrams; inertia of reciprocating parts; elementary cases of balancing; governors and flywheels; friction and efficiency; strength and proportions of machine parts in simple cases.

Group iii. Applications of Electricity:—

Theory and design of continuous- and alternating-current generators and motors, synchronous and induction motors and static transformers; design of generating- and sub-stations and the principal plant required in them; the principal systems of distributing electrical energy, including the arrangement of mains and feeders; estimation of losses and of efficiency; principal systems of electric traction; construction and efficiency of the principal types of electric lamps.

42 Candidates should see, that all their "Forms" are duly completed and passed by the Council of the Institution of Civil Engineers, Great George Street, Westminster, S.W., before 1st January for the February Examination, and before the 1st September for the October Examination. Candidates should, therefore, apply to the Secretary for the "Forms," *at least six months before* these Examinations, to give them time to make due and proper Application, and to thoroughly *Revise* the subjects upon which they are to be examined with an experienced **Guide** and Tuition by Correspondence.

Examinations Abroad.—The papers of the October Examination only will be placed before accepted Candidates in India and the Colonies. To enable the Secretary to make arrangements for the Application Forms and Fees, &c., of these Candidates, their Forms, &c., must be in the Secretary's hands, before the 1st June preceding the October Examinations.

APPENDIX B.

LECTURE I.—ORDINARY QUESTIONS.

1. Steel wire, $\frac{1}{8}$ inch diameter, is supplied in coils, 3 feet diameter. Find the bending moment and the stress induced by coiling the wire on a mandril 2 feet in diameter, keeping the direction of curvature unaltered. You may assume that Young's modulus is 13,000 tons per square inch.

(C. & G., H., Sec. A, 1903.)

2. A crane chain, of sectional area 1 square inch, carries a weight of 1 ton, which is being lowered at a uniform rate of 2 feet per second. When the length of chain unwound is 30 feet, the weight is suddenly pulled up. Estimate the stress induced in the chain due to the sudden stoppage, stating any assumptions involved, and taking Young's modulus for the chain to be 30×10^6 lbs. per square inch.

(C. & G., H., Sec. A, 1904.)

3. Estimate the thickness of a steam pipe of 15 inches internal diameter, so that when the internal pressure is 200 lbs. per square inch the stress in the material does not exceed 4,000 lbs. per square inch. If the pipe be wound initially with a single layer of wire, $\frac{1}{8}$ inch diameter, with a winding stress of 3,000 lbs. per square inch, find the initial stress in the material of the pipe due to winding, and the stresses in the wire and pipe when the internal pressure of 200 lbs. per square inch is applied. For the purposes of the question the pipe may be assumed thin.

(C. & G., H., Sec. A, 1904.)

4. It was necessary to test a certain part of a machine with a load of 30 tons, but it was only possible to apply load by the tightening of a long steel bolt and to measure the load by the lengthening of the bolt. The bolt was $2\frac{1}{2}$ inches diameter; the distance between two shoulders was 102 inches when unloaded. How much was the distance increased when the load was just 30 tons? The Young's modulus of the material was known to be 3×10^7 lbs. per square inch. It was found that the bolt did not take a set.

(B. of E., S. 2, 1904.)

5. A 4-inch steel shaft (Young's modulus 3×10^7 lbs. per square inch), 6 feet long between bearings which control its position but not its direction, has at its middle a wheel weighing $\frac{1}{2}$ ton whose centre of gravity is 0.02 feet away from the axis of the shaft, neglect the inertia of the shaft itself and find the greatest bending moment in the shaft when it makes 800 revolutions per minute. Notice that the balance is more and more untrue at higher speeds. [Formula for deflection of uniform beam of length l , supported at the ends, loaded at the middle $W \frac{l^3}{48 EI}$. Arithmetical error is easy if dimensions are not kept in feet.]

(B. of E., S. 3, 1904.)

6. Explain how a thick cylinder may be strengthened against internal pressure (i.) by chilling the inside layers during cooling, (ii.) by building the cylinder in hoops, each hoop being shrunk over the preceding one. A

hydraulic cylinder, having an internal diameter of 5 inches, is subjected to an internal pressure of 2 tons per square inch. If the greatest safe stress in the material is 4 tons per square inch, estimate the thickness of metal required, (C. & G., H., Sec. A, 1905.)

7. The cylinder of a hydraulic accumulator is 9 inches diameter inside. What thickness of metal would be required for internal pressure of 700 lbs., the maximum tensile stress in the metal being 2,100 lbs.? What is the stress at the outer surface of the cylinder? (C. & G., H., Sec. A, 1906.)

8. A rivet is subjected to a shear stress of 5 tons per square inch, and a tensile stress, due to contraction in cooling, of 3 tons per square inch. Estimate the maximum direct and shear stresses in the rivet.

(C. & G., H., Sec. A, 1906.)

9. Explain the difference between the action of a gradually applied load and one applied suddenly, as, for example, when the load attached to a crane chain is suddenly stopped. A rod, 5 feet long and 1 square inch section, hangs from the top and has a collar at its bottom end. A load of 200 lbs. falls $\frac{1}{2}$ inch on to the collar. Find the maximum stress induced and compare it with the static stress, $E = 29 \times 10^6$ lbs. per square inch.

(C. & G., H., Sec. A, 1907.)

10. A bar of steel, 4 inches wide and $\frac{3}{4}$ of an inch thick, and a bar of brass, 4 inches wide and 1 inch thick, are placed side by side and firmly riveted together at the ends, thus forming a composite bar 4 inches wide and $1\frac{1}{2}$ inches thick. This composite bar is then subjected to a total pull of 16 tons. Determine, in tons, the total load carried by each of the two bars. E , for steel, is 13,500 tons per square inch, for brass, 4,000 tons per square inch. How much would this bar elongate under this load if its original length was 24 inches?

(B. of E., S. 3, 1908.)

11. Show how to determine the distribution of stress in a tension member, when the line of action of the load is parallel to the axis of the specimen. An eye bar for a bridge is 8 inches by 1 inch in cross-section, and the total load upon it is 84,000 lbs. Owing to the ends not being truly bored, the pin centres are both $\frac{1}{8}$ inch out of line on the same side of the axis of the member. Calculate the maximum and minimum stresses on the section, and show by a diagram what the stress is at any point.

(C. & G., H., Sec. A, 1908.)

LECTURE II.—ORDINARY QUESTIONS.

1. State clearly what occurs at a cross-section of a loaded beam. Prove the rule as to the position of the neutral line. (B. of E., S. 2, 1904.)

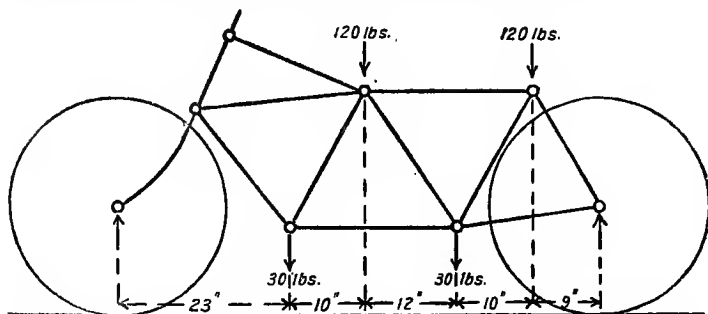
2. What is the relation between the bending moment and the shearing force in the case of a beam subjected to both? Explain how the respective diagrams show this. Draw bending moment and shearing force diagrams for a beam loaded as follows:—A uniformly distributed load of 3 cwts. per foot run covers $\frac{2}{3}$ of the span from one abutment, and the span is 60 feet. Mark on your drawing the position and amount of the maximum bending moment. (B. of E., S. 3, 1904.)

3. A rolled steel joist 12 inches deep, with flanges 4 inches wide, and 1 inch thick, and a web $\frac{1}{2}$ inch thick, supports a wall over a span of 14 feet, the weight of the wall being equivalent to a disturbed load of 1,600 lbs. per foot run. In addition to this load another cross-girder rests on the centre of the rolled joist, and transmits to it a load of 2 tons. Find, neglecting the weight of the joist, (a) the maximum bending moment in this joist in inch-lbs.; (b) the maximum shearing force in lbs.; (c) the

maximum intensity of the tensile and compressive stresses in the flanges of the joist. (B. of E., S. 2, 1906.)

4. State the general geometrical connection between the curves of loads, shearing force and bending moments; and, given the curves of weight and buoyancy for a floating body (such as a ship), explain how a curve of bending moments may be obtained. (C. & G., H., Sec. A., 1906.)

5. The sketch shows the frame of a tandem bicycle. Calculate and



draw bending moment and shear diagrams for this frame when each of the two riders weighs 150 lbs., 30 lbs. of the weight of each rider being assumed to be borne at the driving axle centres. (B. of E., S. 3, 1907.)

6. A beam of 30 feet span, supported at the ends, carries a load which varies uniformly from 1 ton per foot of the beam at the left-hand end to 4 tons per foot of beam at the other end. Determine (a) the supporting forces, (b) the magnitude of the maximum bending moment and the position of the section in which it acts, (c) the magnitude of the maximum shearing force and the position of the section in which it acts. Sketch complete bending moment and shear diagrams for this beam. What do you mean by bending moment? (B. of E., S. 2, 1908.)

LECTURE III.—ORDINARY QUESTIONS.

1. A plate web girder, of 80 feet clear span, is 6 feet deep and has booms which are 3 feet wide. If the total uniformly distributed load which the girder carries is 120 tons, obtain a suitable section for the booms at the middle of the span. Design also a suitable cover plate for one of the boom plates. The safe stress in tension, compression, and shear may be taken as 5 tons per square inch. (C. & G., H., Sec. A, 1903.)

2. A beam of uniform section is built into the walls of a house. If one wall afterwards settles an amount d without disturbing the horizontal direction of the ends of the beam, show that the maximum stress induced in the beam due to the settling is $3 E h d / l^2$, in which h is the depth and l the span of the beam, and E the value of Young's modulus for the beam. (C. & G., H., Sec. A, 1904.)

3. A wooden plank, 12 inches wide and 3 inches thick in section, rests freely on two supports in the same horizontal level which are 20 feet apart. A man weighing 12 stone stands in the middle of this plank carrying on his shoulder a hod of bricks which weighs 84 lbs. Find—(a) The maximum stress at the central section due to this load and the weight of the plank

(1 cubic foot of wood weighs 46 lbs.); (b) the deflection in the centre, if Young's modulus of elasticity is 1,600,000 lbs. per square inch.

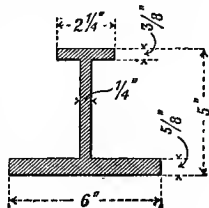
Note.—The numerical coefficient in the usual formula for the deflection of a beam when loaded in the centre is $\frac{1}{48}$, and, when uniformly loaded, $\frac{5}{384}$. (B. of E., S. 2 and 3, 1905.)

4. A ladder, hinged at the top, rests on a smooth horizontal plane. The lengths of the two legs are 6 feet and $6\frac{1}{2}$ feet respectively, and the distance apart of their feet is 4 feet. They are connected by a horizontal string $4\frac{1}{2}$ feet below the apex. If a man, weighing 140 lbs., stands on the longer leg at a distance of 2 feet from the apex, find the pull on the string, and sketch the bending moment diagrams for the two legs.

(C. & G., H., Sec. A, 1905.)

5. Why, in an ordinary cast-iron girder, is the top flange generally made of less sectional area than the bottom flange? A small cast-iron girder is, in section, as shown in the figure. Estimate the ratio of the stresses on the top and bottom fibres, and show, by a sketch, how the bending stress varies across the section.

(C. & G., H., Sec. A, 1905.)



6. A steel cable consists of 6 strands, with 7 wires in each strand, the diameter of the wire being $\frac{1}{16}$ inch. It is wound round a drum 14 feet diameter and carries a cage at its extremity. Calculate the stress due to bending; and if the greatest stress allowed is $11\frac{1}{2}$ tons per square inch, estimate the greatest weight of cage, &c., that ought to be carried. $E = 13,000$ tons per square inch. (C. & G., H., Sec. A, 1906.)

7. A cast-iron water main, 30 inches in internal diameter and $1\frac{1}{2}$ inches thick, is carried across a stream underneath a road bridge. If the main is unsupported for a length of 16 feet, find the maximum intensity of tensile stress in the metal due to the bending produced by the weight of the water and the weight of the pipe itself. If $E = 12,000,000$ lbs. per square inch, how much will the pipe sag in the centre of the 16 feet? Assume the pipe to be a beam fixed at the ends. One cubic inch of cast iron weighs 0.26 lb. (B. of E., S. 2 and 3, 1907.)

8. A steel rolled joist rests on two supports which are 12 feet apart; the depth of the section of the joist is 10 inches and the moment of inertia of the cross-section about the neutral axis is 122 in inch units. From what height must a weight of $\frac{1}{2}$ of a ton fall upon the centre of this beam if the maximum stress produced is not to exceed 20,000 lbs. per square inch. ($E = 30,000,000$ lbs. per square inch.) (B. of E., S. 3, 1907.)

9. A bar of iron, 2 inches diameter, is bent into the arc of a circle by subjection to a bending moment of 5,000 lb.-inches, the value of E being 29×10^6 lbs. per square inch. Find (i.) the greatest stress at any section of the bar, and (ii.) the radius of the circle into which the bar is bent.

(C. & G., H., Sec. A, 1907.)

10. When bending moment is applied to the section of a beam which is already curved, we can calculate the change of curvature. What is the rule? Prove it to be correct. (B. of E., S. 2, 1908.)

11. A rolled steel joist of the following cross-section:—Depth of section, 16 inches; width of flanges, 6 inches; thickness of flanges, $\frac{3}{8}$ inch; thickness of web, $\frac{1}{8}$ inch—is firmly built at either end into masonry walls which are 6 feet apart. Upon the centre of the 6-foot span of the joist, a built-up steel column rests and transmits to it a total load of 50 tons.

Determine the maximum intensity of tensile and compressive stresses in the joist and the deflection in the centre. (B. of E., S. 3, 1908.)

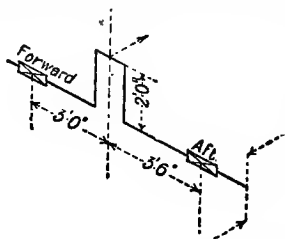
12. A 15-inch Γ beam has a section which may be represented approximately by two rectangular flanges 6 inches by $\frac{3}{4}$ inch connected by a web $\frac{3}{4}$ inch thick. Calculate the moment of resistance of this section for a stress of 12,000 lbs. per square inch. Also determine the increase of strength of the section if a plate 12 inches by $\frac{1}{2}$ inch is riveted to each flange, allowance being made for two rows of $\frac{7}{8}$ -inch rivets in each flange.

(C. & G., H., Sec. A, 1908.)

LECTURE IV.—ORDINARY QUESTIONS.

1. If a shaft 4 inches in diameter will safely withstand a torque of 120,000 lb.-inches, what torque would a 9-inch shaft take? What H.P. would the former shaft transmit at 200 revolutions per minute, and what would the latter transmit at 50 revolutions per minute? What do you mean by shear stress in a shaft? (B. of E., Adv., 1903.)

2. The crank-shaft of a single cylinder engine is as shown in the figure. The work is absorbed at one end and the bearings may be assumed to exercise no constraint on the shaft. When the connecting-rod is perpendicular to the crank-arm, the effective force in the rod is 30 tons. Find the magnitudes of the twisting and maximum bending moments on the crank-pin. If three-quarters of the work is absorbed aft and one-quarter forward, find the same things.



(C. & G., H., Sec. A, 1903.)

3. Design a hollow steel tunnel shaft and coupling to transmit 10,000 horse-power at 120 revolutions per minute. There are eight bolts, and the inside diameter of the shaft has to be $\frac{1}{6}$ times the outside diameter. Choose your own working stresses, &c. The shaft may be assumed subjected to pure twisting, and the ratio of the maximum to the mean twisting moment to be 1.2. (C. & G., H., Sec. A, 1903.)

4. A circular shaft, supported in bearings 12 feet apart, transmits 15 horse-power at 90 revolutions per minute, and carries a belt and pulley 3 feet from one bearing, which produces a side pull of 500 lbs. on the shaft. If the greatest stress induced in the shaft has not to exceed 4 tons per square inch estimate the diameter of the shaft assumed solid.

(C. & G., H., Sec. A, 1904.)

5. Obtain expressions for the maximum shearing stress and for the total twist produced in a round shaft of given dimensions under a given twisting moment. A steel shaft is employed in order to transmit power to a distance of 100 feet, the maximum intensity of shearing stress is not to exceed 4 tons per square inch, and the total twist on the whole length is not to exceed 30° . What size must the shaft be, and how many horse-power will it transmit when running at 90 revolutions a minute? The modulus of shear elasticity of steel is 12,000,000 lbs. per square inch.

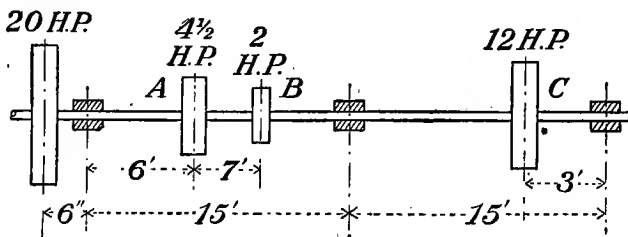
(B. of E., S. 3, 1904.)

6. In a single cranked engine of 4 feet stroke the centre line of the cylinder is 3 feet from the left-hand bearing and 2 feet from the right-hand bearing. The work is taken off equally, as a pure couple, at the two ends

of the shaft. If, when the connecting-rod is perpendicular to the plane of the crank arms and shaft, the thrust in the rod is 10 tons, estimate the twisting moment and maximum bending moment on the crank pin, and also on each crank arm. (C. & G., H., Sec. A, 1905.)

7. A solid steel shaft has to transmit 150 H.P. The number of revolutions of the shaft per minute is 115, and the maximum twisting moment in each revolution exceeds the mean by 25 per cent. If the maximum intensity of the shearing stress is not to exceed 9,000 lbs. per square inch, what must the diameter of this shaft be? (B. of E., S. 2, 1906.)

8. A steel shaft $2\frac{1}{2}$ inches in diameter is driven by a 20-H.P. gas engine at 100 revolutions per minute. The shaft is supported by three bearings, spaced 15 feet apart between centres, and the centre of the driving pulley is 6 inches beyond the centre of one of the end bearings. Pulleys are arranged, as shown on the sketch, to work certain machines, and the

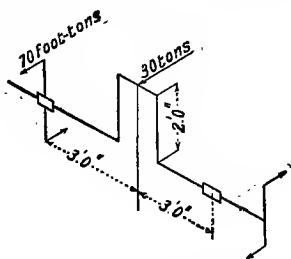


Pulleys:—A is 18 inches, B is 10 inches, and C is 24 inches diameter.

horse-power taken off each of these pulleys is shown on the sketch; in addition each bearing absorbs $\frac{1}{2}$ H.P. Assuming that all loads are applied at the centres of the respective pulleys and bearings, calculate the angle of twist in the shaft at each of these points, reckoning from either end of the shaft. The modulus of rigidity is 12,500,000 lbs. per square inch. (B. of E., S. 3, 1906.)

9. The crank shaft of a three-cylinder engine, 4 feet stroke, and cranks at 120° , is supported on four equi-distant bearings, the common pitch being 6 feet. The centre line of each crank is midway between its bearings. Calculate the maximum bending moment and twisting moment on the after-most crank pin when the after crank is in a horizontal position, the vertical forces on each crank pin being then 20 tons, and each segment of the shaft, from bearing to bearing, being assumed to act as an independent beam. (C. & G., H., Sec. A, 1906.)

10. The after crank of a three-cranked engine is as shown. The turning moment transmitted from the high and intermediate cranks is 70 foot-tons; when the low-pressure connecting-rod is perpendicular to the plane of crank arms the thrust is 30 tons. Find the turning moment transmitted through the remaining part of the shaft, and the bending moment and twisting moment on the crank pin, assuming the crank and shaft arms rigid. (C. & G., H., Sec. A, 1907.)



11. Show how to obtain a formula for the horse-power transmitted by a solid shaft of diameter, D , in the form

$$\text{Horse-power} = c N D^3 f_s,$$

where f_s is the allowable shearing stress, N is the number of revolutions per minute, and c is a constant. Determine the value of c and calculate the horse-power which may be transmitted by a shaft 4 inches in diameter, when turning at 300 revolutions per minute, if the allowable stress is 9,000 lbs. per square inch. (C. & G., H., Sec. A, 1908.)

12. Explain how to determine the maximum intensity of stress in a shaft due to combined bending and twisting moments. A shaft, 3 inches in diameter, transmits 180 horse-power at 200 revolutions per minute. It is supported in bearings 9 feet apart, and a pulley, keyed to the shaft 6 feet from one bearing, has a belt upon it giving a pull equivalent to a load of 270 lbs. Calculate the greatest shearing stress on the shaft due to the twisting moment, and the greatest bending stress due to the pull of the belt, and also determine the maximum intensity of the resultant stress. (C. & G., H., Sec. A, 1908.)

LECTURE V.—ORDINARY QUESTIONS.

1. State precisely how you would make a tensile test of a ductile material, such as wrought iron or mild steel, stating what measurements you would make and how you would reduce the observed results. What are the most important items to obtain from a commercial point of view, and what results would you expect in a specimen of good mild steel. (C. & G., H., Sec. A, 1903.)

LECTURE VI.—ORDINARY QUESTIONS.

1. Write specifications for the steel you would use in the construction of (i.) a steel girder, (ii.) a steel boiler, (iii.) a propeller shaft. Describe, briefly, the apparatus required to carry out the prescribed tests. (C. & G., H., Sec. A, 1904.)

2. A tension bar is held at each end by a turned pin; the bar is rectangular in section, 6 inches wide and $1\frac{1}{2}$ inches thick. Owing to the fact that the eyes for the pins at the ends of the bar are each bored out with its centre $\frac{3}{8}$ of an inch to one side of the centre line of the bar, the line of pull is $\frac{3}{8}$ of an inch to one side of the geometrical axis of the flat side of the bar, but parallel to this axis. Calculate the maximum and minimum stresses in any section at right angles to the line of pull, when the total tensile load on the bar is 40 tons. By means of a diagram roughly drawn to scale, show graphically the distribution of stress in the cross-section. (B. of E., S. 3, 1905.)

3. Describe Wöhler's experiments on the continued repetition of stress and give—quantitatively, if possible—the leading results obtained. What more recent experiments have been made on the same subject, and what are the results obtained? (C. & G., H., Sec. A, 1905.)

4. Explain in what way the tenacity of a plate is affected (i.) by punching, (ii.) by drilling a hole, and how any prejudicial effects may be removed. In the case of a treble-riveted butt-joint with two covering plates, holes drilled, the ultimate tenacity of the steel plates may be taken as 65,000 lbs., and the shearing resistance of the rivets as 50,000 lbs. per square inch. Find the pitch of the rivets to connect plates $\frac{3}{4}$ inch thick, the diameter of

the rivets being 1 inch. What "efficiency" would you expect, in practice, with such a joint? (C. & G., H., Sec. A, 1905.)

5. In a certain method of making shock tests of metal, a heavy rotating disc carries a knife at a point on its circumference. The disc having been set in rapid rotation, the metal specimen to be tested, supported on two knife edges at a certain distance apart, is moved up towards the rotating disc to such a position that the knife in descending strikes the centre of the specimen and fractures it. The work absorbed in fracturing the bar is estimated from the change of velocity of rotation of the disc. In an actual machine the diameter of the disc, which is of steel, is 12 inches, and its thickness is 4 inches (1 cubic inch of steel weighs 0.28 lb.). The angular velocity of the disc before impact with the specimen is 36 radians per second, and, after impact, 24.3 radians per second. Calculate the number of inch-lbs. of work spent in fracturing the specimen. Criticise this method of testing the quality of a specimen of any material.

(B. of E., S. 3, 1906.)

LECTURE VII.—ORDINARY QUESTIONS.

1. Answer only *one* of the following:—(a) Describe an experiment by which you could determine E, Young's modulus of elasticity, by stretching a steel wire. (b) Describe an experiment to measure how the kinetic energy of a flywheel depends upon its speed. (B. of E., Adv., 1903.)

2. Answer only *one* of the following:—(a) Describe an experiment by which you could determine E, Young's modulus of elasticity, by bending an iron bar. What shape of bar would you use, and why? How would you attach the load, and how support the beam? How would you ascertain the deflection of the beam? (b) Describe an experiment to measure how the kinetic energy of a flywheel depends upon its speed. If the time the flywheel took to come to rest were measured, how could you calculate the average H.P. lost in friction? (B. of E., H., Part I., 1903.)

3. Prove the formula you use for calculating the deflection of a beam of rectangular cross-section. A beam 1 inch wide and 1.5 inch deep, is placed upon knife edges 4 feet apart, and the overhanging ends are loaded with weights of 100 lbs. each, placed 18 inches beyond the points of support (i.e., the loads themselves are 7 feet apart), what is the upward deflection of the middle point of the beam ($E = 30,000,000$)? What is the shearing force between the knife edges? (B. of E., H., Part I., 1903.)

4. A wire, 34 inches long and .123 inch diameter, hanging vertically, is clamped at its upper end and carries a light pulley $8\frac{1}{2}$ inches diameter, having its central plane horizontal, at its lower end. When two equal and opposite forces of 1 lb. are applied in a horizontal plane at the opposite extremities of a diameter of the pulley, the angle turned through by the pulley was found to be 70° . Estimate the coefficient of rigidity of the material of the wire. (C. & G., H., Sec. A, 1904.)

5. Describe *one* only of the following, (a) or (b):—(a) Describe how you would carry out an experiment to determine the modulus of tensile elasticity of a piece of round 1-inch mild steel bar. Explain carefully how you would prepare the bar and hold it in the testing machine, how you would measure the extensions, and what precautions you would adopt to secure accuracy. (b) Fifty cast-iron beams, say 12 feet long, are to be delivered; it is specified that each shall stand a certain load, say $\frac{1}{2}$ ton, without taking any set. On behalf of the foundry, you are arranging to have them tested in the presence of the buyers. How would you arrange for this?

Presumably you know that a beam just manufactured will take a set with even a very small load. (B. of E., S. 3, 1904.)

6. The following results were obtained during a tensile test of a mild steel bar $\frac{3}{4}$ inch in diameter :—

Total load on the bar, } in tons, }	0.88	1.76	2.64	3.52	4.40	5.28	6.16	7.04
Elongation on a length } of 8 inches, in inches, }	0.0012	0.0024	0.0035	0.0047	0.0061	0.0075	0.0088	0.0102

Plot a curve on squared paper to show the relation between the load and the elongation. Find the total work done in inch-lbs. upon this 8-inch length of bar during the test. Calculate Young's modulus of elasticity in lbs. per square inch. (B. of E., S. 2, 1905.)

7. Answer only one of the following, (a) or (b) :—(a) You are given a piece of steel wire; explain fully how you would determine its modulus of rigidity (or of *shear* elasticity); give any formulæ you would use in making the final calculations from your experimental results. (b) It is required to determine experimentally the coefficient of discharge for a rectangular weir with sharp edges. How would you carry out such an experiment? Explain carefully how you would measure the head of water over the lip, or sill, of the weir, and what calculations you would have to make to find the numerical values of the coefficients. (B. of E., S. 3, 1905.)

8. The following results were obtained during a complete tensile test to destruction of a mild steel bar $\frac{3}{4}$ inch in diameter :—

Total load on the bar, } in tons, }	1.76	3.52	5.28	7.04	7.48	8.80	10.56	11.44	11.88	12.32	9.46
Elongation on a length of 8 inches, in inches, }	0.0024	0.0047	0.0075	0.0102	0.16	0.26	0.51	0.82	1.18	1.70	2.38 Bar broke.

(a) Calculate and tabulate stress and strain, and plot a complete stress-strain curve on squared paper. (b) Calculate from the plotted figure the total work done in inch-tons per cubic inch in fracturing this specimen. (c) Calculate—(i.) The modulus of direct elasticity in lbs. per square inch; (ii.) the limit of elasticity of the steel in lbs. per square inch; (iii.) the ultimate tenacity of the steel in lbs. per square inch. (B. of E., S. 3, 1905.)

9. In connection with a contract for the supply of cast-iron pipes, certain bending tests were specified on bars (cast at the same time) 40 inches long, 2 inches deep, and 1 inch thick. The following results were obtained when one of these bars was tested on edge on a 36-inch span :—

Load at centre of beam, . . lbs.,	100	400	800	1,200	1,600	2,000	2,400
Deflection at centre of beam, inches,	.012	.043	.098	.150	.204	.256	.314

(a) Plot on squared paper a curve to show the relation between the load at the centre of the beam and the deflection at the centre of the beam; from

your curve determine the load which will be required at the centre of the beam in order to give a deflection of one-eighth of an inch. (b) Calculate in lbs. per square inch Young's modulus of elasticity for this cast iron; the deflection of a rectangular beam—length l , breadth b , depth d , loaded with W at its centre—is $W l^3 \div 4 E b d^3$. (c) Calculate in inch-lbs. the total work done in bending this beam up to a load of 2,400 lbs. in the centre of the span. (d) The beam eventually broke with a load of 3,200 lbs. in the centre; assuming that the ordinary beam formula holds up to the breaking point in cast-iron beams, what was the maximum intensity of tensile stress in the metal at the instant of rupture? (B. of E., S. 2 and 3, 1906.)

10. Describe some instrument by means of which the value of Young's modulus for a ductile material may be very accurately obtained. How is the instrument calibrated? (C. & G., H., Sec. A, 1906.)

11. Describe, in detail, how you would make a test to destruction of a piece of mild steel plate 2 inches by $\frac{1}{2}$ inch in cross-section, and of sufficient size to measure the extension on a 10-inch length. Draw a probable force-extension diagram for such a bar, in which the extensions are doubled and the pull on the specimen is to a scale of 20,000 lbs. to the inch. Also show how to calculate the coefficient of direct elasticity from the results of your observations. (C. & G., H., Sec. A, 1908.)

LECTURE VIII.—ORDINARY QUESTIONS.

1. Give the theory of the laterally loaded strut. What is the effect of eccentric loading, and what is the effect of non-uniformity of material? When is this theory useful? (B. of E., H., Part I., 1903.)

2. A cast-iron column, 5 inches internal and 7 inches external diameter, has a bracket attached to it, and a weight of 20 tons is carried by the bracket at a distance of 12 inches from the axis of the column. Estimate the maximum and minimum stresses induced in the column, and show, by means of a sketch, how the stress varies across the section. (C. & G., H., Sec. A, 1903.)

3. Show that the buckling load of a long strut having rounded ends is $\frac{EI \pi^2}{l^2}$, in which l is the length of the strut, I the geometrical moment of inertia of the cross-section about a line perpendicular to the plane of bending, and E Young's modulus. How is the behaviour of a strut affected when the load is not applied axially? (C. & G., H., Sec. A, 1904.)

4. The floors of a warehouse are supported by means of cast-iron columns. The lowest column in any tier is 12 inches in external diameter, and the metal is $1\frac{1}{4}$ inches thick. The total load transmitted to this column from those above it in the tier and from the portion of the floor immediately above, which it supports, is 125 tons. What is the compressive stress in the metal of this column? What amount will this column be shortened if its length is 15 feet, and if Young's modulus of elasticity is 12,500,000 lbs. per square inch? If the load in tons per square inch which would destroy such a column is given by the equation—

$$\text{Destructive load in tons per square inch} = \frac{36}{1 + \frac{1}{800} \left(\frac{L}{D} \right)^2},$$

where L is the length of the column in inches, and D the external diameter in inches, what factor of safety has this column? (B. of E., S. 2, 1905.)

5. A hollow cast-iron column has to be designed to support a total load of 130 tons; the column is to be 15 feet in length, and the internal diameter is to be four-fifths of the external diameter, and it is desired to have a factor of safety of 10. Find the external and internal diameters of the column if the crushing load in tons per square inch for such a column is given by the formula stated in the previous question. (B. of E., S. 3, 1905.)

6. The cylinder of a vertical engine is entirely supported by one vertical cast-iron column of rectangular section. The centre line of the cylinder is 8 inches from the vertical face of the column, and the column is 10 inches deep and 6 inches wide, and the thickness of the metal all round is 1 inch. If the driving force on the piston when at the end of the stroke is 5 tons, estimate the greatest stress induced in the column, and show, by a sketch, how the stress varies over a section. (C. & G., H., Sec. A, 1905.)

7. The usual form of Gordon's formula as applied to medium-sized struts is buckling load in lbs. per square inch = $\frac{\beta}{1 + \frac{\alpha}{k^2}}$, in which l = length of

strut, k = least radius of gyration, and α, β are empirical coefficients. What is the rational basis of this formula? and find the values of α and β from the following data, which refer to wrought-iron struts, fixed at the ends, each having a section of 1 inch $\times$ 1 inch, namely:— l = 30 inches, buckling load = 11.3 tons; l = 90 inches, buckling load = 4.35 tons.

(C. & G., H., Sec. A, 1906.)

8. An upright timber post, 12 inches in diameter, supports a vertical load of 18 tons, the line of action of which is 3 inches from the vertical axis of the post. Determine the maximum and minimum intensities of stress on a normal cross-section of the post, and show by a diagram how the intensity of normal stress varies across the section. Prove the formula used by you. (B. of E., S. 3, 1907.)

9. Explain how the strength of a long, thin column varies according to the fixing of the ends, and state the relative strengths of a column for the four standard cases of fixing. Obtain the Rankine-Gordon formula for the safe load on a column, and show in what way it is modified to make it applicable to the four cases. (C. & G., H., Sec. A, 1908.)

LECTURE I.—I.C.E. QUESTIONS.

1. A bar of steel 2 inches in diameter is subjected to a tension of 18 tons in a direction parallel to its axis. Find a general expression for the intensity of the normal and tangential stresses on a section inclined at any angle to the axis and calculate them for an angle of 30° .

(I.C.E., Oct., 1903.)

2. Calculate the least radius to which a 1-inch round bar of wrought iron (for which $E = 28,000,000$ lbs. per square inch) may be bent in order that the skin stress may not exceed 15 tons per square inch. What is then the moment of resistance of a section?

(I.C.E., Oct., 1903.)

3. Prove the formula for the stress on a longitudinal section of a thin cylindrical shell exposed to internal pressure. Can this formula be used for thick hollow cylinders? Give reasons for your answer.

(I.C.E., Oct., 1903.)

4. What area and form of section would be suitable for a mild steel roof member subject to the following axial loads (+ signifying tension and - compression): (i.) due to dead load + 3,500 lbs.; (ii.) due to wind on one side + 6,000 lbs.; (iii.) due to wind on the other side - 4,000 lbs.

(I.C.E., Oct., 1903.)

5. Steel rails are welded together and there is no stress at a temperature of 60° F. They are prevented from buckling and they cannot expand or contract. Find the stresses when the temperature is (i.) 20° F., (ii.) 120° F. Steel expands by 0.0012 of its length for a change in temperature of 180° F. Take $E = 30,000,000$ lbs. per square inch. If the elastic limit in tension is 40,000 lbs. per square inch, at what temperature would it be reached?

(I.C.E., Feb., 1904.)

6. A hard drawn copper wire is stretched between two points on the same level, 50 feet apart. Find the sag at the centre (i.) when the greatest stress is 5 tons per square inch; (ii.) when the breaking stress, 26 tons per square inch, is reached. Copper weighs 560 lbs. per cubic foot.

(I.C.E., Feb., 1904.)

7. What would be the smallest size of a drum on which it would be advisable to wind a steel measuring tape $\frac{3}{8}$ of an inch thick? If wound on a drum too small in diameter what effect would you expect to be produced on the length of the tape?

(I.C.E., Oct., 1905.)

8. A piece of material is subjected to a pair of tensile stresses at right angles to each other of 8 and 4 tons per square inch respectively. Calculate the stresses produced on a plane making an angle of 30° degrees to the direction of the stress of 8 tons.

(I.C.E., Feb., 1906.)

9. A 2-inch diameter bolt, 12 feet long, is screwed into the under side of a rigid beam, and hangs vertically. At the lower end of the bolt is a solid head; on the bolt is a weight of 100 lbs., which fits it loosely. The weight falls a distance of 10 feet on to the lower head. Neglecting the work done by the weight falling through the small distance due to the elastic stretch of the bar, calculate the stress produced in the bolt. Take the elastic modulus as 30,000,000 lbs. per square inch.

(I.C.E., Feb., 1906.)

10. The internal diameter of a water main is 30 inches; it is constructed of steel plates $\frac{3}{8}$ inch thick, with double-riveted butt joints. Assuming the tensile strength of the plates to be 30 tons per square inch, and the efficiency of the joints as 95 per cent., what is the bursting pressure of the main?

(I.C.E., Feb., 1906.)

11. What is meant by the resilience of a body? Give an illustration.

(I.C.E., Feb., 1906.)

12. Sketch the eye end of a steel link for a suspension bridge, the body of the link being 6 inches by 3 inches, and show how you determine the size of the pin. (I.C.E., Feb., 1906.)

13. Define the two terms "stress" and "strain." The suspension-rod of a bridge is 30 feet long and stretches $\frac{1}{16}$ inch under the load; what is the strain? (I.C.E., Oct., 1906.)

14. Define the two terms "limit of elasticity" and "modulus of elasticity." A rod 100 feet long has a sectional area of 2 square inches, and, when submitted to a tension of 20 tons, stretches 0.62 inch; what is the modulus of elasticity of the material? (I.C.E., Oct., 1906.)

15. The limit of elasticity of a wrought-iron bar was found to be 20,000 lbs. per square inch, the strain at that point being 0.0006; what was the resilience of the material? (I.C.E., Oct., 1906.)

16. Sketch the forged eye end of a steel tension bar, 4 inches by 1 inch, to some recognised proportion, and state generally what effect the diameter of the pin has upon these proportions. Assume the diameter of the pin to be three-fourths of the width of the bar. (I.C.E., Oct., 1906.)

17. A cylindrical chimney, 6 feet external and 4 feet internal diameter, is subject to a wind pressure equivalent to 30 lbs. per square foot on the vertical projected area; find the limiting height of the chimney to avoid any tension at the base, the weight of the brickwork being 125 lbs. per cubic foot. (I.C.E., Oct., 1906.)

18. A rod of steel 10 feet long with a sectional area of $\frac{1}{4}$ square inch, is kept at the proof strain with a tensile load of 25,000 lbs.; what is the amount of the resilience of the rod when $E = 35,000,000$ lbs. per square inch? (I.C.E., Feb., 1907.)

19. A load of 3 cwts. is suspended by three parallel wires of equal length, each $\frac{1}{16}$ square inch in area, the outer ones being of steel and the inner one of brass, the wires being adjusted so that each has the same stress. A load of 3 cwts. is then added to the original load; what is the final stress on the wires assuming E to be 30,000,000 lbs. per square inch for steel and 10,000,000 lbs. per square inch for brass? (I.C.E., Feb., 1907.)

20. A circular steel tank 20 feet diameter has to be filled with water to a depth of 14 feet. Assuming the efficiency of the joints to be 80 per cent., what must be the thickness of the plates so that a stress of 5 tons per square inch shall not be exceeded? Weight of water = 62.5 lbs. per cubic foot. (I.C.E., Feb., 1907.)

21. A riveted butt joint with two covers has to be made in a tension plate, 16 inches by $\frac{3}{4}$ inch, which has to transmit a stress of 90 tons. Design what you consider to be the best form of joint, and make a neat sketch of the complete joint with covers. Assume the diameter of rivets $\frac{1}{2}$ inch. Working stresses:—tension, 8 tons per square inch; shear, 5 tons per square inch; bearing, 10 tons per square inch. (I.C.E., Oct., 1907.)

22. The flanges of a plate web girder consist of three plates $\frac{1}{2}$ inch thick fixed to a single web $\frac{1}{2}$ inch thick, with two angle bars 4 inches $\times$ 4 inches $\times$ $\frac{1}{2}$ inch. Sketch the jointing of the three plates under one cover for (a) a theoretical joint, (b) a practical joint, giving the diameter and pitch of the rivets on the flange and web plates and the angles. (I.C.E., Oct., 1907.)

23. State and explain fully the conditions which have to be taken into account in deciding on the thickness of the cylinder walls for a steam engine. Obtain a formula embodying these requirements, and compare the results it gives with the following empirical rule:—Thickness of wall = $0.03 \sqrt{\text{diameter of cylinder} \times \text{pressure}}$, when the pressure is 200 lbs. per square inch gauge, and the diameter of the cylinder 30 inches,

and taking 2,000 lbs. per square inch as the allowable stress in tension in cast iron. (I.C.E., Oct., 1907.)

24. A bar, 4 square inches in sectional area, carries a weight of 20 tons. Determine the intensities of the normal and tangential stresses on a plane making an angle of 60° with the cross-section. (I.C.E., Feb., 1908.)

25. Determine the area of cross-section required for a wrought-iron bar 16 feet long, in order that it may resist the energy of a load $\frac{3}{4}$ ton falling through a height of 6 inches, the resultant of the load being along the axis of the bar, and the material just strained to the elastic limit 12 tons per square inch. $E = 12,000$ tons per square inch. (I.C.E., Feb., 1908.)

26. Design a zigzag double-riveted butt joint, with two covers, for two steel plates $\frac{3}{8}$ inch thick. Rivets $\frac{3}{4}$ inch diameter. Assume the following working strengths:—Tensile strength of plates, 7 tons per square inch; shearing strength of rivets, 6 tons per square inch; compressive strength of plates, 12 tons per square inch. Determine the efficiency of the joint. Give a dimensioned plan of a portion of the joint. (I.C.E., Feb., 1908.)

27. A flat bar, 12 inches by 1 inch, is in tension, and has to be jointed with double cover-plates; sketch and fully dimension a joint in which the strength of the member has not been reduced more than 11 per cent. The rivets to be $\frac{7}{8}$ inch diameter; the working stress in bars and covers to be 7 tons per square inch; the bearing stress on rivets, 10 tons per square inch; and the shear stress in rivets, 5 tons per square inch. (I.C.E., Feb., 1908.)

28. A rod of steel 100 feet long, with a sectional area of 2 square inches, stretches $\frac{3}{4}$ of an inch when a load of 15 tons is suspended from the end; determine the stress, the strain, and the modulus of elasticity. (I.C.E., Oct., 1908.)

29. A steel rod 30 feet long, having a sectional area of $\frac{1}{2}$ square inch, has a weight of 1 ton suspended from it; what is the amount of energy stored in the bar owing to its extended condition, when the modulus of elasticity is 13,000 tons per square inch? If the weight were applied suddenly, what would be the amount of the energy stored? (I.C.E., Oct., 1908.)

LECTURE II.—I.C.E. QUESTIONS.

1. A log of wood, 20 feet long, of uniform cross-section, floats in water and carries two weights, each equal to 1 cwt., at distances of 4 feet from each end. Sketch, roughly to scale, the diagrams of shearing force and bending moment on the log. (I.C.E., Oct., 1903.)

2. A girder 20 feet long carries a distributed load of 1 ton per lineal foot over 6 feet of its length, the load commencing at 3 feet from the left-hand abutment. Sketch the shearing-force and bending-moment diagrams, and find, independently, the magnitude of the maximum bending moment and the section at which it occurs. (I.C.E., Feb., 1904.)

3. A girder, 40 feet long, rests on two supports at distances of 10 feet from the ends. It is loaded with a uniformly distributed load of $\frac{1}{2}$ ton per foot run, and a concentrated load of 10 tons at the centre. Draw, to scale, diagrams of bending moments and shearing forces. (I.C.E., Feb., 1904.)

4. A cantilever 30 feet long is built in at one end, and when unloaded touches, but does not exert pressure on, a support at the other end. It is loaded with a uniformly distributed load of 10 cwts. per foot run. Draw diagrams of bending moments and shearing forces, and find the position of the point of contrary flexure. (I.C.E., Feb., 1904.)

5. The tensile strength of a specimen of cast iron is found to be 10 tons per square inch. A bar of the same material 1 inch wide and 2 inches

deep is laid upon supports 3 feet apart, and loaded in the middle. What would be the breaking weight of the beam, if it were calculated from the known tensile strength of the extreme fibre by the theory of elastic flexure, and what would it generally be in practice? (I.C.E., Oct., 1904.)

6. A wire is carried across a series of equal spans a, b, c , &c., of 240 feet each, the weight of the wire in each span being 400 lbs. The span a, b has a dip of 3 feet, and b, c a dip of 2.5 feet, the wire being attached at b to the top of a vertical post 20 feet high. Find the bending moment at the base of the post. (I.C.E., Oct., 1904.)

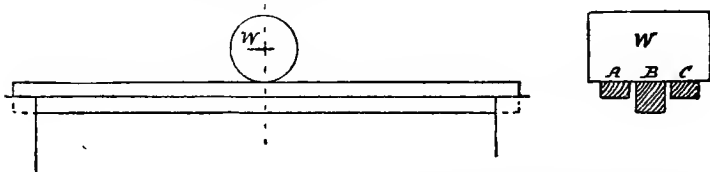
7. A floating beam of timber of uniform section and 20 feet long carries three equal loads of 400 lbs. each, one in the middle of its length, and one at each end. Find the bending moment at the centre, and sketch the whole diagram of bending moments. (I.C.E., Oct., 1904.)

8. A beam 20 feet long is supported on two supports, 3 feet from each end of the beam. Weights of 10 lbs. and 20 lbs. are suspended from the two ends of the beam. Draw, to scale, the bending-moment and shearing-force diagrams; and, in particular, estimate their values at the central section of the beam. (I.C.E., Feb., 1905.)

9. A girder crossing a span of 400 feet is traversed by a railway train having the uniform weight of 1 ton per foot; and the train, whose length is greater than the span, may enter the bridge from either end. Find the greatest positive and negative values of the resulting shearing force at a section 100 feet from either abutment, and sketch the diagram of maxima. (I.C.E., Feb., 1905.)

10. Show how the same question can be determined when the train consists of one or two heavy locomotives followed by lighter carrying stock. (I.C.E., Feb., 1905.)

11. Suppose that three beams or planks, A, B, and C, of the same material are laid side by side across a span $L = 100$ inches, and a load $W = 600$ lbs. is laid across them at the centre of the span so that they must all bend together. The beams are all 6 inches wide, but while A and C have a depth of 3 inches, the depth of the middle beam B is twice as great. How much of the weight W will be carried by each of the three beams, and what will be the extreme fibre-stress in each?



(I.C.E., Feb., 1905.)

12. Sketch approximately to scale bending moment and shear diagrams for a beam supported at its ends and loaded half way along with a uniformly distributed load. Show how to find the maximum bending moment. (I.C.E., Oct., 1905.)

13. The side elevation of a cantilever 6 feet long is triangular, the depth increasing from 0 at the extremity to 12 inches at the wall into which it is fixed, the breadth is 3 inches, and the cross-section is rectangular. Sketch diagrams to show the distribution of tension, compression, and shear at the wall caused by its own weight. (I.C.E., Oct., 1905.)

14. A number of small web-plate girders were designed to carry a distri-

buted test load of 100 tons each; they were tested at the contractor's yard by placing two girders back to back about 3 feet apart, with slings at the ends, and applying a hydraulic jack between the girders at the centre, pressing against the two simultaneously with a pressure of 50 tons. Was this a fair test to all parts of the girder? State the reasons for your answer. (I.C.E., Oct., 1905.)

15. A plank of wood, of specific gravity 0.7, 12 feet by 1 foot by 1 inch thick, rests in water in a horizontal position and carries concentrated loads of 5 lbs. at each end. What is the bending moment in the centre of the plank? (I.C.E., Feb., 1906.)

16. A uniform iron beam 10 feet long, weighing 600 lbs., rests on two rollers, the centres of which are 1 foot and 6 feet from one end. If the beam be rolled forward 3 feet, what will be the pressures on the two rollers in its new position? (I.C.E., Feb., 1906.)

17. A beam 40 feet long resting on the ends is loaded with three weights of 5, 10, and 15 tons placed 10 feet apart, the 15-ton weight being at the centre of the span. Draw the diagrams of the bending moments and the shearing stresses. (I.C.E., Feb., 1906.)

18. A trapezoidal masonry dam, weighing 140 lbs. per cubic foot, is 60 feet high, 8 feet wide at the top, 38 feet wide at the base, and has the up-stream face vertical. Find (i.) its centre of gravity, (ii.) the stress per square foot at each side of the base, assuming that the stress varies uniformly across the base, when the water is level with the crest of the dam. (I.C.E., Oct., 1906.)

19. Find the centre of gravity of a concrete wall, 18 feet high, 10 feet wide at the base, 5 feet wide at the top, with one face vertical and the other in steps of 3 feet high and 1 foot offset. Assuming a uniform variation of stress across the base and a weight of 120 lbs. per cubic foot for the concrete, find the stress per square foot at each side of the base. (I.C.E., Feb., 1907.)

20. A beam, 20 feet span, supported at both ends, carries a uniform load of $\frac{1}{2}$ ton per foot run, distributed over its length; also two concentrated loads of 4 tons and 6 tons, at 4 feet and 12 feet respectively from the right support. Draw the curves of shearing force and bending moment for the whole span. Calculate the value of the bending moment at a section 9 feet from the right support. (I.C.E., Oct., 1907.)

21. A girder, 50 feet span, carries a uniform fixed load of $\frac{1}{2}$ ton per foot run. A moving load, assumed of uniform weight, $1\frac{1}{2}$ tons per foot run (of length greater than the span), travels over the girder from one end. Find the section at which the shearing force changes sign. Is this the section where the bending moment is greatest, and if not, where is it? (I.C.E., Oct., 1907.)

22. A beam of uniform cross-section, 20 feet long, is fixed horizontally at one end, and is supported at the other end by a prop at the same level. It carries a uniform load of $\frac{3}{4}$ ton per foot run. Determine the pressure on the prop, and the position and value of the maximum bending moment. Give a dimensioned sketch of the bending-moment diagram. (I.C.E., Feb., 1908.)

23. A girder A B C D, 50 feet long, rests on two piers at B and C 30 feet apart, the ends A and D overhanging the piers by 10 feet. The 30 feet of girder between the piers B and C is loaded with a uniform load of 1 ton per foot run. At the end A is a weight of 12 tons, and at the end D a load of 8 tons. Draw the bending-moment and shearing-force diagrams for the whole girder. Calculate the bending moment at a section 12 feet from B in the portion B C. (I.C.E., Feb., 1908.)

24. A beam, 24 feet long, is uniformly loaded with 10 tons for three-quarters of its length, beginning at one abutment; find the value and position of the maximum bending moment. (I.C.E., Feb., 1908.)

25. A road bridge of 60 feet effective span carries a travelling load of a vehicle on four axles, A, B, C, and D, A = 5 tons, B = 5 tons, C = 3 tons, and D = 3 tons. The distance between A and B = 7 feet 9 inches, between B and C = 7 feet 9 inches, and between C and D = 7 feet 9 inches, the total wheel-base being 23 feet 3 inches. Determine the position of the vehicle on the span when the bending-moment under B is a maximum. State the amount of the bending-moment. (I.C.E., Oct., 1908.)

26. Draw the bending-moment diagram for a continuous girder of four spans of 100 feet of uniform depth and section, and carrying a uniformly distributed load of 1 ton per lineal foot on each end span, and 2 tons per lineal foot on each of the two spans on each side of the centre. State the amounts of the bending-moments at each pier and at the centre of each span. (I.C.E., Oct., 1908.)

27. A sluice-gate consists of an outer frame 20 feet square, the top and bottom girders of which are connected by vertical ribs spaced 3 feet 4 inches apart, centre to centre, and covered by plating. The highest water-level coincides with the top of the gate. Calculate the load on one rib and determine the position and amount of the maximum bending-moment and the amount of shear at each end. Fresh-water weighs 62½ lbs. per cubic foot. (I.C.E., Oct., 1908.)

LECTURE III.—I.C.E. QUESTIONS.

1. A girder is built of an $\mathbf{I}$ rolled joist; flanges 1 inch thick, 7½ inches wide; web ½ inch thick, 20 inches deep; with two ½-inch plates 12 inches wide riveted to each flange. Calculate the modulus of the section (a) approximately, (b) more accurately. (I.C.E., Oct., 1903.)

2. A girder of section as in previous question is 30 feet long and subject to a distributed load of 1½ tons per foot run. Calculate the shearing stress between the plates, and between the inner plate and the joist at a section close to the supports. Hence show how you would find a suitable diameter and pitch for the rivets. (I.C.E., Oct., 1903.)

3. Find the least radius of gyration for the $\mathbf{I}$ joist referred to in the above questions. Taking a factor of safety of 6, find the working load of such a joist as a column 10 feet high with fixed ends. Use the formula:—

$$P = \frac{25 A}{\frac{\lambda^2}{1 + \frac{30,000}{\lambda^2}}},$$

where P is the buckling load in tons, A the cross-sectional area in square inches, and λ is the ratio of the length to the least radius of gyration. (I.C.E., Oct., 1903.)

4. Construct the bending-moment and shearing-force diagrams for a beam 25 feet long, freely supported at one end and resting on a support 5 feet distant from the other end, the beam being loaded with a uniformly distributed load of ½ ton per foot run, a load of 5 tons at mid-span, and a load of 10 tons at the overhanging end. (I.C.E., Oct., 1903.)

5. Prove an expression for the deflection at mid-span of a beam resting freely on two supports and loaded uniformly per foot run; or describe carefully how you would find it graphically (I.C.E., Oct., 1903.)

6. Explain clearly how you would proceed to obtain graphically the bending-moment diagram for a continuous girder of three equal spans resting on supports at the same level; the dead load being evenly distributed over the whole girder. (I.C.E., Oct., 1903.)

7. A girder of length 60 feet is subjected to a uniformly distributed dead load of $\frac{3}{4}$ ton per foot run, and to a uniformly distributed rolling load of $1\frac{1}{2}$ tons per foot run. Construct diagrams to show for every section of the girder (1) the shearing stress due to the dead load, (2) the maximum positive and negative shearing stresses due to the rolling load as it crosses the girder. (I.C.E., Oct., 1903.)

8. A bridge is carried by two mild steel plate-girders over a span of 50 feet, the depth of the girder being 5 feet. It is to carry an equivalent distributed load of 3 tons per foot run. Selecting suitable data, design the section of the girder at the centre and at the ends. (I.C.E., Feb., 1904.)

9. A load W is applied in the middle of a cantilever of uniform section. Prove, either graphically or mathematically, that the elastic deflection of the free end is $\frac{5}{48} \frac{W l^3}{EI}$, l being the length of the cantilever, I the second moment of its section, and E the Young's modulus of the material. (I.C.E., Feb., 1904.)

10. State clearly the assumptions on which are based the usual formulæ for beams $\frac{M}{I} = \frac{f}{y} = E \frac{d^2 y}{dx^2}$, noting to what extent and within what limits they are true. A cast-iron beam, 2 inches $\times$ 1 inch, when subjected to a transverse test, broke when the tensile stress (calculated from the formula $M = fZ$) was 35,000 lbs. per square inch. A similar specimen tested in tension broke when the stress was 28,000 lbs. per square inch. Why do the results differ? (I.C.E., Feb., 1904.)

11. The pitch or flute of a trough decking () is 32 inches and the depth is 13 inches, the neutral axis being at the half depth. The moment of inertia round the neutral axis is 883 inch-units. Find the distributed load per square foot which can be supported on a span of 30 feet if the stress is not to exceed 5 tons per square inch. (I.C.E., Feb., 1904.)

12. An I joist, 20 inches deep, has flanges $7\frac{1}{2}$ inches wide and 1 inch thick, and web $\frac{1}{2}$ inch thick. The greatest moment of inertia is 1,650 inch-units, and the total shear over a section is 80 tons. Show by a diagram, drawn to scale, the intensity of shear stress at all points of the section. (I.C.E., Feb., 1904.)

13. A beam ABCD, whose length AD is 50 feet, is supported at each end, and carries a weight of 2 tons at B, 10 feet from A, and a weight of 2.5 tons at C, 20 feet from D. Calculate the shearing force at the centre of the span, and sketch the diagram of shearing forces. (I.C.E., Oct., 1904.)

14. Referring to the loaded beam described in the previous question, how much additional load would have to be put on at the point B in order to reduce the shearing force at the centre of the span to zero? (I.C.E., Oct., 1904.)

15. Describe briefly the reasoning and the hypothetical assumptions on which the ordinary theory of transverse flexure is based, and its deductions in regard to transverse strength—illustrating by the case of a simple beam of rectangular section. (I.C.E., Feb., 1905.)

16. Assuming the usual theory of transverse flexure, show how the shearing force is distributed over a vertical section of the rectangular beam. (I.C.E., Feb., 1905.)

17. If any beam of uniform section deflects 1 inch in a span of 100 inches under a central load, what will be the slope of the beam at each end? (I.C.E., Feb., 1905.)

18. A steel girder crossing a span of 120 feet carries a uniform load, and is designed with a uniform depth of 10 feet, but with flanges of varying section adapted to a constant working stress of $6\frac{1}{2}$ tons per square inch. Calculate the deflection, taking $E = 13,000$ tons per square inch. (I.C.E., Feb., 1905.)

19. If the girder in the previous question were designed for the same span and load, and with the same depth, but were made of uniform section with a maximum flange stress of $6\frac{1}{2}$ tons per square inch, what would be its deflection? (I.C.E., Feb., 1905.)

20. A rectangular beam is loaded by a weight at its centre. How will the deflection be changed by (a) doubling the span, (b) doubling the width, (c) doubling the depth, (d) doubling the load? (I.C.E., Oct., 1905.)

21. If a bar of metal 2 inches square be placed upon bearings 3 feet apart, will it carry the greatest load at the centre with the sides vertical or with the diagonal vertical? Give the breaking-load for both positions when the bar is made of cast iron. (I.C.E., Oct., 1905.)

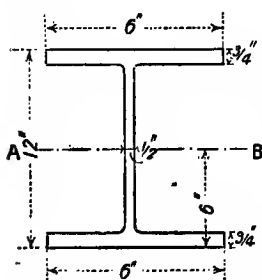
22. State the rule for finding the moment of inertia of a beam section which admits of being split up into a number of rectangular areas, and apply it to the case of a 4-inch $\times$ 4-inch tee-iron whose mean thickness is $\frac{1}{2}$ an inch. (I.C.E., Feb., 1906.)

23. A girder made up of a web plate 2 feet 6 inches deep, $\frac{1}{2}$ inch thick, riveted to flange plates by angle irons, is, near the abutments, subjected to a shearing load of 23 tons. If the pitch of the rivets connecting the angles to the web plate is 4 inches, what size rivets would you put in? Shearing stress 5 tons per square inch, bearing pressure 10 tons per square inch. (I.C.E., Feb., 1906.)

24. A mild-steel joist, total depth 6 inches, flanges 4 inches by $\frac{3}{8}$ inch, and thickness of web $\frac{1}{8}$ inch, rusts away to a depth of $\frac{1}{8}$ of an inch all over the surface. Before rusting the load produced a stress of 8 tons per square inch on the outer layer. What will be the stress after rusting if the load is the same as before? The results need only be given approximately. (I.C.E., Feb., 1906.)

25. Describe with sketches the method you would employ to determine the deflection produced by a passing train in the main girder of a plate girder bridge of about 60 feet span over a road. (I.C.E., Feb., 1906.)

26. Either of the following sections is available for a beam which is required to be as strong and as stiff as possible:—(a) Circular 2 inches in diameter, or (b) rectangular 2 inches deep, 1.178 inches wide. Which would you use? (I.C.E., Feb., 1906.)



27. The deflection of two plate girders is to be ascertained when a load of 10 tons acts at the centre. An hydraulic jack capable of exerting a push of 5 tons is available; it is fitted with a pressure gauge, so that the pressure it exerts can be measured. Sketch the arrangement you would employ to determine the deflection. (I.C.E., Feb., 1906.)

28. A cast-iron water pipe 10 inches external diameter and $\frac{1}{2}$ inch thick rests on supports of 40 feet apart. Calculate the maximum stress in the outer fibre of the material when empty and when full of water, also the corresponding deflections. (I.C.E., Feb., 1906.)

29. Calculate the moment of inertia round the axis A B of the rolled joist of the section shown in the sketch, assuming that the flanges and web are rectangular. (I.C.E., Feb., 1906.)

30. Calculate the moment of resistance of the above joist; and assuming it to be resting on bearings 10 feet apart, what load will it carry at the centre, when the stress in the extreme fibre does not exceed 6 tons per square inch? (I.C.E., Feb., 1906.)

31. A rectangular bar of steel 3 inches deep by 2 inches wide is placed on bearings 5 feet apart with a load of 5 tons suspended from the centre. How much will the bar deflect, assuming the modulus of elasticity to be 30,000,000 lbs. per square inch. (I.C.E., Feb., 1906.)

32. A rectangular bar of steel 3 inches deep by 2 inches wide is placed on bearings 6 feet apart, the ends simply resting on the bearings; assuming the modulus of elasticity to be 30,000,000 lbs. per square inch, how much will the bar deflect when a load of 2 tons is placed at the centre? (I.C.E., Oct., 1906.)

33. A plate girder, 30 feet long and 4 feet deep, carries the equivalent of a uniformly-distributed dead load of 6 tons per foot run; find the necessary thickness of the flanges at the centre, the breadth being 16 inches. The flanges are connected to the web by angles $4\frac{1}{2}$ inches $\times$ $4\frac{1}{2}$ inches $\times$ $\frac{1}{2}$ inch. Show how you allow for the material cut away for the rivet holes, and how to determine the length of the individual plates in the flanges. Take the working intensity of stress for tension 9 tons per square inch, and for compression 7 tons per square inch, and the rivets as $\frac{7}{8}$ inch diameter. (I.C.E., Oct., 1906.)

34. Calculate the necessary thickness for the web of the girder in the last question, and, taking a convenient value, find the number of rivets, $\frac{7}{8}$ inch diameter, required per foot to connect it to the angles at the ends of the girder. Why cannot the full shearing strength of these rivets be developed? (I.C.E., Oct., 1906.)

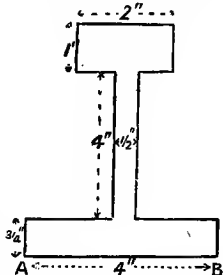
35. What is the least internal radius which a bar of steel 4 inches wide $\times$ $\frac{3}{8}$ inch thick can be bent so that the maximum stress will not exceed 5 tons per square inch? $E = 13,000$ tons per square inch. (I.C.E., Feb., 1907.)

36. A wrought-iron girder is 20 feet long and 2 feet deep, with flanges 9 inches broad and $\frac{3}{4}$ inch thick, the web being $\frac{3}{8}$ inch thick. What uniformly-distributed load will the girder carry with a maximum intensity of stress of 5 tons per square inch in the flanges? What is the total horizontal shear between the web and flange from the centre to the end. (I.C.E., Feb., 1907.)

37. Find the centre of gravity of the section **I** shown, and its moment of inertia round a line through the centre of gravity parallel to A B. (I.C.E., Oct., 1907.)

38. An oak beam, 12 feet span, is supported at both ends; it is 9 inches wide and 12 inches deep in cross-section, and carries a brick wall 9 inches thick. Find to what height the wall can be built with safety, if the greatest permissible stress is 1,000 lbs. per square inch. Weight of brickwork, 120 lbs. per cubic foot. (I.C.E., Oct., 1907.)

39. A piece of Tee-iron consists of a web $\frac{1}{2}$ inch thick and 4 inches deep, combined with a flange 2 inches broad and $\frac{1}{2}$ inch thick. The piece is subject to compression, and the line of action of the compressive force is not axial, but acts in a line parallel to the axis through the centre of depth. Compare its strength under this condition with its strength if the force were axial. (I.C.E., Oct., 1907.)



40. The cross-section of a cast-iron girder consists of a top flange 4 inches wide, 2 inches thick; bottom flange, 10 inches wide, 2 inches thick; web, 8 inches deep, 1 inch thick. Determine its moment of inertia about an axis through the centre of area parallel to the flanges. If the span is 20 feet, determine what distributed load it will carry for a working tensile stress of $1\frac{1}{2}$ tons per square inch. (I.C.E., Oct., 1907.)

41. Find the greatest deflection in inches of a rectangular wooden beam carrying a load of 2 tons at the centre of a span of 20 feet, with a limiting intensity of stress of 1,000 lbs. per square inch. The depth of the beam is 14 inches. Calculate the breadth. Assume $E = 600$ tons per square inch. (I.C.E., Oct., 1907.)

42. Assume the rolled steel joist shown by the diagram to have rectangular flanges and web. Calculate (a) the section modulus, (b) the moment of inertia about the axis A B, (c) the least radius of gyration. (I.C.E., Oct., 1907.)

43. Supposing the joist shown by the diagram to be resting upon supports 20 feet apart, what load would it carry in the centre so as not to exert a greater unit-stress in the extreme fibres than 8 tons per square inch? Neglecting the weight of the beam, what load applied to the centre would deflect it $\frac{1}{2}$ inch? Assume the modulus of elasticity to be 28,000,000 lbs. per square inch. (I.C.E., Oct., 1907.)

44. The cast-iron valve box of a pump has one side 18 inches square inside supported on all four sides by being cast to the body of the box; but it is not stiffened by any ribs. The pressure in the box being 300 lbs. per square inch, what must be the thickness of the side to give a factor of safety of 4? (I.C.E., Oct., 1907.)

45. Find the radius in feet of the smallest circle into which a round bar 4 inches in diameter can be bent, if the longitudinal stress be limited to 4 tons per square inch, and the modulus of elasticity be 12,500 tons per square inch. (I.C.E., Feb., 1908.)

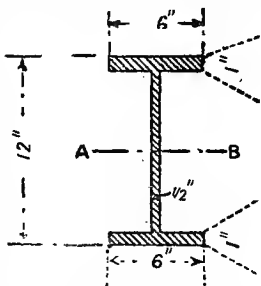
46. A floor is formed of wooden joists 9 inches deep and 3 inches wide, spaced 14 inches apart centre to centre. The clear span is 20 feet. Determine what distributed load per square foot the floor can safely carry if the longitudinal stress is limited to 1,000 lbs. per square inch. (I.C.E., Feb., 1908.)

47. Calculate the following properties of a rolled steel joist, 15 inches deep over all, with flanges 6 inches $\times$ $\frac{3}{4}$ inch, and web $\frac{1}{2}$ inch thick:— (a) The moment of inertia about the horizontal axis, (b) the radius of gyration about the horizontal axis, (c) the section modulus. (I.C.E., Feb., 1908.)

48. A rectangular beam, 12 inches deep and 4 inches wide, rests on bearings 10 feet apart, and is loaded with 10 tons at the centre, what is the maximum stress in the extreme top and bottom fibres? (I.C.E., Feb., 1908.)

49. A rolled steel joist, having a section modulus of 18-inch units, is designed to act as a uniformly-loaded continuous girder over two spans of 20 feet each, the stress in the extreme fibre being 7 tons per square inch. After erection, it was found that the centre support had subsided $\frac{1}{2}$ inch; what was the effect on the stresses of the beam at this point? (I.C.E., Feb., 1908.)

50. A rolled steel joist 12 inches deep over all, with equal flanges 6 inches

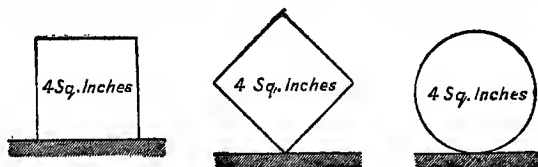


wide, $\frac{7}{8}$ inch thick and web $\frac{1}{2}$ inch thick, is placed upon bearings 20 feet apart. If a load of 10 tons be applied at the centre, what will be the deflection, assuming the modulus of elasticity to be 13,000 tons per square inch?

(I.C.E., Oct., 1908.)

51. Establish the formula for ascertaining the deflection of a rectangular beam with depth a , breadth b when loaded uniformly with a load W over a span l , assuming that the curve of deflection is circular. (I.C.E., Oct., 1908.)

52. Three cast-iron bars have each an area of 4 square inches cross-section, two of them being square, the third one being circular; they are placed upon bearings 5 feet apart in the following positions:—

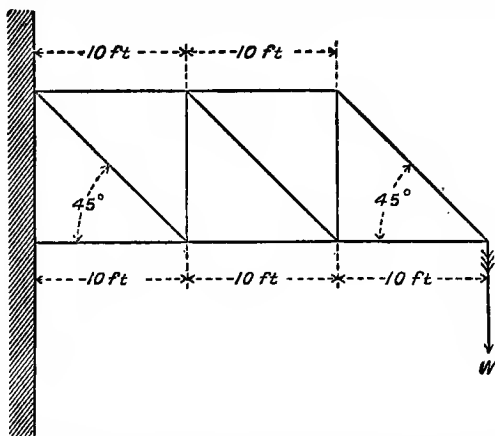


What is the ratio of the respective breaking weights? (I.C.E., Oct., 1908.)

53. What is the bending moment in lb.-inches which can be applied to a concrete beam 16 inches deep by 10 inches wide, reinforced by four steel rods $\frac{3}{4}$ inch in diameter placed with their centres 2 inches above the bottom surface, so that the working-stresses do not exceed 15,000 lbs. per square inch on the steel and 600 lbs. on the concrete? Assume the modulus of elasticity of the steel to be 30,000,000 lbs. per square inch, and that of the concrete 2,000,000 lbs. per square inch; the neutral axis is 6.3 inches below the top surface.

(I.C.E., Oct., 1908.)

54. The cantilever frame as shown carries a load, W , at its point, which load causes a uniform stress of 5 tons per square inch in every member.



Calculate the deflection of the point of the cantilever. What percentage of the total deflection is due to the web members? Assume the modulus of elasticity to be 10,000 tons per square inch.

(I.C.E., Oct., 1908.)

LECTURE IV.—I.C.E. QUESTIONS.

1. A turbine is connected to a dynamo, placed vertically above it, by a shaft 2 feet in diameter made of steel plate $\frac{5}{8}$ inch thick. Calculate the diameter of a solid shaft of the same material to transmit the same power at the same speed with the same maximum skin stress, due to twist. Find the relative weights. (I.C.E., Oct., 1903.)

2. A mild steel shaft transmits 100 H.P. at 120 revolutions per minute, and is subjected to a maximum bending moment of 2,000 lb.-feet. Find a suitable diameter if the maximum resultant stress is not to exceed 6,000 lbs. per square inch. (I.C.E., Oct., 1903.)

3. A shaft, 3 inches in diameter, running at 250 revolutions per minute, transmits 50 H.P. Find the maximum stress and the twist of the shaft in degrees in a length of 100 feet. The rigidity modulus is 12,000,000 lbs. per square inch. (I.C.E., Feb., 1904.)

4. Prove the common formula for the resultant stress in a circular shaft subject to combined bending and twisting. (I.C.E., Feb., 1904.)

5. A shaft S is driven at a speed of 800 revolutions per minute by means of an engine of 300 H.P., the speed of piston being 500 feet per minute. The connection between the shaft S and that of the engine is effected by a belt which embraces a pulley of 8 inches diameter on the shaft S. Neglecting losses by friction, estimate the mean pull and thrust of the piston-rod, the torsion of the shaft S, and the tension of the belt, assuming that of the slacker side to be one-third of that of the tighter. (I.C.E., Oct., 1904.)

6. A shaft 20 feet long has to transmit a twisting moment of 1,000 lb.-feet when revolving at 200 revolutions per minute. Find the required diameter so that the shaft will not twist more than 2 degrees, and that the maximum stress shall not exceed 5 tons per square inch. (I.C.E., Oct., 1905.)

7. A marine engine whose mechanical efficiency is 88 per cent. indicates 1,000 H.P., and runs at 80 revolutions per minute. The speed of the ship is 12 knots. What size would you make the propeller shaft for this engine if solid, allowing 4 tons per square inch? What would be the direct compression stress in it between the propeller and the thrust block, and what would be the stress due to the twisting moment between the engine and the thrust block? (I.C.E., Feb., 1906.)

8. Two steel shafts 6 inches in diameter have to be coupled together by an ordinary coupling. The diameter of the pitch circle of the bolts is 13.5 inches, and the torque is 6,000 foot-lbs. Calculate the number and size of the bolts (steel), and make a hand sketch of the coupling. (I.C.E., Feb., 1906.)

9. What conditions determine the dimensions of the crank pin of a simple overhung crank? (I.C.E., Feb., 1906.)

10. The crank shaft for a gas engine should be much larger in diameter than the shaft of a steam engine of the same power and speed. Why is this? (I.C.E., Feb., 1906.)

11. A wrought-iron crank shaft, 6 inches diameter and 20 feet long, has a load applied to the crank which twists the ends through an angle of 2° ; taking the coefficient of transverse elasticity as 9,000,000 lbs. per square inch, what is the stress at the extreme fibre? (I.C.E., Oct., 1906.)

12. A shaft has to transmit 50 H.P. at 300 revolutions per minute, find the required diameter of the shaft, so that the extreme fibre stress may not exceed 5 tons per square inch. (I.C.E., Feb., 1907.)

13. Determine an expression for the amount of twist in a circular shaft under a given twisting moment. A steel shaft is 3 inches in diameter and 60 feet long; find the angle in degrees through which it will be twisted if subjected at one end to a twisting moment of 20,000 lb.-inches.

(I.C.E., Oct., 1907.)

14. What horse-power can be transmitted by a steel shaft 4 inches in diameter when running at 300 revolutions per minute; the stress in the extreme fibre to be 6 tons per square inch?

(I.C.E., Oct., 1908.)

LECTURE V.—I.C.E. QUESTIONS.

1. Sketch and describe any arrangement in which a weight can be moved along a beam which oscillates about a fixed fulcrum—such as the beam in a testing machine—without exercising any constraint on the oscillatory motion of the beam.

(I.C.E., Oct., 1903.)

2. Specimens of (a) cast iron and (b) mild open-hearth steel are to be tested in direct tension. Sketch the forms of test-piece and of holding shackle that would be suitable in each case, and describe the kind of fracture that may be expected to take place.

(I.C.E., Oct., 1904.)

3. In testing specimens of mild steel, with a given length between gauge-marks, how would the ultimate elongation be affected by the diameter of the test-piece?

(I.C.E., Oct., 1904.)

4. Describe, with sketches, and explain the working of any form of autographic apparatus for producing a stress-strain diagram during the testing of a specimen to destruction in tension.

(I.C.E., Feb., 1908.)

LECTURE VI.—I.C.E. QUESTIONS.

1. Describe carefully a machine for testing specimens under repeated bending or tensile stresses. State briefly the general results of such tests and their bearing on design.

(I.C.E., Feb., 1904.)

2. If the ultimate strength of the above-named specimens (Lecture V., Q. 2) proved to be (a) 10 tons per square inch, and (b) 27 tons per square inch of original section, how much ultimate elongation would you expect to find, and what would the figures indicate in regard to the capability of the material for resisting a blow?

(I.C.E., Oct., 1904.)

3. How would the ultimate elongation be affected in each case if the above-named test-pieces (a) and (b), with a diameter of $\frac{3}{4}$ inch, were cut by a narrow groove $\frac{1}{8}$ inch deep and $\frac{1}{4}$ inch wide without any sharp angles (gauge-marks, say, 8 inches apart)? Give any explanation that you can by reference to a sketch of the stress-strain diagram.

(I.C.E., Oct., 1904.)

4. In designing girder-work of mild open-hearth steel what working stress would generally be considered safe and suitable for the two following cases:—(i.) A wholly dead load producing stress a ? (ii.) a wholly live load (without shock) causing sudden alternations of stress from zero to b —the stress b being statically reckoned?

(I.C.E., Oct., 1904.)

5. Explain the reasons that would influence your own mind in treating these two cases differently, and give some practical rule for determining the safe working stress in intermediate cases, where the stress alternates between a and $a+b$.

(I.C.E., Oct., 1904.)

6. Two bars of steel 12 inches wide by 1 inch thick are required to be joined together to resist tension. Sketch (a) a theoretical joint giving the

maximum efficiency, and (b) a practical joint with a high efficiency, and indicate where the joint would fall. (I.C.E., Oct., 1905.)

7. Two bars of steel 6 inches by 3 inches are joined together by a jaw on the end of one bar and an eye on the other, with a pin passing through the two, to withstand a tensile stress of 120 tons. Assuming the width of the eye to be 3 inches and the sides of the jaw 2 inches each, determine the diameter of the pin so that the unit stress of the extreme fibre shall not exceed $6\frac{1}{2}$ tons per square inch. (I.C.E., Oct., 1905.)

8. What is meant by the "fatigue" of metals? Describe briefly the results of Wöhler's experiments on this subject. (I.C.E., Oct., 1906.)

9. Describe the effect of a compressive load upon cubes of any two of the following materials:—Cast iron, cast steel, timber in the direction of the fibre, sandstone, and concrete; give, if possible, the unit crushing loads. (I.C.E., Oct., 1906.)

10. Describe briefly the effects on the strength and elasticity of the various forms of iron and steel when treated by the following processes:—Annealing, tempering, case-hardening, and chilled casting. (I.C.E., Feb., 1907.)

11. How is the strength, structure, and elasticity of a rod of steel affected when drawn into a fine wire? (I.C.E., Feb., 1907.)

12. When the stresses due to dead load and the maximum tensile and compressive stresses due to live load have been determined for each member of a structure, explain how the cross-section area of the various ties and struts may be determined. (I.C.E., Feb., 1907.)

13. A suspension bridge is composed of two ribs having a parabola for their centre lines; the ribs are connected together by a hinge at the centre and are hinged at the supports; show how to find the tensions in the flanges of the ribs due to a uniform load on the bridge. (I.C.E., Feb., 1907.)

14. Give the conclusions arrived at from the results of Wöhler's experiments regarding the effect of repeated application of loads on the strength of a bar, and explain how they can be utilised in structural and machine design. (I.C.E., Oct., 1907.)

15. A vertical steel suspension-rod of a bridge is 30 feet long and 1 square inch in sectional area; it carries 3,500 lbs. of the roadway, and 3,000 lbs. of the live load. Determine the extension of the rod. $E = 13,000$ tons per square inch. (I.C.E., Feb., 1908.)

LECTURE VII.—I.C.E. QUESTIONS.

I. Sketch neatly and describe an extensometer suitable for the determination of the Young's modulus of the material of a specimen 8 inches or 10 inches long, noting any prominent features in the design and the reason for their adoption. Define "elastic limit" and "yield point." (I.C.E., Oct., 1903.)

2. A square bar of isotropic material, stressed within the elastic limit, is loaded so that the stress intensity over each section is (1) uniform, (2) proportional to the distance from a line drawn through the centre parallel to one side, tension on one side and compression on the other side of the line. Calculate, in terms of the maximum stress and the Young's modulus, the energy stored per unit volume in both cases. (I.C.E., Oct., 1903.)

3. Exhibit, on a diagram drawn approximately to scale, the relation between the stress applied and the resultant strain in mild steel (suitable for bridge work) and annealed copper bars about 10 diameters long. (I.C.E., Oct., 1903.)

4. In a tensile test of a wrought-iron bar, 0.75 inch in diameter and 10 inches between the gauge points, the following results were obtained:—

Stress in lbs. per square inch, }	8,000	16,000	24,000	32,000	38,000	40,000	44,000
Extension per cent.,	0.027	0.056	0.081	0.113	0.135	2.7	4.2
Stress in lbs. per square inch, }	48,000	52,000	55,000				
Extension per cent.,	6.5	10.7	23.7				

The yield point is 38,000 and the specimen broke at the maximum stress, 55,000. Plot the stress-strain diagram. Calculate the Young's modulus and find approximately the work done per cubic inch up to fracture.

(I.C.E., Feb., 1904.)

5. What is meant by the "steelyard drop," or the "yield point"? At what load would you expect to find it in test-pieces of—(a) wrought iron, (b) 27-ton steel?

(I.C.E., Feb., 1905.)

6. Describe briefly the observed effects of a direct compressive load applied to cubes or short cylinders of the following materials:—(a) Sandstone, (b) mild steel, (c) cast iron, (d) timber placed with the grain on end.

(I.C.E., Feb., 1905.)

7. Sketch the form of the stress-strain diagram, showing the plastic elongation of tough mild steel as commonly observed under direct tension.

(I.C.E., Feb., 1905.)

8. Describe and illustrate some of the methods and appliances for the accurate measurement of elastic extensions, and show how Young's modulus is determined.

(I.C.E., Feb., 1905.)

9. How is the strength of the material affected when rivet-holes are punched in thick or thin plates of mild steel, or in the foot of a Vignoles rail of harder steel?

(I.C.E., Feb., 1905.)

10. Sketch approximately to scale stress-strain curves as drawn by an automatic recording apparatus for test bars of wrought iron, mild steel, and soft rolled brass. State what you would expect the load per square inch to be at the limit of elasticity and at fracture; also the maximum stress for each of the bars.

(I.C.E., Oct., 1905.)

11. Describe any instrument with which you are acquainted for measuring the elastic tension in a test-bar.

(I.C.E., Oct., 1905.)

12. Describe some form of "drop weight" or "impact" testing machine.

(I.C.E., Oct., 1905.)

13. How would you prepare the following materials for compression tests:—A block of sandstone about 3 inches cube, a brick, a piece of cast iron 10 inches long and 1 inch square in section?

(I.C.E., Oct., 1905.)

14. Sketch an apparatus suitable for taking the necessary measurements for obtaining the modulus of elasticity of steel, and briefly describe the same.

(I.C.E., Oct., 1906.)

15. When testing a piece of steel in an ordinary testing machine, explain clearly what happens to the test-piece up to the moment of fracture, and draw a stress-strain diagram.

(I.C.E., Oct., 1906.)

16. Define "stress" and "strain." The following data were obtained from a tensile test on a mild steel bar:—Original dimensions: length, 8 inches; area of cross-section, 0.62 square inch. Final dimensions: length, 10.2 inches; area of cross-section, 0.34 square inch. Load at elastic limit, 13 tons; maximum load, 19.6 tons. Mean extension per ton of load up to elastic limit, 0.001 inch. Determine the intensity of stress at elastic limit; the intensity of the breaking stress; the contraction of area per cent.; and the modulus of elasticity.

(I.C.E., Oct., 1907.)

17. Describe a testing machine for making a tensile test to destruction of a piece of mild steel. Sketch the stress-strain diagram you would expect to get, and mark on it the chief points to be noted, giving approximate values. What measurements are made of the test-piece itself, and explain with what objects they are taken? (I.C.E., Oct., 1907.)

18. State the characteristics of the "plastic state" of a material. Describe the observed effects of direct compressive loading applied to cubes or short cylinders of (a) mild steel, (b) cast iron, (c) timber placed with the grain on end. (I.C.E., Feb., 1908.)

19. Draw roughly to scale the load-extension diagram of the tensile test of a steel bar 1 inch in diameter, having a maximum strength of 32 tons per square inch; indicate the points in the diagram where changes in the structure occur. (I.C.E., Oct., 1908.)

20. State briefly what effects the following conditions have upon the testing of steel:—(a) The length of the test-piece, (b) the sectional area of the test-piece, (c) the shape of the test-piece, (d) the time occupied in making the test. (I.C.E., Oct., 1908.)

LECTURE VIII.—I.C.E. QUESTIONS.

1. A vertical mild steel tube of 6 inches external diameter, $\frac{1}{2}$ inch thick, is securely bedded in the ground. Its height above ground is 10 feet, and it is subjected at the upper end to a horizontal pull of 1,500 lbs. Calculate the maximum stress at the ground section and the deflection at the top. Take E as 30,000,000 lbs. per square inch. (I.C.E., Oct., 1903.)

2. Criticise briefly Euler's, Rankine's, and Gordon's formulae for struts. Which do you prefer to use, and for what reasons? (I.C.E., Feb., 1904.)

3. A number of connected loads roll across a bridge. Explain carefully how you would find the maximum bending moment at any section. (I.C.E., Feb., 1904.)

4. A masonry column is circular in section. Show that, if the line of pressure cuts the section at a point which is distant more than $\frac{1}{3}$ of the diameter from the centre, there will be tensile stress over a portion of the section. (I.C.E., Feb., 1904.)

5. A rolled T bar, 4 inches $\times$ 4 inches $\times$ $\frac{1}{2}$ inch, is to be used as a strut. Find the moment of inertia I , and the radius of gyration k of the section about a neutral axis parallel to the top-table, and about another at right angles to the first. (I.C.E., Oct., 1904.)

6. A slender rod of steel, 5 feet long, and either square or cylindrical in section is tested for elastic deflection as a beam supported at the extreme ends. It bends exactly 1 inch under a central load of 20 lbs. Calculate the breaking weight of the same rod when set upright and used as a column with rounded ends (adopting Euler's formula). (I.C.E., Oct., 1904.)

7. In what cases may Euler's formula be rationally and safely used for calculating the breaking weight of a column? In what other cases would the formula be unreliable, and what kind of error would it introduce? (I.C.E., Oct., 1904.)

8. In the case of solid cylindrical columns, what would be your rough estimate of the relative strengths of cast and of wrought iron—(a) when the length of the column is equal to 5 diameters, and (b) when it is 50 diameters? (I.C.E., Feb., 1905.)

9. Find the sectional area A , the moment of inertia I (about the diametral axis), and the radius of gyration R , for a hollow cylindrical column whose external diameter is 8 inches, and its internal diameter 6 inches. (I.C.E., Feb., 1905)

10. A hollow circular column has a projecting bracket on which a load of 1 ton rests. The centre of this load is 2 feet from the centre of the column. External diameter of column is 10 inches and the thickness 1 inch. What is the maximum compression stress? (I.C.E., Oct., 1905.)

11. A solid cast-iron column, 6 inches in diameter and 10 feet high, is fixed firmly at each end. What load will it carry when the unit stress is not to exceed 8 tons per square inch? (I.C.E., Oct., 1905.)

12. A circular mild-steel tie bar, 3 inches in diameter and 30 feet long, is overloaded until it is fractured. Take 26 tons per square inch as the breaking load, and 10 per cent. as the extension. Limit of elasticity 15 tons per square inch. Calculate by an approximate rule the work done in fracturing the bar. (I.C.E., Feb., 1906.)

13. The strut in a framed structure is formed of a steel pipe 6 inches external diameter and $\frac{1}{2}$ inch thick; it is 10 feet long and has pin connections at each end. With a factor of safety of 5, to what load may it be submitted? (I.C.E., Feb., 1906.)

14. In which way would a steel column 30 feet long by 9 inches external and 8 inches internal diameter fail, assuming a load to be applied in the direction of the longitudinal neutral axis? Assuming the ends to be hinged, what load would this column safely carry with a factor of safety of 5, the ultimate strength of the material being 30 tons per square inch, and the modulus of elasticity 30,000,000 lbs. per square inch? (I.C.E., Oct., 1906.)

15. A column 20 feet high, fixed in direction at the ends, has to carry a load of 25 tons, and consists of an I section beam, 12 inches deep and 8 inches wide. Find the necessary thickness for the flanges, neglecting the effect of the web and taking a factor of safety of 3. Given that the breaking load = $\frac{21S}{1 + \frac{L^2}{30,000 k^2}}$, where S = the cross-sectional area, L = the length, and k = the radius of gyration. (I.C.E., Oct., 1906.)

16. A cast-iron strut, 2 feet long, with a sectional area of 8 square inches carries a load of 32 tons. Assuming E = 8,000 tons per square inch, how much is the strut shortened? What is the stress and what is the strain? (I.C.E., Feb., 1907.)

17. A building 20 feet high to the eaves and 50 feet wide is to be constructed of steel columns placed 15 feet apart, with the sides formed of steel framing and plaster slabs. The roof has a rise of 12 feet above the eaves, and its weight is 20 lbs. per square foot. With a wind pressure of 40 lbs. per square foot on the sides and of 2.4 lbs. per square foot normal to the roof, calculate the stresses in the column, and sketch the sections to be adopted; the maximum stresses not to exceed 5 tons per square inch. Assume the base to be rigidly fixed, and the roof truss to be simply carrying its own weight and not assisting the column. (I.C.E., Oct., 1907.)

18. A circular steel chimney 5 feet in diameter outside, is to be built of steel plates $\frac{3}{8}$ inch thick; assuming the diameter of the chimney and thickness of plates to be constant, and the base rigidly fixed to the foundation, how high can the chimney be built to withstand a wind pressure of 40 lbs. per square foot equally distributed over the whole surface, and not to exceed a unit-stress of 6 tons per square inch? (I.C.E., Oct., 1907.)

19. Find by Rankine's formula,

$$P = \frac{fA}{1 + \alpha \frac{l^2}{k^2}}$$

the working load for a mild steel column, 24 feet long, firmly fixed at the ends. The column is of $\mathbf{I}$ cross-section, and consists of two equal flanges 8 inches wide by 1 inch thick. Web, $\frac{1}{2}$ inch thick. Distance between flanges, 10 inches. The constant " α " in formula = $\frac{1}{250000}$; $f = 21$ tons per square inch; $k =$ radius of gyration of section with respect to axis about which I is least. Assume a factor of safety 5. (I.C.E., Feb., 1908.)

20. Assuming the safe unit load for cast iron in direct compression to be 5 tons per square inch, what would be the safe unit load on a cast-iron column 20 feet long, 12 inches external diameter, with metal 1 inch thick, the column being fixed at the ends? Adopt the formula $f = 5 - \frac{1}{60} \frac{l}{r}$; where f is the safe unit load in tons, l is the unit length, and r is the least radius of gyration. (I.C.E., Feb., 1908.)

21. If a solid column, 10 feet long and 4 inches in diameter, is eccentrically loaded, what load, acting vertically at a distance of 1 foot from the centre of the column, would produce an extreme fibre stress of 3 tons per square inch? (I.C.E., Feb., 1908.)

22. A steel bar 1 inch wide and $\frac{1}{4}$ of an inch thick is bent in the form of a semicircle, so that the unit stress is 7 tons per square inch; what is the tension in a wire holding the two ends in position? Assume the modulus of elasticity to be 13,000 tons per square inch. (I.C.E., Oct., 1908.)

23. The revolving portion of a crane, weighing 30 tons, is carried upon a steel centre post 15 inches in diameter. The centre of gravity of this revolving portion is 3 feet 4 inches from the centre of the post. The load to be lifted by the crane is 10 tons at 20 feet radius, on the opposite side of the post from the centre of gravity of the revolving part of the crane. What is the maximum bending moment on the crane post, and what are the maximum and minimum stresses induced in it? Note that the entire revolving weight both of crane and load is carried by the post. (I.C.E., Oct., 1908.)

24. If a column has rounded ends, how is its strength measured according to Euler's Theory, and what conditions are necessary for this theory to be valid? (I.C.E., Oct., 1908.)

25. A column imposes a load of 133 tons uniformly on the top of its foundation. The column base is 3 feet 9 inches square. The foundation has bottom dimensions of 8 $\frac{1}{2}$ feet square, at a depth of 2 $\frac{1}{2}$ feet below the column base plate. Make the necessary calculations and sketch a grillage foundation of two layers of rolled steel joists embedded in concrete. The concrete is not to be taken as contributing to the strength. It will be sufficient to state the moduli of joist sections required, allowing 7 tons per square inch maximum stress. (I.C.E., Oct., 1908.)

APPENDIX B.—(Continued.)

ANSWERS TO QUESTIONS AT END OF LECTURES.

LECTURE I.—ORDINARY.

5. Resilience = 32.4 foot-lbs.
6. Resilience = 391 inch-lbs.
7. Internal diameter of covering hoop = 11.99 inches. Internal diameter of third hoop = 17.99 inches.
10. See p. 7.
12. Working press. = 326.6 lbs. per sq. in. and 752.68 ft. head of water.
13. See pp. 10, &c., for full answer.
14. 140 tons per square inch.
19. Stress in metal of tube due to its load = 14,602 lbs. per square inch; pressure of compressed air = 87.2 lbs. per square inch. Tensile stress at bottom of tube = 29,204 lbs. per square inch.

LECTURE I.—C. & G.

- | | |
|---------------------------|---------------------|
| 1. 4 inches diameter. | 2. Load = 8.5 tons. |
| 3. Extension = .114 inch. | 4. 9 studs. |

LECTURE I.—INST. C. E.

1. See pp. 16-19 for complete solution.
2. Stress = 4,991 lbs. per square inch; $E = 10,695$ tons per square inch.
3. Length of rod = 322.6 feet.
5. 569 tons; 500 tons; 283.2 tons.
8. Extension = .18 inch.
9. 5,832 lbs. per square inch; $E = 16.9$ tons per square inch.
10. $E = 27,428,571$ lbs. per square inch.
15. Elongation = .75 inch.
16. Tensile stress at centre = 5,184 lbs. per square inch.
18. (a) 28,143,000 lbs. per square inch. (b) 34,000 lbs. per square inch.
- (c) 38.6 per cent. (d) 6,725 inch-lbs.

19. Length = 102.6 feet. Area = 9.5 square inches. Weight = 1.47 tons. Maximum pull = 38 tons.

21. Tension at middle of chain = 48.83 tons. Tension at ends of chain = 79.3 tons.

	Members in Tension.	Stress in Cwts.	Members in Compression.	Stress in Cwts.
22.	ED	16.0	AE	25.5
	FG	20.5	BE	25.5
	GD	4.5	CH	17.5
	GH	2.0	CI	17.5
	ID	5.5	EF	18.0
			HI	0.0

24. 13.7 by Gordon's formula.

25. Tension stress = 5.6 tons per sq. inch. Elongation = .124 of an inch.

26. Load = 3,142 lbs.

27. 180° F.

LECTURE II.—ORDINARY.

7. (1) Maximum BM = 1,000 lb.-feet, SF = 200 lbs. (2) Maximum BM = 1,000 lb.-feet, SF = 400 lbs.

8. BM at 10 feet = $133\frac{1}{3}$ ton-feet; BM at 15 feet = 150 ton-feet.

9. SF at 25 feet = $3\frac{1}{2}$ tons; SF at 50 feet = $12\frac{1}{2}$ tons.

11. D = $10\frac{1}{2}$ inches.

13. Breadth = $3\frac{1}{2}$ inches, depth = 6 inches, web = $\frac{1}{4}$ inch, flanges = $\frac{3}{8}$ inch thick.

15. BM at C = $356\frac{2}{3}$ ton-inches; BM at D = $2,370\frac{2}{3}$ ton-inches; BM at E = $683\frac{1}{3}$ ton-inches; SF at C = $14\frac{1}{2}$ tons; SF between C and D = $4\frac{1}{2}$ tons; SF between E and B = $8\frac{3}{4}$ tons; SF between E and D = $1\frac{3}{4}$ tons; total at D = 6 tons.

16. BM under 3 tons = 780,444 lb.-inches; BM under centre of girder = 1,122,660 lb.-inches; BM under $7\frac{1}{2}$ tons = 1,212,480 lb.-inches.

17. BM at centre of span = 9.25 ton-feet; BM under concentrated load = 8.8 ton-feet.

LECTURE II.—INST.C.E.

2. 5.25 tons on each span.

4. SF = 112.8 tons.

7. SF at 10 feet from end = $1\frac{1}{2}$ cwts.

12. Maximum BM at centre = 34 ton-feet; maximum SF = 3.4 tons.

13. Stress on the material = 2.2 tons per square inch.

15. Distribution of pressure on base = .96 to 1.63 tons per square inch.

16. R = 5,973 $\frac{1}{2}$ lbs.; BM = 1,400 lb.-feet.

18. BM at centre = 9 ton-feet; BM at each support = 12 ton-feet.

19. L inch-cwt, or 112 L inch-lbs.

20. B M at A = B M at C = $W L + \frac{1}{2} w l^2 = 3,000$ ton-feet;
 B M at B = $3,000 - \frac{w l^2}{8} = -2,400$ ton-feet.
22. 80 feet.
24. Shearing force at a section 30 feet from either support = ± 25 tons.
25. Maximum B M = 50,625 ton-feet.
26. B M at centre = 2,700 ton-feet; B M at 30 and 90 feet = 2,025 ton-feet. S F at centre = $22\frac{1}{2}$ tons; S F at 30 and 90 feet = $50\frac{1}{2}$ tons.
27. Reaction at ends = 33,750 lbs. Reaction at centre = 112,500 lbs. B M at centre = -506,250 lb.-feet. Maximum intermediate B M = 284,766 lb.-feet.
28. Upward force = $\frac{5}{8} W$.
29. S F = 208 $\frac{1}{2}$ lbs. B M = 780.4 lb.-feet.
30. Maximum B M at 10 feet from end = 3,666 $\frac{2}{3}$ lb.-feet; maximum S F at the same point = $\pm 533\frac{1}{3}$ lbs.
31. Maximum B M = 302.5 ton-feet; maximum S F = 22.5 tons.
33. Ratio of loads, 1 : 5. 34. Reactions = $37\frac{1}{2}$ and $72\frac{1}{2}$ tons.
35. Planes of principal stresses are at $19^\circ 20'$ and $70^\circ 40'$ to the vertical. Intensities of stress 5.7 and -0.7 tons per square inch.
36. Approximately the same weight.

LECTURE III.—ORDINARY.

5. Stress at outer fibres = 14,000 lbs. per square inch.
6. Stress for circular section = 5 tons per square inch; for rectangular section = 5 tons per square inch.
12. $20\frac{1}{2}$ inches depth; area of top flange 7.5 square inches; area of bottom flange = $18\frac{3}{4}$ square inches.
13. $W = 18\frac{1}{2}$ tons, or $w = 1.54$ tons per foot run.
18. $W = 45,702$ lbs., or 1305.8 lbs. per foot run.
19. Ratio of R M, 4.56 : 1.
20. B M = 284.6 lb.-inches, and $f_c = 4,171$ lbs. per square inch.
22. B M = 17.5 ton-feet. Points of inflexion 4 feet from each end.
26. $W = 5,625$ lbs. $\delta = .453$ inch.
29. Max. B M = $32\frac{3}{4}$ ton-feet; max. S F = $8\frac{1}{2}$ tons. f_t and $f_c = 44,400$ lbs. per square inch. Max. $f_s = 5,460$ lbs. per square inch.
30. Deflection = .032 inch. 32. $W = 3.42$ tons.
34. $E = 14,199,691$ lbs. per square inch; $f = 151,200$ lbs. per square inch.

LECTURE III.—INST.C.E.

3. 104 $\frac{1}{2}$ tons; 108.8 tons; 147.2 tons; 542 tons.
6. Max. stress = 7.5 tons per square inch; $\Delta = .21$ inch.
10. Slope 3° due to its own weight, and $2^\circ 52'$ due to central load.
11. $W L^3 \div 12 E I$.
12. Neutral axis = .69 inch from base; R M = 4.68 ton-inches.
13. 410.5 tons; 540 tons; 649 tons.
14. Resultant force = 390.7 tons at $66^\circ 48'$.
17. Max. B M = 500 ton-feet under trailing axle of locomotive.
18. Max. S F at right abutment = 47.4 tons, and max. S F at centre = 17.6 tons.

20. Ratio of deflection of girders, 3 : 2.
21. Relative deflections of girders, 6 : 5.
22. $WL^3 \div 4EI$.
23. (a) 0; (b) 2.4 : 1.
24. $\frac{5}{8}$ total load at centre and $\frac{3}{8}$ at ends.
25. Approx. $9\frac{1}{2}$ feet from the bottom.
26. $f = 21\frac{1}{2}$ tons per square inch.
32. Ratio of strength of beams, 1 : 1.44.
33. Compressive stress = 18,432 cwt.
36. $I = 1,514$ units.
37. Max. stress is 4.28 tons per square inch.
41. $\Delta = 3.75$ inches.
42. $\Delta = 2.25$ inches.
43. B M at B and C = 432 lb.-ft.; B M at centre of span B C = -108 lb.-ft.
44. $E = 5,832$ tons per square inch.
45. Max. tensile stress = 9.6 tons per square inch.
46. $\Delta = .59$ inch.
47. $\Delta = .72$ inch.
48. $\Delta w_1 : \Delta c :: 8 : 13$.
49. Tension on the string = 4 lbs.
50. Breaking load = 4,741 lbs.
51. Max. stress = 4.42 tons per square inch.
52. (a) $\frac{3}{8}$ inch; (b) 5 inches.
54. (a) Max. deflection = $\frac{1}{384} \frac{WL^3}{EI} = .1812$; (b) Points of contrary flexure = .289 L on either side of centre.
55. Distributed load = 100 lbs. per square foot.
56. $E = 15,950,000$ lbs. per sq. inch; max. stress = 17.2 tons per sq. inch.
57. Supporting forces $5\frac{1}{2}$ and $4\frac{1}{2}$ tons; max. B M under the 6-ton load = $9\frac{3}{8}$ ton-feet.
58. Let the safe flange stress be 6 tons per square inch, then it is possible to use a plate 12 inches $\times$ $\frac{5}{8}$ inch and two 4-inch $\times$ 4-inch $\times$ $\frac{1}{4}$ -inch angle-irons, riveted together with $\frac{7}{8}$ -inch rivets having a pitch of 4 inches.
59. Assuming a dip of 30 feet, then the horizontal component of the pull in the wire ropes = 60 tons.
60. $\Delta = .077$ inch.
64. Weight = .8 ton; $f_c = 1.93$ tons per square inch.
65. Rolled steel joist, 6 inches $\times$ 3 inches $\times$ $\frac{3}{8}$ inch.
66. Work done = 705.6 inch-lbs., assuming $E = 2 \times 10^6$ lbs. per sq. inch.

LECTURE IV.—ORDINARY.

2. T R of hollow shaft : T R of solid shaft : : 1.26 : 1.
3. Ultimate strength of wrought-iron shaft = 806 $\frac{3}{4}$ lbs.; cast-iron shaft = 476 $\frac{3}{4}$ lbs.
7. Diameter = $68.5 \sqrt[3]{\frac{HP}{Nf_1}}$. Where N = revolutions per minute, f_1 = the intensity of stress where it is greatest—i.e., at the outside of the shaft.
8. H.P. = 82 $\frac{3}{4}$.
11. Greatest stress F = 4,184 lbs. per square inch (compression).
12. D = 13.75 inches, d = 8.5 inches.
13. Twisting moment = 1,413 lb.-feet.
15. T M = 740 lb.-inches; $f = 3,240$ lbs. per square inch.
16. Torque = 22,800 lb.-inches; H.P. = 54.2.
17. Diameter of shaft = 3.25 inches; H.P. = 123.
18. Strengths 1 : 2.1 and costs 1 : 1.65.
20. Diameter, $\frac{3}{4}$ to 1 inch.
21. D = 13.5 inches.
22. H.P. = 2,827.7; $\theta = 15.3^\circ$.
23. Shearing stress $f_s = 3,258$ lbs. per square inch;
Tensile stress $f_t = 4,887$ lbs. per square inch.

LECTURE IV.—INST.C.E.

2. $1.55 : 1$; $1.076 : 1$.
 9. $D = 4$ inches.
 4. 10.45 H.P.
 10. $f = 9,000$ lbs. per square inch ; $D = 2.3$ inches.
 11. 810 lbs.
 12. $\theta = 32^{\circ} 42'$ if $C = 12 \times 10^6$ lbs. per square inch.
 13. $.78$ inch or $\frac{3}{4}$ -inch bolts.
 14. Thrust = 4.5 tons.
 15. Strain = $.001636$.
 16. $TM = .622$ ton-inch.
 17. Tensile stress = 2.5 tons per square inch.
-

ANSWERS TO QUESTIONS IN APPENDIX B.

LECTURE I.—INST.C.E.

1. $f_n = f \sin^2 \theta$; $f_t = \frac{1}{2} f \sin 2\theta$, where θ is the angle which the plan makes with the axis. When $\theta = 20^\circ$; $f_n = 1.43$ tons per square inch, and $f_t = 2.48$ tons per square inch.
2. Least radius = 34.7 feet; and moment of resistance of a section = 1.472 ton-inches.
4. 1 inch diameter.
5. Stresses in the steel rails are (i.) 8,000 lbs. per square inch; (ii.) 12,000 lbs. per square inch. Temperature at which the induced stress reaches the elastic limit = 260° F.
6. Sag at centre of span (i.) 15.6 inches; (ii.) 3 inches.

LECTURE II.—INST.C.E.

4. Bending moments at built-in end = 56½ ton-feet; shearing force at built-in end = 9½ tons; shearing force at end support = 5½ tons; point of contrary flexure = 7½ feet from wall.
5. $W = \frac{2}{7}$ tons.

LECTURE III.—INST.C.E.

1. (b) Modulus of section = 500 (inch-units)³.
2. Shearing stress between the plates = .336 ton per square inch; shearing stress between the inner plate and the joint close to the supports = 1.046 tons per square inch. Suitable diameter and pitch for the rivets = 1 and 12 inches respectively.
3. Least radius of gyration for the joist, $k = 2\frac{3}{4}$ inches. Working load on column = 225 tons.
11. Distributed load = 422.6 tons per square foot.
12. Intensity of shear stress at neutral axis = 8.44 tons per square inch.
17. Tangent of angle of slope = .03.
18. Deflection = $2\frac{4}{15}$ inches.
19. Deflection = 1.08 inches.

LECTURE IV.—INST.C.E.

1. Diameter of solid shaft = 13·86 inches. Relative weights of the two shafts, '304 : 1.
2. Diameter of mild steel shaft = $4\frac{1}{2}$ inches.
3. Maximum stress = 2,378 lbs. per square inch. Twist in degrees of 100 feet length of shafting = $18^{\circ} 5'$.

LECTURE VII.—INST.C.E.

2. Energy stored per unit volume, (i.) $\frac{f^2}{2E}$, and (ii.) $\frac{f^2}{6E}$.

LECTURE VIII.—INST.C.E.

1. Maximum stress at the ground section = 7·3 tons per square inch; deflection at top of tube = 0·875 inch.
 5. Moment of inertia of section about its centre of gravity = 5·56 (inch-units)⁴; moment of inertia for the axis at right angles to the former, $I = 2·7$ (inch-units)⁴.
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APPENDIX C.

THE INSTITUTION OF CIVIL ENGINEERS' EXAMINATION, FEBRUARY, 1909.

ELECTION OF ASSOCIATE MEMBERS.

STRENGTH AND ELASTICITY OF MATERIALS.

Not more than EIGHT questions to be attempted by any Candidate.

1. State concisely the meaning of the following terms:—Elastic limit, yield point, modulus of elasticity, resilience.

2. A rolled steel joist 12 inches deep has a moment of inertia about the neutral axis parallel to the flanges of 375·6; what is its “section modulus,” and what distributed load could it carry when supported upon bearings 10 feet apart if the stress in the extreme fibre did not exceed 8 tons per square inch?

3. A hard steel rod $\frac{3}{4}$ inch diameter and 20 inches long was shortened by 0·007 inch by a load of 2 tons; what was the stress, the strain, and the modulus of elasticity?

4. Derive from first principles the formula $f = \frac{My}{I}$ when f = the unit stress within the elastic limit, M = the bending moment, y = the distance of the extreme fibre from neutral axis, and I = moment of inertia. State any assumption you make.

5. A ball of steel falls 38·82 feet upon a horizontal slab of the same material. Assuming the coefficient of restitution (or resilience) to be 0·7, how high will the ball rise after impact? What is the ratio of kinetic energy just before and just after impact?

6. A steel bar 1 inch square is placed upon bearings 30 inches apart; what load placed in the centre of the bar will deflect it 0·25 inch? Assume the modulus of elasticity to be 30,000,000 lbs. per square inch.

7. Sketch the appearance of the fracture of the following materials when crushed to destruction:—(a) A cube of cast iron, (b) a cube of wrought iron, (c) a cube of sandstone, (d) a piece of pine 2 inches long by 1 inch square placed on end.

8. A steel shaft has to transmit 100 horse-power when running at 300 revolutions per minute; what must be its diameter if the stress in the extreme fibre does not exceed 6 tons per square inch?

9. It is frequently stated that the line of resistance in a masonry structure must not pass outside the “middle third”: (a) To what form of section does this apply? (b) What are the assumptions upon which the

statement is based? (c) What is the corresponding limit of deviation in a structure of circular section? Give the proof.

10. Fill in the spaces left in the following table, indicating the properties of the various materials :—

Material.	Breaking Load.		Extension in 8 Inches.	Modulus of Elasticity.	Limits of Elasticity.
	Tensile.	Compressive.			
Steel bars, . . .					
Wrought-iron bars,					
Cast iron, . . .					
Brass, ...					
Copper, ...					
Concrete, ...					
Portland cement, .					
Pitch pine, . . .					

11. Sketch a machine for making tensile tests of steel up to 100 tons ; mark the leading dimensions.

12. State the advantages and the disadvantages of the use of reinforced concrete for structural purposes, and explain the duty of each material in the compound material.

City and Guilds of London Institute.

DEPARTMENT OF TECHNOLOGY.

TECHNOLOGICAL EXAMINATIONS, 1909.

FOR GENERAL INSTRUCTIONS, SEE APPENDIX A.

46.—MECHANICAL ENGINEERING.

HONOURS GRADE (Written Examination).

The Candidate is not expected to answer more than *eight* of the following questions, which must be selected from *two* sections *only*.

Section B is given in the Author's *Text-Book on Steam and Steam Engines, including Turbines and Boilers*, and Section C is given in Vol. IV., "Hydraulics."

SECTION A.

[STRENGTH OF MATERIALS AND THEORY OF STRUCTURES.]

1. Give a brief account of the effects of stress beyond the elastic limit upon the structure of wrought iron and steel as shown by the microscope. Explain why it is that a specimen will ultimately fracture under the effects of repeated stresses, slightly exceeding the elastic limit of the material.

(C. & G., 1909, H., Sec. A.)

2. Show how to determine the maximum and minimum stresses on a section subjected to a direct stress and a bending moment. In a punching machine the punch has an overhang of 30 inches, and the section of the frame at the level of the punch is a box casting 20 inches deep and 10 inches wide, with a thickness of metal of 1 inch. Determine the maximum and minimum stresses on the section for a maximum pressure at the punch of 10 tons.

(C. & G., 1909, H., Sec. A.)

3. Explain why it is that a small amount of steel reinforcement, suitably applied, enormously increases the strength of a concrete beam. A reinforced concrete beam, having a breadth of 8 inches and a depth of 20 inches, is reinforced by three round steel rods 1 inch in diameter, set in the concrete at a distance of $2\frac{1}{2}$ inches from the lower side. Taking the working stress of the concrete in compression as 500 lbs. per square inch, and neglecting its tensile resistance, determine approximately the moment

of resistance of the section. The modulus of the concrete is 2×10^6 and the modulus for steel is 3×10^7 , both expressed in inches and lbs.

(C. & G., 1909, H., Sec. A.)

4. Determine the deflection of a beam of uniform cross-section, and of length l , when supported at both ends and loaded at the centre by a weight w . Also determine the deflection if this load is distributed uniformly along the beam. From your results prove that the load on a prop at the centre of a uniformly-loaded beam is five-eighths of the whole load when the central section of the beam is at the same level as the ends.

(C. & G., 1909, H., Sec. A.)

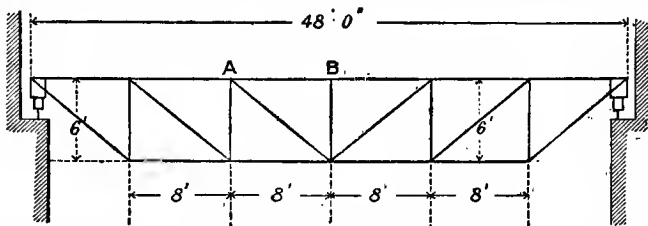
5. Show that when a shaft of length l and diameter d is twisted by a couple T , the angle of twist θ is expressed by the formula—

$$\theta = \frac{32 T l}{\pi d^4 c},$$

where c is the modulus of rigidity of the material. A shaft 4 inches in diameter and turning at 400 revolutions per minute is found to twist through an angle of 1° in a length of 10 feet. Determine the horse-power transmitted if the value of c is 12×10^6 in lbs. and inch-units.

(C. & G., 1909, H., Sec. A.)

6. An overhead crane, to lift 40 tons, consists of two equal girders, one of which is shown in outline by the sketch. Determine the stresses in all



the members of this girder, if the loading is taken to consist of two equal loads of 10 tons applied at the points A and B respectively.

(C. & G., 1909, H., Sec. A.)

7. Show that the hoop stress f at any radius r of a thick cylinder having an internal radius r_1 and an external radius of r_2 is expressed by the formula—

$$f = \frac{p_1 r_1^2}{r_2^2 - r_1^2} \left(\frac{r_2^2}{r^2} + 1 \right),$$

where p_1 is the internal pressure. Draw a curve showing the hoop stress at any radius for a cylinder having an internal diameter of 8 inches and an external diameter of 12 inches when the internal pressure is 3,000 lbs. per square inch.

(C. & G., 1909, H., Sec. A.)

APPENDIX D.

EXAMINATION TABLES.

USEFUL CONSTANTS.

1 Inch = 25·4 millimetres.

1 Gallon = ·1605 cubic foot = 10 lbs. of water at 62° F. ∴ 1 lb. = ·01605 cubic foot.

1 Knot = 6080 feet per hour. 1 Naut = 6080 feet.

Weight of 1 lb. in London = 445,000 dynes.

One pound avoirdupois = 7000 grains = 453·6 grammes.

1 Cubic foot of water weighs 62·3 lbs.

1 Cubic foot of air at 0° C. and 1 atmosphere, weighs ·0807 lb.

1 Cubic foot of Hydrogen at 0° C. and 1 atmosphere, weighs ·00557 lb.

1 Foot-pound = $1·3562 \times 10^7$ ergs.

1 Horse-power-hour = 33000 × 60 foot-pounds.

1 Electrical unit = 1000 watt-hours.

Joule's Equivalent to suit Regnault's H, is $\begin{cases} 774 \text{ ft.-lbs.} = 1 \text{ Fah. unit.} \\ 1393 \text{ ft.-lbs.} = 1 \text{ Cent. "} \end{cases}$

1 Horse-power = 33000 foot-pounds per minute = 746 watts.

Volts × amperes = watts.

1 Atmosphere = 14·7 lb. per square inch = 2116 lbs. per square foot = 760 m.m. of mercury = 10^6 dynes per sq. cm. nearly.

A Column of water 2·3 feet high corresponds to a pressure of 1 lb. per square inch.

Absolute temp., $t = \theta^\circ \text{C.} + 273^\circ\text{7.}$

Regnault's H = 606·5 + ·305 $\theta^\circ \text{C.} = 1082 + ·305 \theta^\circ \text{F.}$

$p u^{1·0846} = 479$

$\log_{10} p = 6·1007 - \frac{B}{t} - \frac{C}{t^2}$

where $\log_{10} B = 3·1812$, $\log_{10} C = 5·0871$,

p is in pounds per square inch, t is absolute temperature Centigrade,

u is the volume in cubic feet per pound of steam.

$" = 3·1416 = \frac{22}{7} = \frac{355}{113} = 10(\sqrt{3} - \sqrt{2}).$

One radian = 57·3 degrees.

To convert common into Napierian logarithms, multiply by 2·3026.

The base of the Napierian logarithm is $e = 2·7183$.

The value of g at London = 32·182 feet per second per second.

TABLE OF LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0048	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 16 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 6 8	11 13 16	18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	6 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7

TABLE OF LOGARITHMS.—Continued.

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56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	5 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
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89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
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98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
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TABLE OF ANTILOGARITHMS.

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·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
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·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
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·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
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·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 3 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 3 4	5 6 6

TABLE OF ANTILOGARITHMS.—Continued.

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•51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
•52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
•53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
•54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
•55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
•56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
•57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
•58	3802	3811	3819	3828	3837	3845	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
•59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
•60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
•61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
•62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
•63	4266	4276	4286	4295	4305	4315	4326	4335	4345	4355	1	2	3	4	5	6	7	8	9
•64	4365	4375	4385	4395	4405	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
•65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
•66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
•67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	6	7	8	9
•68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	5	6	7	8	9
•69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
•70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
•71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
•72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
•73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
•74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
•75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	6	7	8	9	10
•76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
•77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
•78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
•79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
•80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
•81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
•82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
•83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
•84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
•85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	16
•86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	16
•87	7418	7436	7454	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
•88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
•89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
•90	7943	7962	7980	7998	8017	8036	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
•91	8128	8147	8166	8186	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
•92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
•93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
•94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
•95	8913	8933	8954	8974	8996	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
•96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
•97	9338	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
•98	9550	9572	9594	9616	9638	9661	9683	9706	9727	9750	2	4	7	9	11	13	16	18	20
•99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	4	7	9	11	14	16	18	20

TABLE OF FUNCTIONS OF ANGLES.

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees.	Radians.								
0°	0	000	0	0	∞	1	1.414	1.5708	90°
1	.0175	.017	.0175	.0176	57.2900	.9998	1.402	1.5533	89
2	.0349	.035	.0349	.0349	28.8383	.9994	1.389	1.5359	88
3	.0524	.052	.0523	.0524	19.0811	.9988	1.377	1.5184	87
4	.0698	.070	.0698	.0699	14.3007	.9976	1.364	1.5010	86
5	.0873	.087	.0872	.0875	11.4301	.9962	1.351	1.4835	85
6	.1047	.105	.1046	.1051	9.5144	.9945	1.338	1.4661	84
7	.1222	.122	.1219	.1228	8.1443	.9926	1.325	1.4488	83
8	.1396	.140	.1392	.1405	7.1154	.9903	1.312	1.4312	82
9	.1571	.157	.1564	.1584	6.3138	.9877	1.299	1.4137	81
10	.1745	.174	.1738	.1768	5.6713	.9843	1.286	1.3963	80
11	.1929	.192	.1908	.1944	5.1446	.9816	1.272	1.3788	79
12	.2094	.209	.2079	.2128	4.7046	.9781	1.259	1.3614	78
13	.2269	.228	.2250	.2309	4.3315	.9744	1.245	1.3439	77
14	.2443	.244	.2419	.2493	4.0108	.9703	1.231	1.3265	76
15	.2618	.261	.2588	.2679	3.7321	.9659	1.218	1.3090	75
16	.2793	.278	.2758	.2867	3.4874	.9613	1.204	1.2915	74
17	.2967	.296	.2924	.3057	3.2709	.9563	1.190	1.2741	73
18	.3142	.313	.3090	.3249	3.0777	.9511	1.176	1.2566	72
19	.3316	.330	.3256	.3443	2.9042	.9455	1.161	1.2392	71
20	.3491	.347	.3420	.3640	2.7475	.9397	1.147	1.2217	70
21	.3665	.364	.3584	.3839	2.6051	.9338	1.133	1.2043	69
22	.3840	.382	.3748	.4040	2.4761	.9272	1.118	1.1868	68
23	.4014	.399	.3907	.4245	2.3559	.9205	1.104	1.1694	67
24	.4189	.418	.4087	.4452	2.2480	.9135	1.089	1.1519	66
25	.4363	.433	.4226	.4663	2.1445	.9063	1.075	1.1345	65
26	.4538	.450	.4384	.4877	2.0503	.8988	1.060	1.1170	64
27	.4712	.467	.4540	.5095	1.9626	.8910	1.045	1.0996	63
28	.4887	.484	.4695	.5317	1.8807	.8829	1.030	1.0821	62
29	.5061	.501	.4848	.5543	1.8040	.8748	1.015	1.0647	61
30	.5236	.518	.5000	.5774	1.7321	.8660	1.000	1.0472	60
31	.5411	.534	.5150	.6009	1.6643	.8572	.985	1.0297	59
32	.5585	.551	.5299	.6249	1.6003	.8480	.970	1.0123	58
33	.5760	.568	.5448	.6494	1.5399	.8387	.954	.9948	57
34	.5934	.585	.5592	.6745	1.4828	.8290	.939	.9774	56
35	.6109	.601	.5738	.7002	1.4281	.8192	.923	.9599	55
36	.6283	.618	.5878	.7265	1.3764	.8090	.908	.9425	54
37	.6458	.635	.6018	.7536	1.3270	.7988	.892	.9250	53
38	.6632	.651	.6157	.7813	1.2799	.7880	.877	.9076	52
39	.6807	.668	.6293	.8098	1.2349	.7771	.861	.8901	51
40	.6981	.684	.6428	.8391	1.1918	.7660	.845	.8727	50
41	.7156	.700	.6561	.8693	1.1504	.7547	.829	.8552	49
42	.7330	.717	.6691	.9004	1.1106	.7431	.813	.8378	48
43	.7505	.733	.6820	.9325	1.0724	.7314	.797	.8203	47
44	.7679	.749	.6947	.9657	1.0355	.7193	.781	.8029	46
45°	.7854	.765	.7071	1.0000	1.0000	.7071	.765	.7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

THE INSTITUTION OF CIVIL ENGINEERS' EXAMINATION, OCTOBER, 1909.

ELECTION OF ASSOCIATE MEMBERS.

STRENGTH AND ELASTICITY OF MATERIALS.

Not more than EIGHT questions to be attempted by any Candidate.

1. A timber beam of 12 feet span is supported at both ends; it is 8 inches deep and 14 inches wide, and carries a brick wall 14 inches thick. Find what height of wall it can carry if the maximum stress in the beam is to be 800 lbs. per square inch, and the weight of brickwork is 125 lbs. per cubic foot.

2. Draw to scale the stress-strain diagram of a tensile test of a steel bar, 1 inch square, and having an ultimate strength of 30 tons per square inch. Explain the meaning of the different parts of the diagram.

3. A load of 3 tons is hung from a round steel rod $\frac{3}{4}$ inch in diameter and 40 feet long. How much will the rod stretch if its modulus of elasticity be 13,000 tons per square inch?

4. A rolled steel joist, 8 inches deep, whose moment of inertia is 55·7 (in inch units), is laid on bearings 10 feet apart, and carries a load of 7 tons uniformly distributed. What is its maximum fibre stress, and what is its central deflection if the modulus of elasticity be 14,000 tons per square inch?

5. In a certain girder a steel flat bar tension member, 10 inches wide and $\frac{1}{2}$ inch thick, carries a load of 30 tons; it is attached to a gusset by a butt joint with two cover plates, rivets of $\frac{3}{4}$ inch diameter being used. Give a dimensioned sketch of the riveting you would propose, and explain fully your reason for the number of rivets, pitch, lap, and thickness of cover plate which you adopt. Assume the following as limiting stresses, 5·5 tons per square inch in shearing for rivets, 11·0 tons per square inch in bearing for plates, 6·5 tons per square inch in tension for plates.

6. Derive the formula for the torsional resisting moment of a solid cylindrical shaft. What will be the maximum fibre stress of a 3-inch shaft transmitting 100 H.P. at 300 revolutions per minute?

7. Rankine's formula for the crippling load P of a round-ended column of cross-sectional area A is

$$\frac{P}{A} = \frac{f}{1 + a \frac{l^2}{k^2}},$$

where l denotes the length of the column and k the radius of gyration of

the section, also $\alpha = \frac{1}{7,500}$ and $f = 48,000$ lbs. per square inch for mild steel.

Use this formula to find the crippling load of a round-ended column made of a 10×8 inch rolled steel joist whose least moment of inertia is 71.6 in inch units, and whose area is 20.6 square inches.

8. A cylindrical steel tank, 30 feet in diameter, is to be filled with water to a depth of 12 feet. What is the thickness of plate and what riveting in the vertical joints would you adopt for the bottom ring of plate? (Weight of water = $62\frac{1}{2}$ lbs. per cubic foot.)

9. What is the least internal radius to which a flat bar $\frac{1}{2}$ inch thick can be bent so that the maximum stress does not exceed 7 tons per square inch, taking the modulus of elasticity at 14,000 tons per square inch?

10. Derive from first principles the moment of resistance of a rectangular beam. (Breadth = b , depth = d .)

11. Describe the effects produced by the several processes of hardening, tempering and annealing upon the physical properties of steel containing various percentages of carbon, from the point of view of the use of the material for engineering purposes.

12. State the physical conditions and phenomena to be determined or observed in the practical examination of Portland cement, and the methods of testing it, giving an outline sketch of some common form of machine for determining its tensile strength.

THE INSTITUTION OF CIVIL ENGINEERS' EXAMINATION, FEBRUARY, 1910.

ELECTION OF ASSOCIATE MEMBERS.

STRENGTH AND ELASTICITY OF MATERIALS.

Not more than EIGHT questions to be attempted by any Candidate.

1. State concisely the meaning of the following terms :—Stress ; strain ; modulus of elasticity ; yield point. How is the yield point of steel affected by repeated straining and by annealing ?

2. Give a general description of a modern 50-ton testing-machine ; and sketch out an extensometer for use in such a machine.

3. A 12-inch $\times$ 12-inch balk of timber spans an opening of 14 feet. What will be the weight per foot run of an evenly distributed load which produces a maximum fibre stress of 700 lbs. per square inch ? and how much will the balk deflect under this load if its modulus of elasticity be 1,500,000 lbs. per square inch ?

4. A round steel suspension-rod 42 feet 4 inches long and $1\frac{3}{8}$ inch diameter is stretched 0.253 inch by a certain load. What is this load if the modulus of elasticity be 28,750,000 lbs. per square inch ?

5. A 10-inch $\times$ 6-inch steel bulb tee, whose moment of inertia is 122.278 (in inch units) and centre of gravity is 3.881 inches from the outside face of its table, spans an opening of 14 feet 6 inches. What load must be concentrated at its mid span in order to produce a maximum fibre stress of $6\frac{1}{2}$ tons per square inch ?

6. A steel cylindrical air-receiver of 67 inches diameter is to carry a pressure of 90 lbs. per square inch ; what thickness of plate would you use for the shell, and what diameter, pitch, and arrangement of rivets in the longitudinal joints, if these are lap joints, double riveted, and if the stresses are limited to 14,500 lbs. per square inch in tension, 12,500 lbs. in shear, and 25,000 per square inch in bearing area ? Sketch out your arrangement of riveting, and state the actual stresses you would have in tension, shear, and bearing.

7. A solid cylindrical steel shaft of 4 inches diameter and running at 200 revolutions per minute transmits 150 H.P. What will be its angle of twist (in degrees) in a length of 46 feet, if the modulus of shear be 11,500,000 lbs. per square inch ?

8. A 10-inch $\times$ 6-inch steel-rolled joist used as a strut is subjected to a compression stress of 50 tons acting along an axis lying in the centre line

of its web, and 3 inches from the outer face of one flange. Find the maximum fibre stress produced. Moment of inertia = 211.614 (in inch units) and area = 12.358 square inches.

9. Describe the effect produced upon the ultimate strength of steel by repeated stresses (*a*) when reversed from compression to tension, (*b*) when varied between two tensions.

10. A steel bolt $1\frac{3}{4}$ inch diameter is passed through a cast-iron pipe of 2 inches internal and 3 inches external diameter, and 20 inches long, and the nut is screwed up until head and nut bear lightly on the pipe. Find how much more the nut must travel along the bolt in order to put a compression of 4 tons per square inch on the pipe, taking the modulus of elasticity at 29,000,000 lbs. for steel and 14,000,000 lbs. for cast iron.

11. If a cast-iron test bar exactly 2 inches deep, 1 inch wide and 36 inches between points of support breaks with a central load of 3,340 lbs., what should be the breaking weight of another test bar of the same quality but 1.92 inches deep, 0.97 inch wide, and 35.4 inches between points of support?

12. A wrought-iron rod 50 feet long and $\frac{3}{4}$ inch diameter has to arrest a weight of 600 lbs. falling through a distance of 6 inches. What will be the maximum stress per square inch imposed thereby on the tie-rod? Modulus of elasticity = 27,000,000 lbs. per square inch.

1910 MAY EXAMINATION ON SUBJECT VII. APPLIED MECHANICS.*

(a) MATERIALS AND STRUCTURES.

BY THE BOARD OF EDUCATION SECONDARY BRANCH,
SOUTH KENSINGTON, LONDON.

FOR GENERAL INSTRUCTIONS, SEE APPENDIX A.

STAGE 2.†

INSTRUCTIONS.

You must not attempt more than EIGHT questions in all, including Nos. 31 and 32—that is to say, although 31 and 32 are not compulsory, you are not allowed to take more than six questions in addition to Nos. 31 and 32.

31. Describe, with the help of good sketches, only one of the following, (a), (b), (c), or (d):—(a) A mortar-mixing machine. (b) The method of securing the cutting chuck into the cutter blocks of a wood-planing machine. (c) Any form of friction clutch suitable for use with a speed cone or reversing pulleys. (d) Any form of vernier calliper suitable for measuring the dimensions of a test-bar to the nearest thousandth of an inch.

(B. of E., S. 2, Div. a, 1910.)

32. Answer only one of the following, (a), (b), or (c):—(a) Two of the tests specified in order to determine the quality of Portland cement are the determination of the tensile strength of (i.) a mortar of neat cement, (ii.) a mortar with sand. Describe carefully how the specimens would be made and tested. (b) You are supplied with a length of steel wire $\frac{1}{8}$ inch in diameter. You are asked to find (i.) Young's modulus for the material, (ii.) the limit of elasticity, and (iii.) the breaking stress. Explain how you would carry out the test. (c) You wish to know the strength and stiffness of a large timber beam. It is to be built in at the ends; to be about 20 feet long between the supports; to be, say, 10 inches broad and 12 inches deep. You, therefore, test a small beam of the same kind of timber; describe exactly how you would make the test, and how you would use your results.

(B. of E., S. 2, Div. a, 1910.)

33. Prove that the tendency of a long cylindric boiler or pipe to burst laterally is twice as great as its tendency to burst endwise

(B. of E., S. 2, Div. a, 1910.)

35. Let the length of a strut divided by the diameter of its section be called x . Tests were made on a set of cast-iron struts all of the same section

* Students are referred to Vols. I., IV., and V. for Questions under Div. (b) *Machines and Hydraulics*.

† See Vol. III.—*Theory of Structures*—for Questions 34, 36, 40 to 45, 53, 54, 56, 57, and 59.

but of different lengths, with the following results. W is the maximum load carried :—

x	10	15	20	25	30
W	64,000	53,500	44,800	33,700	24,100

Plot a curve showing how the strength depends upon x . What is the maximum load when the length is 18 times the diameter? We see that the strength of a strut does not depend merely on its cross-section; in this a strut differs from a tie-rod; what is the reason for this difference?

(B. of E., S. 2, Div. a, 1910.)

37. State briefly, giving figures, what is known about the fatigue of materials—that is, the effect of repeated loading and unloading or the effect of continually reversing the nature of the loading.

(B. of E., S. 2, Div. a, 1910.)

38. A shaft of 3 inches diameter gets a twist of 10° for a twisting moment 12, and it yields with a twisting moment 18; what angular twist will be produced in a shaft of the same length, of 4 inches diameter, by a twisting moment 10, and what twisting moment will cause yielding?

(B. of E., S. 2, Div. a, 1910.)

39. Answer *one* only of the following questions, (a) or (b):—(a) Describe briefly, sketches are hardly needed, how Portland cement is manufactured. Give a reason for each part of the process. What is your notion of what occurs (i.) when cement sets, (ii.) when it slowly hardens as it gets older? (b) Describe briefly, sketches are hardly needed, how any kind of steel used for girders is manufactured. Give a reason for each part of the process.

(B. of E., S. 2, Div. a, 1910.)

STAGE 3.

You must not attempt more than EIGHT questions in all, including Nos. 51 and 52—that is to say, although 51 and 52 are not compulsory, you are not allowed to take more than six questions in addition to Nos. 51 and 52.

51. Describe, with the help of neatly drawn sketches, which should be roughly to scale, only *one* of the following, (a), (b), (c), or (d):—(a) Any form or vernier calliper, suitable for measuring dimensions to the nearest $\frac{1}{1000}$ th inch. Show carefully how the vernier is divided. (b) A mortar or concrete mixing machine driven by steam power. (c) Any form of crane suitable for use in connection with the erection of lofty buildings. Explain carefully how the jib is luffed and slewed. (d) Any form of multiple drilling machine, such as is used in bridge-building yards. Explain how motion is transmitted to the drills, and how the pitch of the holes can be varied.

(B. of E., S. 3, Div. a, 1910.)

52. Answer only *one* of the following, (a), (b), or (c):—(a) You are given a strip of mild steel plate, and you are requested to carry out the ordinary commercial tensile tests; state exactly what apparatus you would need, what measurements you would make, and what results you would expect to obtain. (b) State carefully how you would prepare the briquettes for

tensile tests of neat cement, and of sand and cement mortar. If the briquettes were broken seven days after having been made, what results would you expect to obtain? Explain also the method of making the "blowing" test. (c) You are given a piece of 1-inch diameter mild steel bar, and you are requested to determine the modulus of rigidity for the steel; explain how you would do this. What numerical value would you expect to obtain? (B. of E., S. 3, Div. *a*, 1910.)

55. Give Euler's theory of the strength of struts. How do experimental results agree with the theory? Why is there any discrepancy? How is it that we have fairly good working formula for the strength of struts in spite of this discrepancy? (B. of E., S. 3, Div. *a*, 1910.)

58. A vertical shaft, 2 inches diameter, of length 5 feet, has a wheel of 1,610 lbs. keyed to its middle. The distance of the centre of gravity of the wheel from the axis of rotation being called x , let x be .001 foot when the wheel is not rotating. What is x when the speed is α radians per second? What is the critical value of α ? When the shaft is run at 10 per cent., 400 per cent., and 900 per cent. greater speeds than the critical, what is x ? Take Young's modulus as 3×10^7 lbs. per square inch. Neglect the stiffness of the fastening of wheel and shaft. (B. of E., S. 3, Div. *a*, 1910.)

60. A round shaft is subjected to a bending moment M , what is the greatest compressive stress? It is subjected to a twisting moment T , what is the greatest shear stress? What are the principal stresses p_1 and p_2 in the material? There is a theory which asserts that for any material the condition of yield is $p_1 + ap_2 = b$, where a and b are constants for a particular material; what is this condition when we express it in terms of M and T ? (B. of E., S. 3, Div. *a*, 1910.)

61. A rectangular timber beam, supported at the ends, is of uniform section from end to end, and it carries a uniformly distributed load. If the working intensity of stress in the wood is not to exceed 2,000 lbs. per square inch, and if the modulus of elasticity of the wood is 1,700,000 lbs. per square inch, determine the ratio of depth of cross-section of beam to span of beam in order that the deflection may not exceed $\frac{1}{800}$ th part of the span. (B. of E., S. 3, Div. *a*, 1910.)

62. A ferro-concrete column is 14 inches square in cross-section; the main reinforcement consists of four longitudinal 2-inch diameter round steel rods, one rod being placed close to each angle of the cross-section. The value of E (Young's modulus) for the steel is 29,000,000 lbs. per square inch, and for the concrete 3,000,000 lbs. per square inch. If a gross compressive load of 60 tons is supported by this column, what is the gross load and the compressive stress per square inch in (a) the concrete, (b) the reinforcing bars? (B. of E., S. 3, Div. *a*, 1910.)

	PAGE		PAGE
Effect of allowing a factor of safety for both fatigue and for impact, ...	151	F	
Effects of small and medium shocks on metals, ...	151	FACTOR of safety for a mixed load, ...	6
Elastic compression of short blocks—Extensometer applied to measure the, ...	179	Factor of safety for both fatigue and for impact, ...	151
„ extension of rods—		Factors of safety, ...	4
Testing machine with		Falling or drop tests, ...	151
extensometer attached		Fatigue of chains, ...	14
for measuring the, ...	176	Fatigue of metals, ...	141-149
„ limits, ...	144	„ —microscopic, ...	144
„ strain diagrams—Auto-		Fidler's formula for columns, ...	196
matically-drawn, ...	167	Fifty-ton single-lever vertical testing machine, ...	123
„ strength of a material		Flanged girder, ...	61
from the stress-strain		Flexure of beams, ...	54
diagram, ...	166	Floating „	38
Elastioity by aid of extenso-		Folding, doubling, or cross-	
meters—Measure-		bending tests, ...	132
ment of Young's		Force—Shearing, ...	29
modulus of, ...	173, 176	French methods of testing	
„ by deflections of		materials of construction, ...	135
beams or canti-		Functions of angles, ...	255
vers—Measure-			
ment of Young's		G	
modulus, ...	180-182	GENERAL Instructions—	
„ —Coefficient of, ...	3	Board of Education Exams., ...	204
„ —Limit of, ...	3	City and Guilds „	205
„ —Modulus of, ...	3	Civil Engineers „	206
Engine—Stresses to which the		Gerber's formula, ...	143
steel parts are subjected, ...	119	Girder—Flanged, ...	61
Equivalent bending and twist-		Girders—Cast-iron, ...	61
ing moments, ...	102	„ —Strength of, ...	28
Euler's formula for long columns		„ —Thin wrought-iron, ...	63
—Proof of, ...	194	Glenfield & Kennedy's trans-	
Ewing's conclusions on exami-		verse bar-testing machine, ...	131
nation of the inter-molecular		Gordon's formula for columns	
crystalline action in metals		—Proof of, ...	192
due to fatigue, ...	145	„ —Table of con-	
Ewing's extensometer, ...	174	stants for, ...	193
Examination questions—Latest,		Greenwood & Batley's testing	
246-249		machine, ...	123
„ tables, ...	250		
Extensometer—Bauschinger's,	178	H	
„ —Ewing's, ...	174	HAMMERING to a point or edge, ...	141
„ —Kennedy's, ...	161	Hardness—Definition of, ...	155
„ —Searle's, ...	173	„ temper, and brittle-	
Extensometers—Measurement		ness tests, ...	154-157
of Young's modulus of elas-		Heating and rest relieve the	
ticity by means of, ...	176	molecular stress, ...	144

	PAGE		PAGE
Method of obtaining and regulating the pressure and flow of water to the ram of a testing machine, . . .	128	P	
Methods of testing materials of construction—French, . .	135	PLANING and punching tests, .	140
Micrographs, . . .	116	Plasticity of a material, . .	167
Microscopic examination of the inter - molecular crystalline action in metals due to fatigue, . . .	144	Power that steel shafting will transmit at various speeds, .	107
„ tests of engineering materials, . .	116	Proof stress on chains, . .	6, 14
Mixed load, . . .	6	Proportion of test-bars, and their attachment in tensile tests, ...	129
Modulus of elasticity, . .	3	Pull—Sudden, ...	7
„ elasticity by aid of extensometers — Measurement of the, 176		Q	
„ elasticity by aid of Searle's apparatus, 173		QUESTIONS—Answers to numerical, ...	239-245
„ elasticity by deflection of beams or cantilevers, 180-182		R	
„ resilience, . . .	6	RANKINE'S rules for columns, .	190
„ rigidity by the torsion of wires, .	183	—Table of constants to be used in, ...	193
„ rigidity by torsional oscillations, 184		Recorder for drawing stress-strain diagram, . .	123-127
„ sections—Table of, 78		Resilience, ...	6
„ the section, . . .	57	Resistance of beams to flexure, 54	
Molecular stress in metals relieved by heating and rest, .	144	„ —Torsional, . . .	94
Moment of inertia, . . .	57	Rest and heating relieve the molecular stress, . . .	144
„ —Table of, 78		Results of mechanical tests of cast and annealed steels, .	117
„ —Turning, . . .	94	Reversal stresses—Determination of the tensile, compressive, and range of, . .	148
„ —Twisting, . . .	94	Reynolds' and Dr. J. H. Smith's method of testing metals, .	145
N		Rigidity by torsional oscillations —Measurement of the modulus of, .	184
NECESSITY for a prior investigation of the kind of stresses to which structures may be subjected, ...	119	„ the torsion of wires —Measurement of the modulus of, .	183
Normal and tangential stresses, 2		Rings—Shrunk, ...	7
Notes on the fatigue of metals —Historical, ...	141	Rules and Syllabus of the Institution of Civil Engineers Examinations, . . .	206
O		Rules for testing materials of machinery—Admiralty, .	133
OPEN—Link chain, . . .	13		
Overhead travelling crane, .	42		

S		PAGE	Strength of suspended chains		PAGE
SAFETY—Factors of, . . .	4		and wires, . . .	16	
„ for both fatigue and for impact — Effect of allowing a factor of, .	151		„ thick cylinders, . .	10	
Scratch method of determining hardness, . . .	156		„ thin cylinders, . .	8	
Seams—Helical, . . .	9		„ —Ultimate, . . .	4	
Searle's apparatus for determining the modulus of elasticity of wires, . . .	173		Stress, . . .	1	
Seaton and Jude's impact testing machine, . . .	150		„ and strain, as shown by the diagram—Influence of time on, . . .	170	
Section—Modulus of the, . .	57		„ —Compressive, . . .	2	
Sections—British standard, .	76		„ —Intensity of, . . .	1	
„ —Table of modulus of, .	78		„ on chains—Proof, working, and ultimate, . .	14	
Shaft—Strain in a, . . .	94		„ —Shearing, . . .	2	
Shafts—Power transmitted at various speeds by steel, . . .	107		„ —Tensile, . . .	1	
„ —Stiffness of, . . .	104		Stresses—Normal and tangential, .	2	
„ subject to both bending and twisting, . . .	100		„ on metals—Determination of the tensile, compressive, and range of reversal, .	148	
„ —Torsional strength of, .	94		„ to which structures may be subjected—Necessity for a careful investigation of the kind, . . .	119	
Shaw's apparatus for automatically drawing stress-strain diagrams, . . .	163		„ to which the steel parts of a reciprocating steam engine are subjected, .	119	
Shear and tension combined, .	101		Stress-strain diagrams—Auto-graphic apparatus for recording, .	160-164	
Shearing force, . . .	29		„ „ giving the elastic, working, and ultimate strengths of materials, .	166	
„ stress, . . .	2		Stud-link chains, . . .	14	
„ „ on planes at right angles, .	101		Sudden pull, . . .	7	
„ test, . . .	139		Suspended chains and wires, .	16	
Shocks on metals—Effects of, .	151		Syllabus and Rules of examinations for election of Associate Members of the Inst.C.E., .	206	
Shrunk rings, . . .	7		Symbols, abbreviations, &c., .	xvi	
Single-lever vertical testing machine, . . .	122-125				
Smith's and Prof. Reynolds' method of testing metals, .	145				
Standard sections (Note), .	76				
Steel casting—Chemical analysis, micrograph and mechanical tests, . . .	117				
Stiffness of shafts, . . .	104				
Strain, . . .	3				
„ in a shaft, . . .	94				
Strength of beams and girders, .	28				
„ materials (table), .	77				
„ shafts under torsion, .	94				
„ shafts under torsion and bending, .	100				

T	
TABLES—	
Constants for Rankine's and Gordon's formulæ on columns, . . .	193

Tables—	PAGE	Testing machine—	PAGE
Factors of safety, ...	5	With extensometer attached	
Logarithms, ...	251	for measuring the elastic	
Moments of inertia and		extension of rods, ...	176
moduli of sections, ...	78	Wöhler's, ...	142
Power that steel shafting		Testing machines—History of,	
will transmit at various		113-185	
speeds, ...	107	Testing materials }	118-157
Showing a comparison of		of machinery, }	
the hardness of metals by		„ Admiralty rules for, ...	133
indentation and scratch		„ French methods, ...	135
methods, ...	157	„ List of papers, &c.,	
Sizes, twisting and bending		on, ...	136
tests for the iron sheathing		Testing—Advantages of, ...	115
wires of submarine cables, ...	154	Tests—Different mechanical,	
Strength of materials, ...	77	118-157	
Strength and stiffness of		„ of engineering materials	
beams, ...	74	—Mechanical, chemical,	
Strength and weight of close-		and microscopic, ...	116
and studded-link crane		Test-pieces and their attach-	
chains, and size of equi-		ment in tensile tests—Pro-	
valent hemp cable, ...	15, 16	portion of, ...	129
Ultimate and working stress		Thick cylinders, ...	10
of materials when in ten-		Thin cylinders, ...	8
sion, compression, and		„ wrought-iron girders, ...	63
shearing, ...	5	Thirty-ton single-lever vertical	
Tangential and normal stresses,	2	testing machine, ...	123
Technological Examinations—		Time on stress and strain, as	
General Instructions, ...	205	shown by the diagram—	
Temper, brittleness, and hard-		Influence of, ...	170
ness—Tests of metals for, ...	154-157	Torsional resistance, ...	94
Tensile stress, ...	1	„ strength of shafts, ...	94
Tension and shear combined, ...	101	„ tests, ...	152, 170
Testing machine—		Torsion, rods and wires,	
Buckton & Co.'s 300-ton, ...	125	153, 183, 185	
Fifty-ton single-lever vertical,		Transverse bar testing machine, ...	131
for torsional tests, ...	152	„ tests, ...	130
Glenfield & Kennedy's trans-		Travelling-crane problem, ...	42
verse bar, ...	131	„ loads—Concentrated, ...	40
Greenwood & Batley's, ...	123	„ „ —Distributed, ...	45
Kirkaldy's, ...	114	„ „ —Two at a dis-	
Prof. Barr's wire, ...	120	tance apart, ...	42
„ Kennedy's method of		Turner's scratch method of	
working the, ...	127	determining hardness, ...	156
„ „ transverse, ...	130	Turning moment, ...	94
Proportions of test-bars and		Twist—Angle of, ...	104
their attachment in, ...	129	Twisting combined with bend-	
Seaton & Jude's impact, ...	150	ing, ...	100
Werder's, ...	121	„ moment, ...	94
Wicksteed's, ...	122, 124-127	„ „ —Equivalent, ...	102
„ 50-ton single-		Two loads at a fixed distance	
lever vertical, ...	124	apart, ...	42

U		PAGE	Wöhler's experiments—General		PAGE
ULTIMATE strength, ...	4		deductions from, . . .		143
„ strength of a material			„ testing machines, . .		142
from the stress-strain diagram, . .	166		Work done in stretching a bar, . .		6
„ stress on chains, . . .	14		„ done per cubic inch in		
Uniform beam on three supports, .	68		fracturing the test-		
„ „ fixed at one end			piece by a tension test, .		168
and supported			„ done per cubic inch in		
at other, ...	68		fracturing the test-		
Unwin's automatic test-record-			piece by a torsion test, .		172
ing apparatus, ...	161		Working load, ...		4
„ test and Turner's scratch			„ strength of a material		
method of determin-			from the stress-strain		
ing hardness, ...	158		diagram, ...		166
„ test for hardness, . . .	157		„ stress on chains, . . .		14
Useful constants, ...	250		Workshop test for hardness, . .		156
			Wrought-iron girders, . . .		63

W			Y		
WEIGHT of chains, ...	15		YOUNG's modulus of elasticity, .		4
Werder testing machine, . . .	121		„ by aid of Bausching-		
Wicksteed's automatic test-			er's extensometer, . . .		178
recording appar-			„ by aid of Searle's		
atus, ...	162		apparatus, ...		173
„ testing machines, . . .			„ in wires—Measure-		
122, 124-127			ment of, ...		176
Wires—Measurement of the			„ by deflection of		
elastic extension of, . . .	177		beams or canti-		
„ —Measurement of Young's			vers — Measure-		
modulus in, ...	176		ment of, ...		180-182
„ —Strength of suspended, .	16		„ of specimens by		
Wire-testing machine — Prof.			Prof. Ewing's ex-		
Barr's and Kelvin's, . . .	120, 121		tensometer, ...		174

